GCE

Mathematics

Advanced GCE

Unit **4736:** Decision Mathematics 1

Mark Scheme for June 2012

Oxford Cambridge and RSA Examinations

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

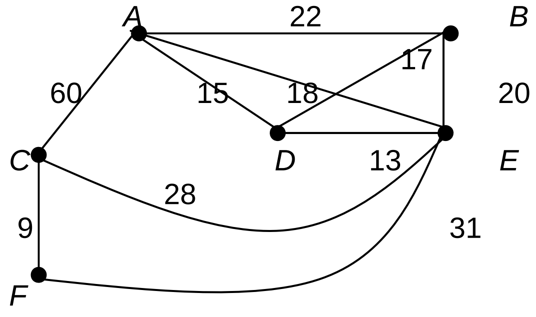
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

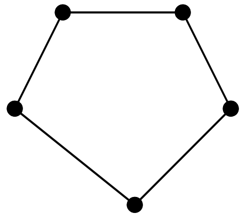
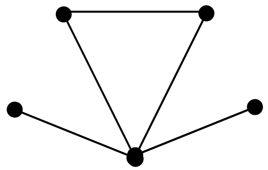
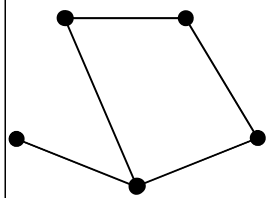
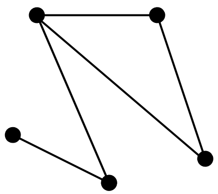
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

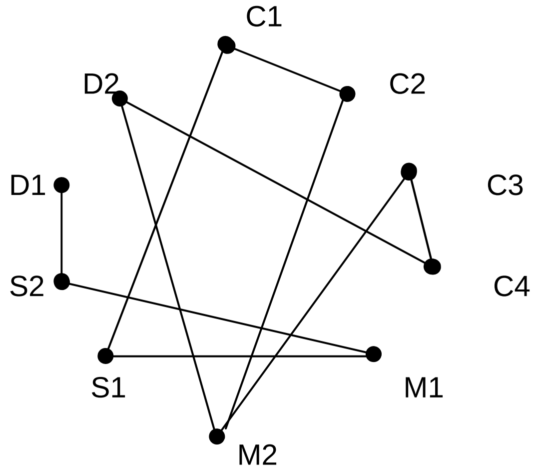
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)			M1 A1 **[2]**	Correct graph (exactly this graph, with 10 arcs) (may draw *AE* so that it does not cross *BD*) All arcs correctly weighted Condone weights that are ambiguously placed	
1	(ii)		A: 1 | 0 B: 4 | 22; 22 C: 5 | 46; 60 46 D: 2 | 15; 15 E: 3 | 18; 18 F: 6 | 49; 49 *A* – *E* – *F*	 M1 A1 B1 B1 B1 **[5]**	Do not allow boxes interchanged condone extra temp labels so we do not need to ft from (i) explicitly <u>Updating</u> from 60 at *C* (60 as temp and a smaller value as temp or perm) Permanent label 46 at *C* (seen as perm, not implied) Permanents labels correct at *B*, *D*, *E* and *F* (seen as perm, not implied) (cao) Order of labelling correct at <u>all six</u> vertices, follow through their labels (permanent label, or smallest temporary label if no permanent label shown at a vertex) cao	
1	(iii)		$3\times\left(\frac{4000}{200}\right)^2 = 3\times400 = 1200$ or $4000^2\times0.000075 = 1200$ or equivalent 1200 seconds = 20 minutes	M1 A1 **[2]**	Correct method, including squaring (may be implied from 1200 or from the final answer being 20, www) May use $\frac{3}{60} = 0.05$ and work in minutes 20 (cao) www	$4000\div200 = 20$ only $\Rightarrow$ M0

