Thursday 24 May 2012 – Morning

AS GCE MATHEMATICS

4736 Decision Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4736
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

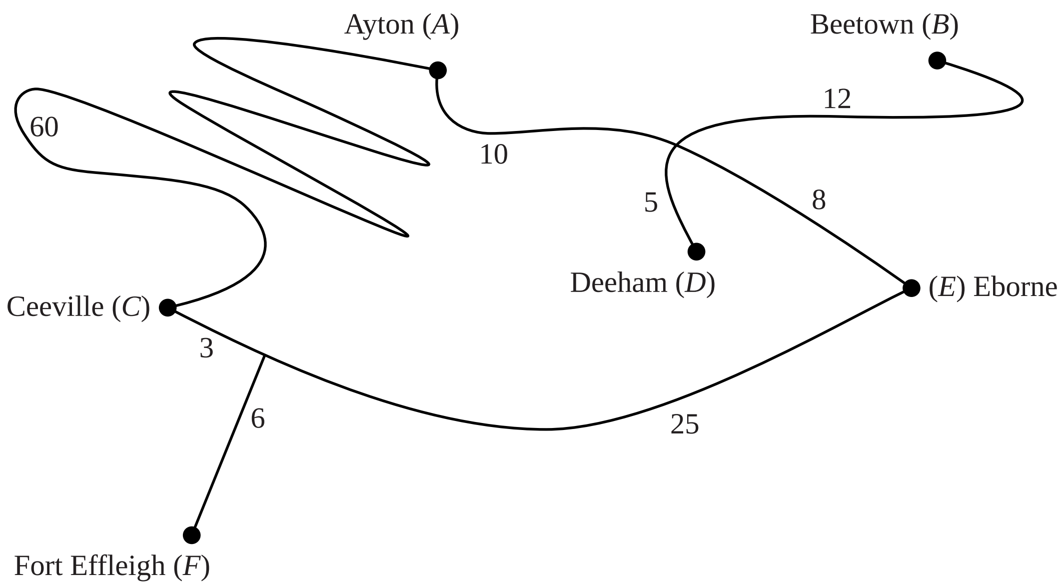
This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 Satellite navigation systems (sat navs) use a version of Dijkstra's algorithm to find the shortest route between two places. A simplified map is shown below. The values marked represent road distances, in km, for that section of road (from a place to a road junction, or between two places).



(i) Use the map to construct a network with exactly 10 arcs to show the direct distances between these places, with no road junctions shown. For example, there will need to be an arc connecting A to B of weight 22, and also arcs connecting A to C, D, and E. There is no arc connecting A to F (because there is no route from A to F that does not pass through another place). **[2]**

(ii) Apply Dijkstra's algorithm, starting at A, to find the shortest route from A to F. **[5]**

Dijkstra's algorithm has quadratic order (order n^2).

(iii) If it takes 3 seconds for a certain sat nav to find the shortest route between two places when it has to process 200 places, calculate approximately how many minutes it will take when it has to process 4000 places. **[2]**

2 A *simple* graph is one in which any two vertices are directly connected by at most one arc and no vertex is directly connected to itself. A *connected* graph is one in which every vertex is joined, directly or indirectly, to every other vertex. A *simply connected* graph is one that is both simple and connected.

(i) (a) Draw a simply connected Eulerian graph with exactly five vertices and five arcs. **[1]**

(b) Draw a simply connected semi-Eulerian graph with exactly five vertices and five arcs, in which one of the vertices has order 4. **[1]**

(c) Draw a simply connected semi-Eulerian graph with exactly five vertices and five arcs, in which none of the vertices have order 4. **[1]**

A teacher is organising revision classes for her students. There will be ten revision classes scheduled into a number of sessions. Each class will run in one session only. Each student has chosen two classes to attend. The table shows which classes each student has chosen.

