RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE

Unit **4723:** Core Mathematics 3

Mark Scheme for June 2012

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep* or dep*M	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
A2	Accuracy mark awarded 2

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			Attempt process for finding critical values	M1	squaring both sides, 2 linear eqns, ineqs, …	If using quadratic, need to go as far as factorising or substituting in formula for M1; if using two linear eqns or ineqs, signs of $2x$ and x must be same in one, different in the other for M1
			Obtain $\frac{4}{3}$	A1		
			Obtain 6	A1		
			Attempt process for inequality involving two critical values	M1	sketch, table, …; implied by plausible soln	
			Obtain $x < \frac{4}{3}$, $x > 6$	A1	A0 for use of $\le$ and/or $\ge$	
				[5]		
2	(i)		<u>EITHER</u>			
			Attempt use of at least one logarithm property correctly applied to $\ln(\frac{ep^2}{q})$	M1	not including $\ln e = 1$; such as $\ldots = \ln ep^2 - \ln q$ for example	
			Obtain 261 legitimately with necessary detail seen	A2	AG; award A1 if nothing wrong but not quite enough detail or if there is one slip on way to 261	
				[3]		
			<u>OR</u>			
			Express $\dfrac{ep^2}{q}$ in form e^n	M1	with correct treatment of powers	
			Obtain e^{261} and hence 261	A2	AG; award A1 if nothing wrong but not quite enough detail to be fully convincing	
2	(ii)		Introduce logarithms and bring power down	M1	relating $n\ln 5$ to a constant; if using base 5 or base 10, no powers must remain on right-hand side	
			Obtain $n\ln 5 > 580$	A1	or equiv (such as $n > 580\log_5 e$ or $n\log 5 > 580\log e$); allow eqn at this stage	
			State single integer 361	A1	not $n > 360$ nor $n \ge 361$	
				[3]		

Question			Answer	Marks	Guidance	
3	(i)		Use $\sec\theta = \dfrac{1}{\cos\theta}$ Attempt to express in terms of $\tan\theta$ only Obtain $\tan^2\theta = 36$ and hence $\tan\theta = 6$	B1 M1 A1 **[3]**	AG; necessary detail needed (but no need to justify exclusion of $\tan\theta = -6$)	
3	(ii)	(a)	Substitute 6 in attempt at formula Obtain $\frac{5}{7}$	M1 A1 **[2]**	of form $\dfrac{\tan\theta \pm \tan 45°}{1 \mp \tan\theta\tan 45°}$ with different signs in numerator and denominator or exact equiv	any apparent use of angle 80.5.. means M0 answer only: 0/2
3	(ii)	(b)	Substitute 6 in attempt at formula Obtain $-\frac{12}{35}$	M1 A1 **[2]**	of form $\dfrac{\tan\theta + \tan\theta}{1 \pm \tan\theta\tan\theta}$ or exact equiv; allow $\frac{12}{-35}$	any apparent use of angle 80.5.. means M0 answer only: 0/2
4	(a)		Obtain integral of form $k(6x+1)^{\frac{1}{2}}$ Obtain $6(6x+1)^{\frac{1}{2}}$ Substitute both limits and subtract Obtain 30 – 6 and hence 24	*M1 A1 M1 A1 **[4]**	any constant k or (unsimplified) equiv dep *M AG; necessary detail needed	
4	(b)		Attempt expansion of integrand Integrate e^{kx} to obtain $\frac{1}{k}e^{kx}$ Obtain $\frac{1}{2}e^{2x} + 4e^{x} + 4x$ Obtain $\frac{1}{2}e^{2} + 4e - \frac{1}{2}$	M1 M1 A1 A1 **[4]**	to obtain (at least) 3 terms for any constant k other than 1 allow $+ c$ at this stage or equiv in terms of e simplified to three terms; no $+ c$ now	

Question			Answer	Marks	Guidance
5	(i)		Sketch (more or less) correct $y = 14 - x^2$ Sketch (more or less) correct $y = k\ln x$ Indicate one root ('blob' on sketch or written reference to one intersection or …)	B1 B1 B1 **[3]**	assessed separately from other graph; must exist in all four quadrants; ignore any intercepts given assessed separately from other graph; must exist in first and fourth quadrants; if clearly meets y-axis award B0; if clear maximum point in first quadrant award B0 dependent on both curves being correct in first quadrant and there being no possibility, from their graphs, of further points of intersection elsewhere
5	(ii)	(a)	Calculate values for at least 2 integers Obtain correct values for $x = 3$ and $x = 4$ State 3 and 4	M1 A1 A1 **[3]**	 $14 - x^2 - 3\ln x$: 1.7 – 6.2 $14 - x^2$, $3\ln x$: 5, 3.3 –2, 4.2 following correct calculations
5	(ii)	(b)	Obtain correct first iterate Attempt iteration process Obtain at least 3 correct iterates in all Obtain 3.24	B1 M1 A1 A1 **[4]**	having started with any positive value; B1 available if 'iteration' never goes beyond a first iterate; implied by plausible sequence of values showing at least 2 d.p. answer required to exactly 2 d.p; not given for 3.24 as the final iterate in a sequence, i.e. needs an indication (perhaps just underlining) that value of α found [$3 \rightarrow 3.27172 \rightarrow 3.23173 \rightarrow 3.23743 \rightarrow 3.23661$ $3.5 \rightarrow 3.20027 \rightarrow 3.24196 \rightarrow 3.23596 \rightarrow 3.23682$ $4 \rightarrow 3.13706 \rightarrow 3.25118 \rightarrow 3.23465 \rightarrow 3.23701$]