Question			Answer	Marks	Guidance	
2	(i)	(a)	eg 	B1 **[1]**	Arcs could cross over, but each vertex must have order 2 and the arcs form a Hamilton cycle	
2	(i)	(b)	eg 	B1 **[1]**	A simply connected graph with one vertex of order 4, two of order 2 and two of order 1	
2	(i)	(c)	eg  or 	B1 **[1]**	A simply connected graph with one vertex of order 3, three of order 2 and one of order 1	

Question			Answer	Marks	Guidance	
2	(ii)	(a)		M1 A1 **[2]**	At least six arcs correct, even with extras cao (arcs may be drawn so that they do not cross)	
2	(ii)	(b)	C1, C4, M1, M2 and D1 in one session C2, C3, S1, S2 and D2 in the other	M1 A1 **[2]**	Two sessions, one with C1 and the other with C2, neither has both C1 and C2 cao	
2	(ii)	(c)	M1 and D1 would need to be in different sessions and S2 cannot be in the same session as either of these	M1 A1 **[2]**	A reasonable attempt, involving at least two of D1, M1 and S2 Condone 'M1 and D1 are in the same session' for M1 A wholly convincing attempt, <u>explicitly</u> involving all three of D1, M1 and S2 Condone 'D1, M1 and S2 would form a cycle'	'M1 and D1 in same session so cannot be done by student at same time' is not enough for the A mark

Question			Answer	Marks	Guidance	
3	(i)		$3x + 2y \geq 6$ or $y \geq 3 - 1.5x$ o.e. $x + y \leq 4$ or $y \leq 4 - x$ o.e. $x \leq 3y$ <u>and</u> $y \leq 2x$ or $y \geq \frac{1}{3}x$ <u>and</u> $y \leq 2x$ o.e.	M1 M1 M1 A1 **[4]**	Check these inputs, tolerance $\pm$ 0.1 on y values $x = 2 \Rightarrow y = 0$, $x = 0 \Rightarrow y = 3$ $x = 3 \Rightarrow y = 1$, $x = 1 \Rightarrow y = 3$ $x = 0 \Rightarrow y = 0$, $x = 3 \Rightarrow y = 1$ <u>and</u> $x = 0 \Rightarrow y = 0$, $x = 1 \Rightarrow y = 2$ All <u>inequalities</u> correct, and no extras	For M marks only need boundary lines correct, (= $<$ $\leq$ $>$ or $\geq$) Only check inequalities if all three M marks achieved
3	(ii)		$y = \frac{1}{3}x$ meets $y = 4 - x$ at (3, 1) $y = 2x$ meets $y = 4 - x$ at $(\frac{4}{3}, \frac{8}{3})$ $y = \frac{1}{3}x$ meets $y = 3 - 1.5x$ at $(\frac{18}{11}, \frac{6}{11})$ $y = 2x$ meets $y = 3 - 1.5x$ at $(\frac{6}{7}, \frac{12}{7})$	B1 B1 M1 A1 **[4]**	(3, 1) cao $(\frac{4}{3}, \frac{8}{3})$ o.e. (cao), in fractions sca <u>using simultaneous equations</u> to find points near both (1.6, 0.5) <u>and</u> (0.9, 1.7) as decimals or fractions ($\pm$ 0.1 on each coordinate), but not just reading from graph Both $(1\frac{7}{11}, \frac{6}{11})$ o.e. <u>and</u> $(\frac{6}{7}, \frac{12}{7})$ o.e. (cao) as <u>fractions</u>	In (ii), condone 'x = …, y = …' instead of coordinates M1 may be implied from either answer <u>correct</u> as <u>fractions</u>, even if second is wrong or not attempted
3	(iii)		$(3, 1) \Rightarrow P = 7$ $(\frac{4}{3}, \frac{8}{3}) \Rightarrow P = 12$ $(\frac{18}{11}, \frac{6}{11}) \Rightarrow P = 3\frac{9}{11}$ (3.82) $(\frac{42}{11})$ $(\frac{6}{7}, \frac{12}{7}) \Rightarrow P = 7\frac{5}{7}$ (7.71) $(\frac{54}{7})$ Optimum point is $(\frac{4}{3}, \frac{8}{3})$ with $P = 12$	M1 A1 B1 **[3]**	Calculating $P = x + 4y$ correctly at two of their vertices (vertices may be implied) All four correct P values shown, as fractions <u>or</u> to $\geq$ 2 dp (cao) $(\frac{4}{3}, \frac{8}{3})$ and 12 (written down) Condone (1.3 to 1.4, 2.6 to 2.7) 11.75 to 12.25	Optimal point in (x, y) form <u>and</u> 12 (within tolerance) written
3	(iv)		(2, 2) with $P = 10$ The only other integer-valued feasible points are (1, 2), (2, 1) and (3, 1) (giving P = 9, 6 and 7, respectively, but the P values need not be seen)	M1 A1 **[2]**	$x = 2$, $y = 2$ and 10 (written down, whether identified as optimum point and value or not) (2, 2) and 10 identified as optimum and a valid explanation that recognises the four integer-valued <u>points</u> that fall in the feasible region	(2, 2) in (x, y) form, could 'identify' using an arrow or star