Revision classes

Student number	C1	C2	C3	C4	M1	M2	S1	S2	D1	D2
1	✓	✓								
2				✓						✓
3			✓	✓						
4			✓			✓				
5								✓	✓	
6		✓				✓				
7					✓			✓		
8	✓						✓			
9						✓				✓
10					✓		✓			

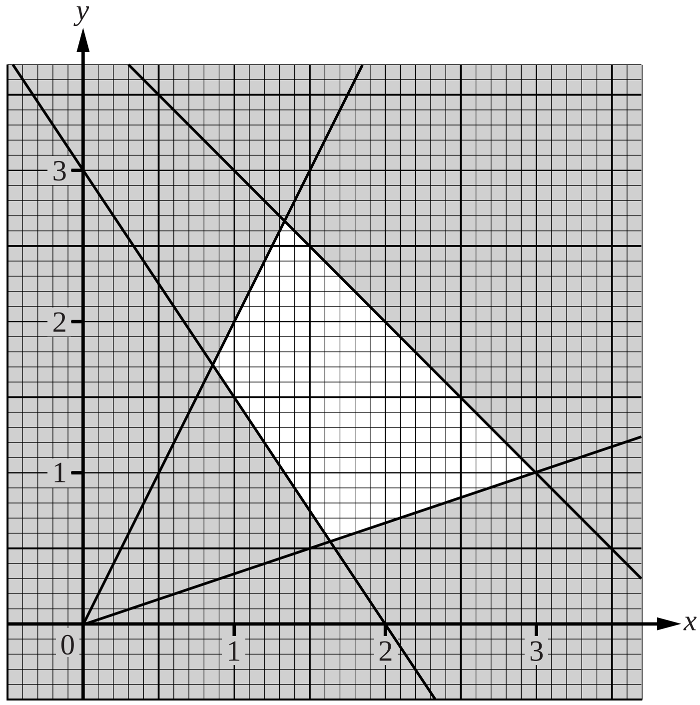
(ii) (a) Draw a graph to show this information. Each vertex represents a class. Each arc links the two classes chosen by a student. **[2]**

(b) Show how the teacher can arrange the classes in just two sessions, which satisfy all student choices. For example, C1 and C2 cannot be in the same session. **[2]**

An extra student joins the group. This student chooses to attend the revision classes in M1 and D1.

(c) Explain why the teacher cannot now arrange the classes in just two sessions. Do **not** amend your graph from part **(ii)(a)**. **[2]**

3 The constraints of a linear programming problem are represented by the graph below. The feasible region is the unshaded region, including its boundaries.



(i) Obtain the four inequalities that define the feasible region. **[4]**

(ii) Calculate the coordinates of the vertices of the feasible region, giving your values as fractions. **[4]**

The objective is to maximise $P = x + 4y$.

(iii) Calculate the value of P at each vertex of the feasible region. Hence write down the coordinates of the optimal point, and the corresponding value of P. **[3]**

Suppose that the solution must have integer values for both x and y.

(iv) Find the coordinates of the optimal point with integer-valued x and y, and the corresponding value of P. Explain how you know that this is the optimal solution. **[2]**

4 Consider the following linear programming problem.

Maximise $P = -5x - 6y + 4z$,

subject to
$$\begin{aligned} 3x - 4y + z &\leqslant 12, \\ 6x + 2z &\leqslant 20, \\ -10x - 5y + 5z &\leqslant 30, \\ x \geqslant 0,\ y \geqslant 0,\ z &\geqslant 0. \end{aligned}$$

(i) Use slack variables s, t and u to rewrite the first three constraints as equations. What restrictions are there on the values of s, t and u? **[2]**

