Mark Scheme (Results)

Summer 2013

GCE Core Mathematics 3 (6665/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x =

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x =

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to x =...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme	Marks
1	$$\begin{array}{r} 3x^2-2x+7 \\ x^2(+0x)-4\overline{)3x^4-2x^3-5x^2+(0x)-4} \\ \underline{3x^4+0x^3-12x^2} \\ -2x^3+7x^2+0x \\ \underline{-2x^3+0x^2+8x} \\ 7x^2-8x-4 \\ \underline{7x^2+0x-28} \\ -8x+24 \end{array}$$	
By Division		
	$a=3$	**B1**
	Long division as far as $$\begin{array}{r} 3x^2-2x....... \\ x^2(+0x)-4\overline{)3x^4-2x^3-5x^2+(0x)-4} \\ \underline{3x^4+0x^3-12x^2} \\ -2x^3+............... \\ \underline{-2x^3+................} \end{array}$$	**M1**
	Two of $b=-2 \quad c=7 \quad d=-8 \quad e=24$	**A1**
	All four of $b=-2 \quad c=7 \quad d=-8 \quad e=24$	**A1**
		(4 marks)

Notes for Question 1

B1 Stating $a=3$. This can also be scored by the coefficient of x^2 in $3x^2-2x+7$

M1 Using long division by x^2-4 and getting as far as the 'x' term. The coefficients need not be correct. Award if you see the whole number part as $...x^2+...x$ following some working. You may also see this in a table/ grid.
Long division by $(x+2)$ will not score anything until $(x-2)$ has been divided into the new quotient. It is very unlikely to score full marks and the mark scheme can be applied.

A1 Achieving two of $b=-2$ $c=7$ $d=-8$ $e=24$.
The answers may be embedded within the division sum and can be implied.

A1 Achieving all of $b=-2$ $c=7$ $d=-8$ and $e=24$

Accept a correct long division for 3 out of the 4 marks scoring B1M1A1A0

Need to see a=…, b=…, or the values embedded in the rhs for all 4 marks

Question Number	Scheme	Marks
Alt 1 **By Multiplication**	* $3x^4 - 2x^3 - 5x^2 - 4 \equiv (ax^2 + bx + c)(x^2 - 4) + dx + e$	
	Compares the x^4 terms $a = 3$	B1
	Compares coefficients to obtain a numerical value of one **further** constant $-2 = b, \quad -5 = -4a + c \Rightarrow c = ..,$	M1
	Two of $b = -2 \quad c = 7 \quad d = -8 \quad e = 24$	A1
	All four of $b = -2 \quad c = 7 \quad d = -8 \quad e = 24$	A1
		(4 marks)

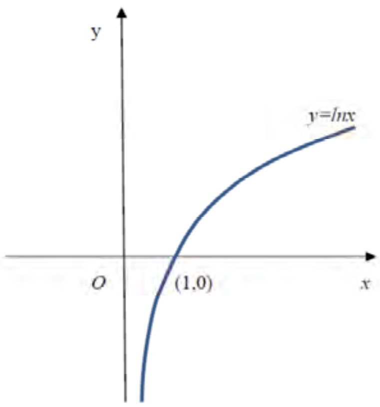
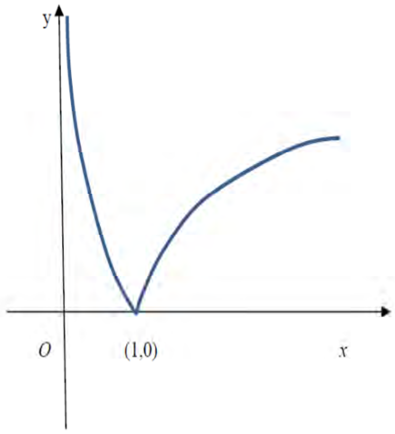
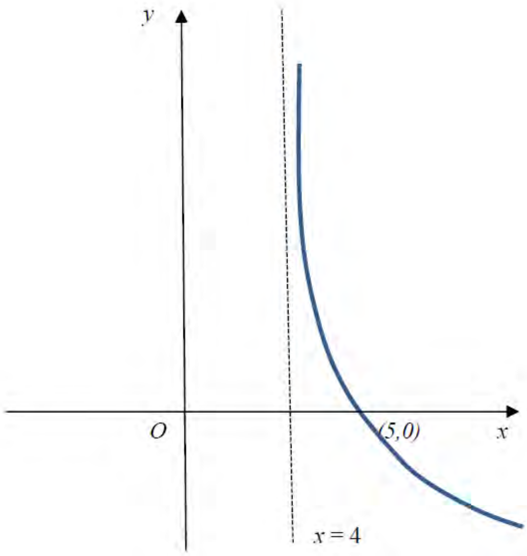
Notes for Question 2