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>4</td><td>(i)</td><td></td><td>$3x - 4y + z + s = 12$
$6x + 2z + t = 20$
$-10x - 5y + 5z + u = 30$

$s \geq 0$, $t \geq 0$, $u \geq 0$</td><td>B1

B1

[2]</td><td>Correct use of slack variables (s, t and u may be in any order)
May scale through (e.g. $-2x - y + z + u = 6$)

s, t and u non-negative (s, t, u 'positive' $\Rightarrow$ B0)
Condone $0 \leq s \leq 12$ etc.</td><td>Must be $+s$ etc. constraints correct, must have =

… non-negative integers $\Rightarrow$ B0</td></tr>
<tr><td>4</td><td>(ii)</td><td></td><td><table>
<tr><td>P</td><td>x</td><td>y</td><td>z</td><td>s</td><td>t</td><td>u</td><td>RHS</td></tr>
<tr><td>1</td><td>5</td><td>6</td><td>–4</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>3</td><td>–4</td><td>1</td><td>1</td><td>0</td><td>0</td><td>12</td></tr>
<tr><td>0</td><td>6</td><td>0</td><td>2</td><td>0</td><td>1</td><td>0</td><td>20</td></tr>
<tr><td>0</td><td>–10</td><td>–5</td><td>5</td><td>0</td><td>0</td><td>1</td><td>30</td></tr>
</table></td><td>
M1

A1

[2]</td><td>Order of rows and columns may be changed

4×8 table of values (ignore ratio column if shown) <u>and</u> (their) constraint rows correct

Objective row correct
Assume blanks in initial tableau mean zero</td><td></td></tr>
<tr><td>4</td><td>(iii)</td><td></td><td>(z is the) <u>only</u> column with a negative in objective row

$12 \div 1 = 12$
$20 \div 2 = 10$
$30 \div 5 = 6$

6 is the least positive ratio so pivot on 5 (third constraint, as given in the question)</td><td>B1

B1

[2]</td><td>x and y have a positive value in the obj row $\Rightarrow$ B1
Need a negative in obj row <u>therefore</u> z column $\Rightarrow$ B1
But 'z most negative in objective row' $\Rightarrow$ B0
'z is negative in objective row' $\Rightarrow$ B0
-5, -6, 4 and (z or 4 is) positive $\Rightarrow$ B0

For the constraint rows, calculate the ratios of the entry in the column for RHS $\div$ entry in (their) pivot column and choose the row with the smallest positive ratio
(allow sight of 12, 10, 6 and 'smallest' as minimum answer)
12, 10, 6 and an arrow pointing at 6 gets B0</td><td>Need to refer to objective (top) row

Do not credit if this appears in part (iv)</td></tr>
</table>

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>4</td><td>(iv)</td><td></td><td>New pivot row = row 4 ÷ 5
New row 1 = row 1 + 4 × new pivot row
New row 2 = row 2 – new pivot row
New row 3 = row 3 – 2 × new pivot row