(ii) Represent the problem as an initial Simplex tableau. **[2]**

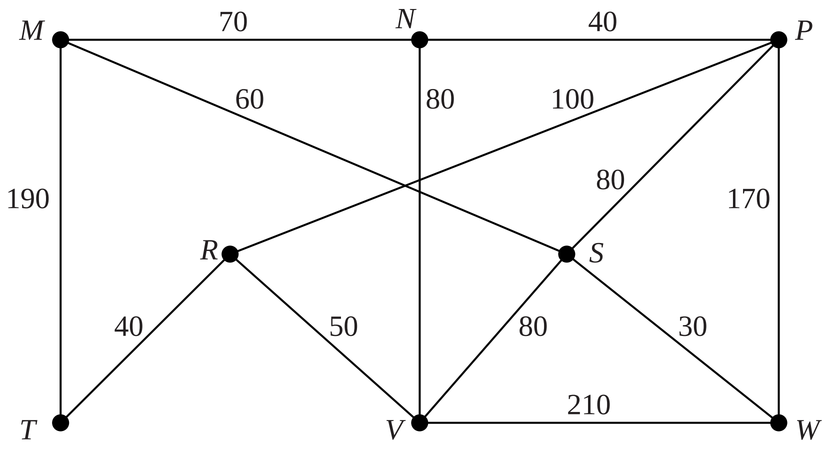
(iii) Show why the pivot for the first iteration of the Simplex algorithm must be the coefficient of z in the third constraint. **[2]**

(iv) Perform one iteration of the Simplex algorithm, showing how the elements of the pivot row were calculated and how this was used to calculate the other rows. **[3]**

(v) Perform a second iteration of the Simplex algorithm and record the values of x, y, z and P at the end of this iteration. **[3]**

(vi) Write down the values of s, t and u from your final tableau and explain what they mean in terms of the original constraints. **[2]**

5 Jess and Henry are out shopping. The network represents the main routes between shops in a shopping arcade. The arcs represent pathways and escalators, the vertices represent some of the shops and the weights on the arcs represent distances in metres.



The total weight of all the arcs is 1200 metres.

The table below shows the shortest distances between vertices; some of these are indirect distances.

	M	N	P	R	S	T	V	W
M	–	70	110	190	60	190	140	90
N	70	–	40	130	120	170	80	150
P	110	40	–	100	80	140	120	110
R	190	130	100	–	130	40	50	160
S	60	120	80	130	–	170	80	30
T	190	170	140	40	170	–	90	200
V	140	80	120	50	80	90	–	110
W	90	150	110	160	30	200	110	–

(i) Use a standard algorithm to find the shortest distance that Jess must travel to cover every arc **in the original network**, starting and ending at M. **[3]**

(ii) Find the shortest distance that Jess must travel if she just wants to cover every arc, but does not mind where she starts and where she finishes. Which two points are her start and finish? **[2]**

Henry suggests that Jess only needs to visit each shop.

(iii) Apply the nearest neighbour method **to the network**, starting at M, to write down a closed tour through all the vertices. Calculate the weight of this tour. What does this value tell you about the length of the shortest closed route that passes through every vertex? **[4]**

Henry thinks that Jess does not need to visit shop W. He uses the table of shortest distances to list all the possible connections between M, N, P, R, S, T and V by increasing order of weight. Henry's list is given in your answer book.