B1 Stating $a = 3$. This can also be scored for writing $3x^4 = ax^4$

M1 Multiply out expression given to get *. Condone slips only on signs of either expression.
Then compare the coefficient of any term (other than x^4) to obtain a numerical value of one further constant. In reality this means a valid attempt at either *b* or *c*
The method may be implied by a correct additional constant to *a*.

A1 Achieving two of $b = -2$ $c = 7$ $d = -8$ $e = 24$

A1 Achieving all of $b = -2$ $c = 7$ $d = -8$ and $e = 24$

Question Number	Scheme		Marks
2(i)		*ln* graph crossing *x* axis at (1,0) and asymptote at *x*=0	B1
2(ii)		Shape including cusp	B1ft
		Touches or crosses the *x* axis at (1,0)	B1ft
		Asymptote given as *x*=0	B1
2(iii)		Shape	B1
		Crosses at (5, 0)	B1ft
		Asymptote given as *x*=4	B1
			(7 marks)

Notes for Question 2

(i) B1 Correct shape, correct position and passing through (1, 0).
Graph must 'start' to the rhs of the y - axis in quadrant 4 with a gradient that is large. The gradient then decreases as it moves through (1, 0) into quadrant 1. There must not be an obvious maximum point but condone 'slips'. Condone the point marked (0,1) on the correct axis. See practice and qualification for clarification. **Do not with hold this mark if (x=0) the asymptote is incorrect or not given**.

(ii) B1ft Correct shape **including the cusp** wholly contained in quadrant 1.
The shape to the rhs of the cusp should have a decreasing gradient and must not have an obvious maximum.. The shape to the lhs of the cusp should not bend backwards past (1,0)
Tolerate a 'linear' looking section here but not one with incorrect curvature (See examples sheet (ii) number 3. For further clarification see practice and qualification items.
Follow through on an incorrect sketch in part (i) as long as it was above and below the x axis.

B1ft The curve touches or crosses the x axis at (1, 0). Allow for the curve passing through a point marked '1' on the x axis. Condone the point marked on the correct axis as (0, 1)
Follow through on an incorrect intersection in part (i).

B1 Award for the asymptote to the curve given/ marked as $x = 0$. Do not allow for it given/ marked as 'the y axis'. There must be a graph for this mark to be awarded, and there must be an asymptote on the graph at $x = 0$. Accept if x=0 is drawn separately to the y axis.

(iii)

B1 Correct shape.
The gradient should always be negative and becoming less steep. It must be approximately infinite at the *lh* end and not have an obvious minimum. The lh end must not bend 'forwards' to make a C shape.
The position is not important for this mark. See practice and qualification for clarification.

B1ft The graph crosses (or touches) the x axis at (5, 0). Allow for the curve passing through a point marked '5' on the x axis. Condone the point marked on the correct axis as (0, 5)
Follow through on an incorrect intersection in part (i). Allow for $\left((i)+4,0\right)$

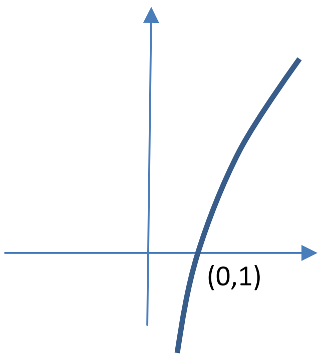
B1 The asymptote is given/ marked as $x = 4$. There must be a graph for this to be awarded and there must be an asymptote on the graph (in the correct place to the rhs of the y axis).