<table>
<tr><th>P</th><th>x</th><th>y</th><th>z</th><th>s</th><th>t</th><th>u</th><th>RHS</th></tr>
<tr><td>1</td><td>–3</td><td>2</td><td>0</td><td>0</td><td>0</td><td>0.8</td><td>24</td></tr>
<tr><td>0</td><td>5</td><td>–3</td><td>0</td><td>1</td><td>0</td><td>–0.2</td><td>6</td></tr>
<tr><td>0</td><td>10</td><td>2</td><td>0</td><td>0</td><td>1</td><td>–0.4</td><td>8</td></tr>
<tr><td>0</td><td>–2</td><td>–1</td><td>1</td><td>0</td><td>0</td><td>0.2</td><td>6</td></tr>
</table></td><td>B1

M1

A1

[3]</td><td>Correct calculations seen for all four rows, <u>including pivot</u> (accept reasonable abbreviations, eg ÷ 5, + 4×pr, but not just + 4)
Pivot row may be shown as individual calculations
Some candidates rewrite initial tableau first
Some candidates use an interim tableau with new pivot row followed by the full augmented tableau

A new tableau with 4 basis columns (or 3 with P missing) in which P has not decreased and all entries in RHS column are ≥ 0
cao (rows and columns may be in any order, condone omission of P column)
Follow through scaled constraints, but not errors, from part (i)</td><td>Any column all 0’s ⇒ M0</td></tr>
<tr><td>4</td><td>(v)</td><td></td><td>Pivot on 10 in x column

<table>
<tr><th>P</th><th>x</th><th>y</th><th>z</th><th>s</th><th>t</th><th>u</th><th>RHS</th></tr>
<tr><td>1</td><td>0</td><td>2.6</td><td>0</td><td>0</td><td>0.3</td><td>0.68</td><td>26.4</td></tr>
<tr><td>0</td><td>0</td><td>-4</td><td>0</td><td>1</td><td>-0.5</td><td>0</td><td>2</td></tr>
<tr><td>0</td><td>1</td><td>0.2</td><td>0</td><td>0</td><td>0.1</td><td>–0.04</td><td>0.8</td></tr>
<tr><td>0</td><td>0</td><td>-0.6</td><td>1</td><td>0</td><td>0.2</td><td>0.12</td><td>7.6</td></tr>
</table>
$x = 0.8$, $y = 0$, $z = 7.6$ <u>and</u> $P = 26.4$</td><td>M1

A1

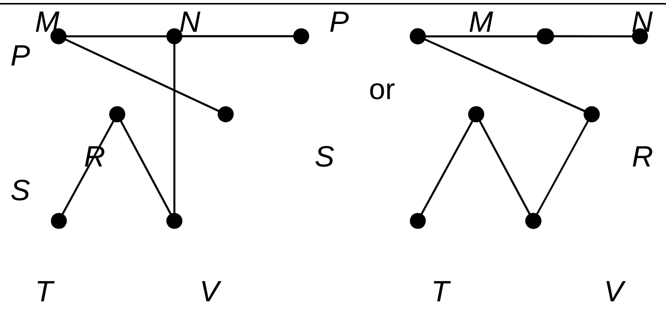
B1
[3]</td><td>A new tableau with 4 basis columns (or 3 with P missing) in which P has not decreased and all entries in RHS column are ≥ 0
cao (rows and columns may be in any order, condone omission of P column)
Follow through scaled constraints, but not errors, from part (i)
Strict ft their final <u>tableau</u> for non-negative x, y, z <u>and</u> positive P (not P as zero)</td><td>Any column all 0’s ⇒ M0</td></tr>
<tr><td>4</td><td>(vi)</td><td></td><td>$s = 2$, $t = 0$, $u = 0$