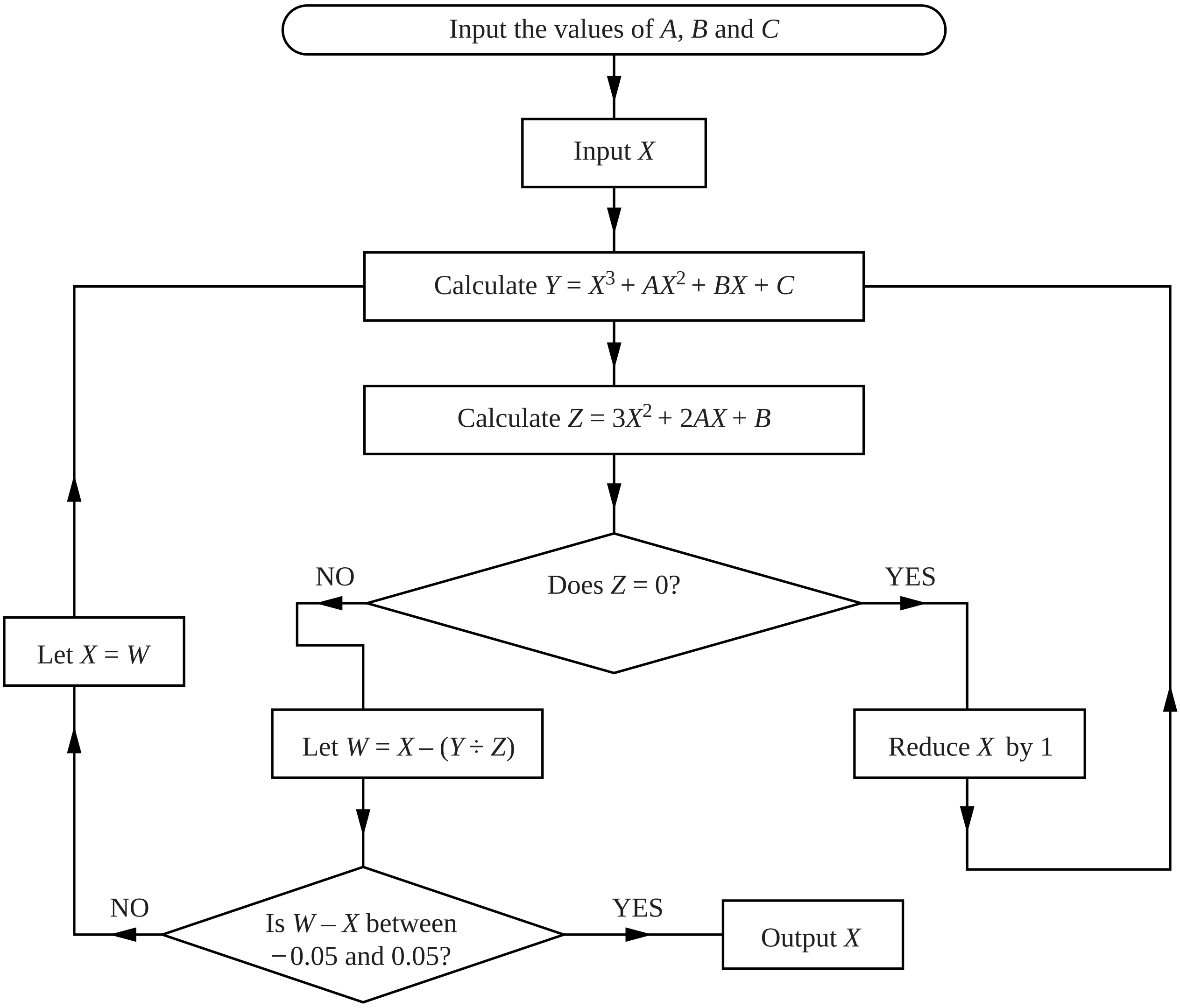
(iv) Use Kruskal's algorithm on Henry's list to find a minimum spanning tree for M, N, P, R, S, T and V. Draw the tree and calculate its total weight. **[2]**

Jess insists that they must include shop W.

(v) Use the weight of the minimum spanning tree for M, N, P, R, S, T and V, and the table of shortest distances, to find a lower bound for the length of the shortest closed route that passes through all eight vertices. **[2]**

[Question 6 is printed overleaf.]

6 The following flow chart has been written to find a root of the cubic equation $x^3 + Ax^2 + Bx + C = 0$, given a starting value X that is thought to be near the root.



(i) Work through the algorithm, recording the values of X, Y, Z and W each time they change, for the equation $x^3 - 4x^2 + 5x + 1 = 0$, with a starting value of $X = 0$. **[6]**

(ii) Show what happens when the algorithm is used for the equation $x^3 - 4x^2 + 5x + 1 = 0$, with a starting value of $X = 1$. **[2]**

(iii) Show what happens when the algorithm is used for the equation $x^3 - 4x^2 + 5x + 1 = 0$, with a starting value of $X = -1$. **[5]**

(iv) Identify a possible problem with using this algorithm. **[1]**