Question			Answer	Marks	Guidance	
6	(i)		Attempt use of chain rule	*M1	to obtain derivative of form $kh(3h^2+4)^n$, any non-zero constants k and n condone retention of -8	
			Obtain $9h(3h^2+4)^{\frac{1}{2}}$	A1	or (unsimplified) equiv; no – 8 here	
			Substitute 0.6 in attempt at first derivative	M1	dep *M; condone retention of – 8 here; implied by their value following wrong derivative if no working seen	
			Obtain 12.17	A1 **[4]**	or greater accuracy	
6	(ii)		State or imply that $\frac{dh}{dt}=-0.015$ or 0.015	B1	implied by use in calculation with part (i) answer	
			Carry out multiplication of $(\pm)0.015$ and answer from part (i)	M1		
			Obtain 0.18 or -0.18 (whatever this value is claimed to be)	A1 **[3]**	or greater accuracy; condone absence or misuse of negative signs throughout; ignore units; allow for answer rounding to 0.18 following slight inaccuracy due to use of 12.18 or 12.2 or …	
7			Show composition of functions	M1	the right way round; or equiv	
			Obtain $2\sqrt[3]{12-a}+5=9$	A1	or equiv	
			Obtain $a=4$	A1		
			<u>EITHER</u>			
			Attempt to find $g(x)$	*M1	obtaining px^3+q or py^3+q form	
			Obtain $(2x+5)^3+4=68$	A1ft	following their value of a	
			Attempt solution of equation	M1	dep *M; earned at stage $2x+5=...$; if expanding to produce cubic equation, earned with attempt at linear and quadratic factors	
			Obtain $-\frac{1}{2}$	A1 **[7]**	and no others; dependent on correct work throughout	
			<u>OR</u>			
			State or imply $f(x)=g^{-1}(68)$	B2		
			Attempt solution of equation of form $2x+5=\sqrt[3]{68-4}$	M1		
			Obtain $-\frac{1}{2}$	A1		

Question			Answer	Marks	Guidance	
8	(i)		State $R = 5$ Attempt to find value of α Obtain 53.1	B1 M1 A1 **[3]**	 implied by correct value or its complement allow $\tan^{-1}\frac{4}{3}$	
8	(ii)	(a)	Attempt to find at least one value of $\theta + \alpha$ Obtain 1 correct value of θ (–64.7 or 138) Attempt correct process to find the second value Obtain second value of θ (138 or –64.7)	M1 A1 M1 A1 **[4]**	(should be –168.5 or –11.5 or 191.5 or …) allow ± 0.1 in answer and greater accuracy involving a positive value of $\sin^{-1}(-\frac{1}{5})$ and subtraction of their α allow ± 0.1 in answer and greater accuracy; and no others between –180 and 180	 note that 138 needs to be obtained legitimately from positive value of $\sin^{-1}(-\frac{1}{5})$ and not from $180 - 41.6$ answers only: 0/4
8	(ii)	(b)	Use –1 as minimum or 1 as maximum value of $\sin(\theta + \alpha)$ Relate $-5k + c$ to -37 and $5k + c$ to 43 Attempt solution of pair of linear eqns Obtain $k = 8$ and $c = 3$	*M1 A1 M1 A1 **[4]**	 as equations or inequalities dep *M; must be equations now **SC**: both $k = 8$ and $c = 3$ obtained with no working or from unconvincing working, award B2 (i.e. max 2/4)	 Note that alternative solutions may occur. If mathematically sound, all 4 marks are available; if work is not fully convincing, apply SC

Question			Answer	Marks	Guidance	
9	(i)		Attempt use of product rule to produce the form $\ln 2y + y \times \frac{a}{by}$	M1		Note that product rule may be applied to expression in form $y(\ln 2y - 1)$
			Obtain correct $\ln 2y + y \times \frac{2}{2y}$ ...	A1	or equiv	
			Obtain complete $\ln 2y + 1 - 1$ and confirm	A1	AG; necessary detail needed	
				[3]		
9	(ii)		Attempt to rearrange eqn to $x = \ldots$or $x^2 = \ldots$	M1	obtaining form $p \ln qy$	
			Obtain $x = \sqrt{\ln 2y}$ or $x^2 = \ln 2y$	A1		
			State or imply volume is $\int \pi \ln 2y \, dy$	A1ft	following their $x = \ldots$ or $x^2 = \ldots$; condone absence of dy; condone presence of dx; no need for limits here; π may be implied by its first appearance later in solution	
			Integrate using result of part (i)	M1		
			Attempt to use limits $\frac{1}{2}$ and $\frac{1}{2}e^4$ correctly with expression involving y	M1		
			Obtain $\frac{1}{2}\pi(3e^4 + 1)$	A1	or equiv involving two terms; dependent on correct work throughout part (ii)	
				[6]		
9	(iii)		Subtract answer to part (ii) from $2\pi e^4$ …	M1	… or its decimal equivalent	
			Obtain $\frac{1}{2}\pi(e^4 - 1)$	A1	or exact equiv involving two terms	
				[2]		

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