If the graphs are not labelled as (i), (ii) and (iii) mark them in the order that they are given.

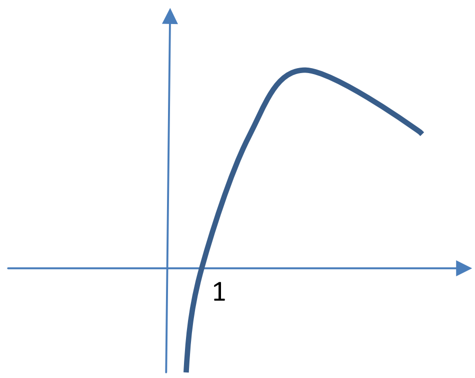
<u>Examples of graphs in number 2</u>

Part (i)

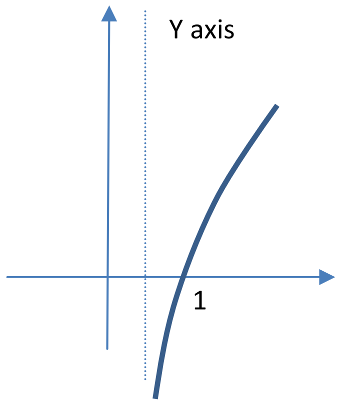
Condoned



Not condoned

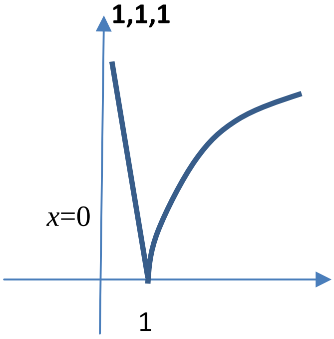


Condone

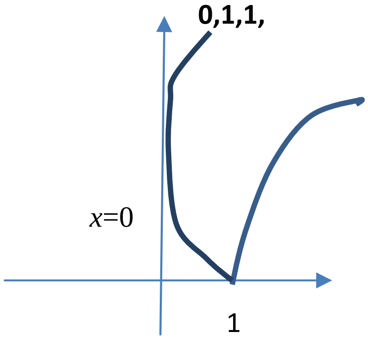


Part (ii)

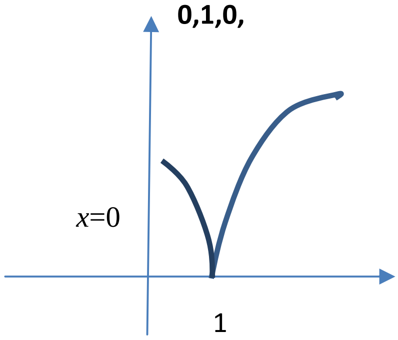
1,1,1



0,1,1,

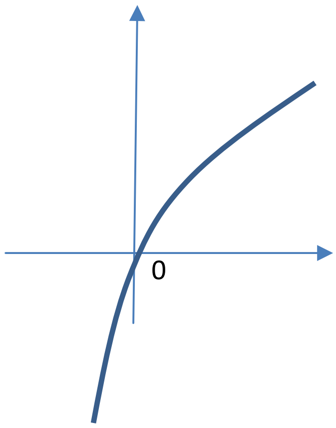


0,1,0,

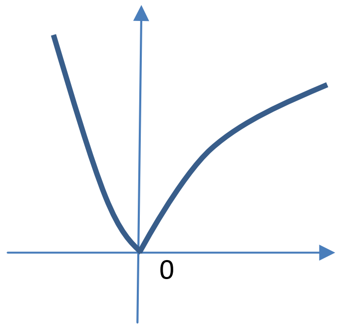


Example of follow through in part (ii) and (iii)