$3x – 4y + z = 10$ which is 2 short of 12

($6x + 2z = 20$ and $–10x – 5y + 5z = 30$)</td><td>B1

B1

[2]</td><td>Strict ft their final tableau for non-negative s, t, u (all three written down, do not imply blank = 0)
Showing what (their) positive value(s) of slack variable(s) mean for the constraint(s)</td><td>s, t, u all zero (or any of them negative) ⇒ B0 for explanation</td></tr>
</table>

Question			Answer	Marks	Guidance	
5	(i)		Odd vertices: M, N, R, W MN = 70 MR = 190 MW = 90 RW = $\underline{160}$ NW = $\underline{150}$ NR = $\underline{130}$ 230 340 220 Total distance = 1200 + 220 = 1420 metres	M1 A1 B1 **[3]**	At least one correct pairing with its weights or total weight (e.g. MN = 70 and RW = 160, or MN, RW = 230) All three pairings seen with correct total weights (cao) www 1420, condone no units (cao)	
5	(ii)		Just repeat MN = 70 Total distance = 1200 + 70 = 1270 metres R and W are still odd so these must be the start and finish vertices	 M1 A1 **[2]**	 1270, condone no units (cao) R and W (from 1270 seen)	
5	(iii)		$M – S – W – P – N – V – R – T – M$ 60 + 30 + 170 + 40 + 80 + 50 + 40 + 190 = 660 metres Upper bound	M1 A1 B1 B1 **[4]**	$M – S – W – P – N – V –$ ... (not a diagram or table) Tour (cycle) listed (cao) with M at each end 660, condone no units (cao) (not 600, which uses WP = 110, from table) length ≤ (their) 660 (but < ⇒ B0)	
5	(iv)		 Weight = 340	 B1 B1 **[2]**	 Correct tree Allow SV used instead of NV 340 (cao)	Vertices may be shown in different positions to these
5	(v)		Lower bound = 340 + 30 + 90 = 460 metres	M1 A1 **[2]**	Their 340 + 30 + either 90 or 170 May be implied from 460 as final answer 460, condone no units (cao)	

Question			Answer	Marks	Guidance	
6	(i)		A = -4, B = 5, C = 1, X = 0 Y = 1, Z = 5 W = -0.2 X = -0.2 Y = -0.168, Z = 6.72 W = -0.175 Output -0.2	B1 M1 A1 M1 A1 B1 **[6]**	Initial values for A, B, C and X Y = 1, Z = 5 W = -0.2 (exact, not rounded) Updating Y, Z, W (showing new values, even if numerically wrong) W = -0.175 (-7/40) (exact, not rounded) www -0.2 stated (as final X or as output)	Note: if initial value of Z = 0 then first value of W occurs when Z first becomes non-zero and the next value is the first update
6	(ii)		A = -4, B = 5, C = 1, X = 1 Y = 3, Z = 0 X is then reduced to 0 and the algorithm is as (i) leading to an answer of -0.2	M1 A1 **[2]**	Y = 3, Z = 0 Describing what then happens (‘X = 0, then as above’, even if part (i) did not lead to -0.2. If values from (i) are written out must get to a second value of W).	
6	(iii)		A = -4, B = 5, C = 1, X = -1 Y = -9, Z = 16 W = -0.4375, X = -0.4375 Y = -2.04, Z = 9.07 W = -0.213, X = -0.213 Y = -0.256, Z = 6.84 W = -0.176 Root is approximately -0.213	M1 A1 M1 M1 dep A1 **[5]**	Y = -9, Z = 16 W = -0.4375 (-7/16) (allow -0.437 or -0.438, but not values that round to these) Updating Y, Z and W first time (showing new values, even if numerically wrong) Updating Y, Z and W second time (showing new values, even if numerically wrong) Stopping at -0.213 (or anything that rounds to this) without wrong working seen	Note: if initial value of Z = 0 see comment above Dependent on previous M1 Dependent on both method marks
6	(iv)		May not converge May not terminate May get stuck in a loop	B1 **[1]**	Allow ‘could take a long time’, ‘takes a long time’ ‘does not stop’, ‘gets stuck in a loop’ Any one of these, or similar	

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