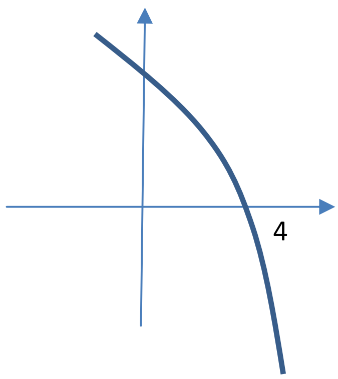
(i) B0



(ii) B1ftB1ftB0



(iii) B0B1ftB0



Question Number	Scheme	Marks
3(a)	$2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$	M1
	$\sin x(\cos 40 + 2\sin 50) = \cos x(2\cos 50 - \sin 40)$	
	$\div\cos x \Rightarrow \tan x(\cos 40 + 2\sin 50) = 2\cos 50 - \sin 40$	M1
	$\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$, (or numerical answer awrt 0.28)	A1
	States or uses cos50 = sin40 and cos40 =sin50 and so $\tan x^{\circ} = \frac{1}{3}\tan 40^{\circ}$ * cao	A1 *
		(4)
(b)	Deduces $\tan 2\theta = \frac{1}{3}\tan 40$	M1
	$2\theta =$ 15.6 so $\theta =$ awrt 7.8(1) One answer	A1
	Also $2\theta =$ 195.6, 375.6 , 555.6 $\Rightarrow \theta = ..$	M1
	$\theta =$ awrt 7.8 , 97.8, 187.8, 277.8 All 4 answers	A1
		(4)
		[8 marks]
Alt 1 3(a)	$2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$	M1
	$2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$	
	$\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$	M1
	$\tan x = \dfrac{\sin 40}{3\cos 40}$(or numerical answer awrt 0.28), $\Rightarrow \tan x = \dfrac{1}{3}\tan 40$	A1,A1
Alt 2 3(a)	$2\cos(x+50) = \sin(x+40) \Rightarrow 2\sin(40-x) = \sin(x+40)$	
	$2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$	M1
	$\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$	M1
	$\tan x = \dfrac{\sin 40}{3\cos 40}$(or numerical answer awrt 0.28), $\Rightarrow \tan x = \dfrac{1}{3}\tan 40$	A1,A1

Notes for Question 3

(a)

M1 Expand both expressions using $\cos(x+50) = \cos x \cos 50 - \sin x \sin 50$ and $\sin(x+40) = \sin x \cos 40 + \cos x \sin 40$. Condone a missing bracket on the lhs.

The terms of the expansions must be correct as these are given identities. You may condone a sign error on one of the expressions.

Allow if written separately and not in a connected equation.

M1 Divide by $\cos x$ to reach an equation in $\tan x$.

Below is an example of M1M1 with incorrect sign on left hand side

$2\cos x \cos 50 + 2\sin x \sin 50 = \sin x \cos 40 + \cos x \sin 40$

$\Rightarrow 2\cos 50 + 2\tan x \sin 50 = \tan x \cos 40 + \sin 40$

This is independent of the first mark.

A1 $\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$

Accept for this mark $\tan x =$ awrt 0.28... as long as M1M1 has been achieved.

A1* States or uses $\cos 50 = \sin 40$ and $\cos 40 = \sin 50$ leading to showing

$\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50} = \dfrac{\sin 40}{3\cos 40} = \dfrac{1}{3}\tan 40$

This is a given answer and all steps above must be shown. The line above is acceptable.

Do not allow from $\tan x =$ awrt 0.28...

(b)

M1 For linking part (a) with (b). Award for writing $\tan 2\theta = \frac{1}{3}\tan 40$

A1 Solves to find one solution of θ which is usually (awrt) 7.8

M1 Uses the correct method to find at least another value of θ. It must be a full method but can be implied by any correct answer.

Accept $\theta = \dfrac{180 + their\alpha}{2}, (or)\dfrac{360 + their\alpha}{2}, (or)\dfrac{540 + their\alpha}{2}$

A1 Obtains all four answers awrt 1dp. $\theta = 7.8, 97.8, 187.8, 277.8$.

Ignore any extra solutions outside the range.

Withhold this mark for extras inside the range.

Condone a different variable. Accept x= 7.8, 97.8, 187.8, 277.8

Answers fully given in radians, loses the first A mark.

Acceptable answers in rads are awrt 0.136, 1.71, 3.28, 4.85

Mixed units can only score the first M 1

Question Number	Scheme	Marks
4(a)	$f'(x)=50x^2e^{2x}+50xe^{2x}$ oe.	M1A1
	Puts $f'(x)=0$ to give x = -1 and x = 0 or one coordinate	dM1A1
	Obtains $(0,-16)$ and $(-1,\ 25e^{-2}-16)$ CSO	A1
		(5)
(b)	Puts $25x^2e^{2x}-16=0 \Rightarrow x^2=\frac{16}{25}e^{-2x} \Rightarrow x=\pm\frac{4}{5}e^{-x}$	B1*
		(1)
(c)	Subs $x_0=0.5$ into $x=\frac{4}{5}e^{-x} \Rightarrow x_1=$ awrt 0.485	M1A1
	$\Rightarrow x_2=$ awrt 0.492, $x_3=$ awrt 0.489	A1
		(3)
(d)	$\alpha=0.49$	B1
	$f(0.485)=-0.487,\ f(0.495)=(+)0.485$, sign change and deduction	B1
		(2)
		(11 marks)

Notes for Question 4

No marks can be scored in part (a) unless you see differentiation as required by the question.

(a)

M1 Uses $vu'+uv'$. If the rule is quoted it must be correct.
It can be implied by their $u=..,v=...,u'=...,v'=...$ followed by their $vu'+uv'$
If the rule is not quoted nor implied only accept answers of the form $Ax^2e^{2x}+Bxe^{2x}$

A1 $f'(x)=50x^2e^{2x}+50xe^{2x}$.
Allow un simplified forms such as $f'(x)=25x^2\times 2e^{2x}+50x\times e^{2x}$

dM1 Sets $f'(x)=0$, factorises out/ or cancels the e^{2x} leading to at least one solution of x
This is dependent upon the first M1 being scored.

A1 Both x = -1 and x = 0 or one complete coordinate . Accept $(0,-16)$ and $(-1,\ 25e^{-2}-16)$ or $(-1,\ awrt -12.6)$

A1 CSO. Obtains both solutions from differentiation. Coordinates can be given in any way.
x= - 1, 0 $y=\frac{25}{e^2}-16,\ -16$ or linked together by coordinate pairs $(0,-16)$ and $(-1,\ 25e^{-2}-16)$ but the 'pairs' must be correct and exact.

Notes for Question 4 Continued

(b)

B1 This is a show that question and all elements must be seen

Candidates must 1) State that f(x)=0 or writes $25x^2e^{2x}-16=0$ or $25x^2e^{2x}=16$

2) Show at least one intermediate (correct) line with either

x^2 or x the subject. Eg $x^2=\frac{16}{25}e^{-2x}$, $x=\sqrt{\frac{16}{25}e^{-2x}}$ oe

or square rooting $25x^2e^{2x}=16 \Rightarrow 5xe^x=\pm 4$

or factorising by DOTS to give $(5xe^x+4)(5xe^x-4)=0$

3) Show the given answer $x=\pm\frac{4}{5}e^{-x}$.

Condone the minus sign just appearing on the final line.

A 'reverse' proof is acceptable as long as there is a statement that f(x)=0

(c)

M1 Substitutes $x_0=0.5$ into $x=\frac{4}{5}e^{-x} \Rightarrow x_1=....$

This can be implied by $x_1=\frac{4}{5}e^{-0.5}$, or awrt 0.49

A1 $x_1=$ awrt 0.485 3dp. Mark as the first value given. Don't be concerned by the subscript.

A1 $x_2=$ awrt 0.492, $x_3=$ awrt 0.489 3dp. Mark as the second and third values given.

(d)

B1 States $\alpha=0.49$

B1 Justifies **by**

either calculating correctly $f(0.485)$ and $f(0.495)$ to awrt 1sf or 1dp,

$f(0.485)=-0.5, f(0.495)=(+)0.5$ rounded

$f(0.485)=-0.4, f(0.495)=(+)0.4$ truncated

giving a reason – accept change of sign, >0 <0 or $f(0.485)\times f(0.495)<0$

and giving a minimal conclusion. Eg. Accept hence root or $\alpha=0.49$

A smaller interval containing the root may be used, eg f (0.49) and f (0.495). Root = 0.49007

or by stating that the iteration is oscillating

or by calculating by continued iteration to at least the value of x_4= awrt 0.491 and stating (or seeing each value round to) 0.49

Question Number	Scheme	Marks
5(a)	$\frac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y = \left(6\sec^2 3y \tan 3y\right)$ $\left(\text{oe } \frac{6\sin 3y}{\cos^3 3y}\right)$	M1A1 **(2)**
(b)	Uses $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$ to obtain $\frac{dy}{dx} = \frac{1}{6\sec^2 3y \tan 3y}$	M1
	$\tan^2 3y = \sec^2 3y - 1 = x - 1$	B1
	Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ to get $\frac{dy}{dx}$ or $\frac{dx}{dy}$ in just x.	M1
	$\Rightarrow \frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$ CSO	A1* **(4)**
(c)	$\frac{d^2y}{dx^2} = \frac{0 - [6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$	M1A1
	$\frac{d^2y}{dx^2} = \frac{6-9x}{36x^2(x-1)^{\frac{3}{2}}} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$	dM1A1
		(4)
		(10 marks)
Alt 1 to 5(a)	$x = (\cos 3y)^{-2} \Rightarrow \frac{dx}{dy} = -2(\cos 3y)^{-3} \times -3\sin 3y$	M1A1
Alt 2 to 5 (a)	$x = \sec 3y \times \sec 3y \Rightarrow \frac{dx}{dy} = \sec 3y \times 3\sec 3y \tan 3y + \sec 3y \times 3\sec 3y \tan 3y$	M1A1
Alt 1 To 5 (c)	$\frac{d^2y}{dx^2} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$	M1A1
	$= \frac{1}{6}x^{-2}(x-1)^{-\frac{3}{2}}[x(-\frac{1}{2}) + (-1)(x-1)]$	dM1
	$= \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ oe	A1
		(4)

Notes for Question 5

(a)

M1 Uses the chain rule to get $A\sec 3y \sec 3y \tan 3y = \left(A\sec^2 3y \tan 3y\right)$.

There is no need to get the lhs of the expression. Alternatively could use the chain rule on $(\cos 3y)^{-2} \Rightarrow A(\cos 3y)^{-3}\sin 3y$

or the quotient rule on $\dfrac{1}{(\cos 3y)^2} \Rightarrow \dfrac{\pm A\cos 3y \sin 3y}{(\cos 3y)^4}$

A1 $\dfrac{dx}{dy} = 2\times 3\sec 3y \sec 3y \tan 3y$ or equivalent. There is no need to simplify the rhs but both sides must be correct.

(b)

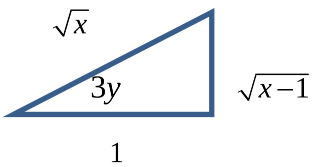
M1 Uses $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$ to get an expression for $\dfrac{dy}{dx}$. Follow through on their $\dfrac{dx}{dy}$

Allow slips on the coefficient but not trig expression.

B1 Writes $\tan^2 3y = \sec^2 3y - 1$ or an equivalent such as $\tan 3y = \sqrt{\sec^2 3y - 1}$ and uses $x = \sec^2 3y$ to obtain either $\tan^2 3y = x - 1$ or $\tan 3y = (x-1)^{\frac{1}{2}}$

All elements **must be present.**

Accept



$\cos 3y = \dfrac{1}{\sqrt{x}} \Rightarrow \tan 3y = \sqrt{x-1}$

If the differential was in terms of $\sin 3y, \cos 3y$ it is awarded for $\sin 3y = \dfrac{\sqrt{x-1}}{\sqrt{x}}$

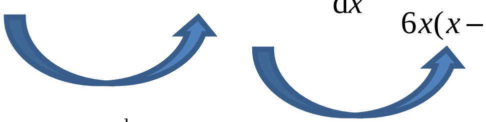
M1 Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ or equivalent to get $\dfrac{dy}{dx}$ in just x. Allow slips on the signs in $\tan^2 3y = \sec^2 3y - 1$.

It may be implied- see below

A1* CSO. This is a given solution and you must be convinced that all steps are shown.

Note that the two method marks may occur the other way around

Eg. $\dfrac{dx}{dy} = 6\sec^2 3y \tan 3y = 6x(x-1)^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{6x(x-1)^{\frac{1}{2}}}$



Scores the 2nd method

Scores the 1st method

The above solution will score M1, B0, M1, A0

Notes for Question 5 Continued

Example 1- Scores 0 marks in part (b)

$$\frac{dx}{dy}=6\sec^2 3y\tan 3y \Rightarrow \frac{dy}{dx}=\frac{1}{6\sec^2 3x\tan 3x}=\frac{1}{6\sec^2 3x\sqrt{\sec^2 3x-1}}=\frac{1}{6x(x-1)^{\frac{1}{2}}}$$

Example 2- Scores M1B1M1A0

$$\frac{dx}{dy}=2\sec^2 3y\tan 3y \Rightarrow \frac{dy}{dx}=\frac{1}{2\sec^2 3y\tan 3y}=\frac{1}{2\sec^2 3y\sqrt{\sec^2 3y-1}}=\frac{1}{2x(x-1)^{\frac{1}{2}}}$$

(c) **Using Quotient and Product Rules**

M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u=1$ and $v=6x(x-1)^{\frac{1}{2}}$ **and** achieving

$u'=0$ and $v'=A(x-1)^{\frac{1}{2}}+Bx(x-1)^{-\frac{1}{2}}$.

If the formulae are quoted, **both** must be correct. If they are not quoted nor implied by their working allow expressions of the form

$$\left(\xcancel{\frac{d^2y}{dx^2}}\right)=\frac{0-[A(x-1)^{\frac{1}{2}}+Bx(x-1)^{-\frac{1}{2}}]}{\left(6x(x-1)^{\frac{1}{2}}\right)^2} \quad \text{or} \quad \left(\xcancel{\frac{d^2y}{dx^2}}\right)=\frac{0-A(x-1)^{\frac{1}{2}}\pm Bx(x-1)^{-\frac{1}{2}}}{Cx^2(x-1)}$$

A1 Correct un simplified expression $\frac{d^2y}{dx^2}=\frac{0-[6(x-1)^{\frac{1}{2}}+3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$ oe

dM1 Multiply numerator and denominator by $(x-1)^{\frac{1}{2}}$ producing a linear numerator which is then simplified by collecting like terms.

Alternatively take out a common factor of $(x-1)^{-\frac{1}{2}}$ from the numerator and collect like terms from the linear expression

This is dependent upon the 1st M1 being scored.

A1 Correct simplified expression $\frac{d^2y}{dx^2}=\frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe

Notes for Question 5 Continued
(c) Using Product and Chain Rules M1 Writes $\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}} = Ax^{-1}(x-1)^{-\frac{1}{2}}$ and uses the product rule with u or $v = Ax^{-1}$ and v or $u = (x-1)^{-\frac{1}{2}}$. If any rule is quoted it must be correct. If the rules are not quoted nor implied then award if you see an expression of the form $(x-1)^{-\frac{3}{2}} \times Bx^{-1} \pm C(x-1)^{-\frac{1}{2}} \times x^{-2}$ A1 $\frac{d^2y}{dx^2} = \frac{1}{6}[x^{-1}(-\frac{1}{2})(x-1)^{-\frac{3}{2}} + (-1)x^{-2}(x-1)^{-\frac{1}{2}}]$ dM1 Factorises out / uses a common denominator of $x^{-2}(x-1)^{-\frac{3}{2}}$ producing a linear factor/numerator which must be simplified by collecting like terms. Need a single fraction. A1 Correct simplified expression $\frac{d^2y}{dx^2} == \frac{1}{12}x^{-2}(x-1)^{-\frac{3}{2}}[2-3x]$ *oe* (c) **Using Quotient and Chain rules Rules** M1 Uses the quotient rule $\frac{vu'-uv'}{v^2}$ with $u = (x-1)^{-\frac{1}{2}}$ and $v = 6x$ **and** achieving $u' = A(x-1)^{-\frac{3}{2}}$ and $v' = B$. If the formulae is quoted, it must be correct. If it is not quoted nor implied by their working allow an expression of the form $\left(\frac{d^2y}{dx^2}\right) = \frac{Cx(x-1)^{-\frac{3}{2}} - D(x-1)^{-\frac{1}{2}}}{Ex^2}$ A1 Correct un simplified expression $\frac{d^2y}{dx^2} = \frac{6x \times -\frac{1}{2}(x-1)^{-\frac{3}{2}} - (x-1)^{-\frac{1}{2}} \times 6}{(6x)^2}$ dM1 Multiply numerator and denominator by $(x-1)^{\frac{3}{2}}$ producing a linear numerator which is then simplified by collecting like terms. Alternatively take out a common factor of $(x-1)^{-\frac{3}{2}}$ from the numerator and collect like terms from the linear expression This is dependent upon the 1st M1 being scored. A1 Correct simplified expression $\frac{d^2y}{dx^2} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$ oe $\frac{d^2y}{dx^2} = \frac{(2-3x)x^{-2}(x-1)^{-\frac{3}{2}}}{12}$

Notes for Question 5 Continued	
(c)	**Using just the chain rule**
M1	Writes $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{6x(x-1)^{\frac{1}{2}}}=\frac{1}{(36x^3-36x^2)^{\frac{1}{2}}}=(36x^3-36x^2)^{-\frac{1}{2}}$ and proceeds by the chain rule to $A(36x^3-36x^2)^{-\frac{3}{2}}(Bx^2-Cx)$.
M1	Would automatically follow under this method if the first M has been scored

Question Number	Scheme	Marks
6(a)	$\ln(4-2x)(9-3x), = \ln(x+1)^2$	M1, M1
	So $36-30x+6x^2 = x^2+2x+1$ and $5x^2-32x+35=0$	A1
	Solve $5x^2-32x+35=0$ to give $x=\frac{7}{5}$ oe (Ignore the solution $x=5$)	M1A1
		(5)
(b)	Take $\log_e$'s to give $\ln 2^x + \ln e^{3x+1} = \ln 10$	M1
	$x\ln 2+(3x+1)\ln e = \ln 10$	M1
	$x(\ln 2+3\ln e) = \ln 10 - \ln e \Rightarrow x=..$	dM1
	and uses lne = 1	M1
	$x = \frac{-1+\ln 10}{3+\ln 2}$	A1
		(5)
	Note that the 4th M mark may occur on line 2	**(10 marks)**

Notes for Question 6

(a)

M1 Uses addition law on lhs of equation. Accept slips on the signs. If one of the terms is taken over to the rhs it would be for the subtraction law.

M1 Uses power rule for logs write the $2\ln(x+1)$ term as $\ln(x+1)^2$. Condone invisible brackets

A1 Undoes the logs to obtain the 3TQ =0. $5x^2-32x+35=0$. Accept equivalences. The equals zero may be implied by a subsequent solution of the equation.

M1 Solves a quadratic by any allowable method.
The quadratic cannot be a version of $(4-2x)(9-3x)=0$ however.

A1 Deduces x = 1.4 or equivalent. Accept both x=1.4 and x=5. Candidates do not have to eliminate $x=5$.
You may ignore any other solution as long as it is not in the range $-1<x<2$. Extra solutions in the range scores A0.

Notes for Question 6 Continued
(b) M1 Takes logs of both sides **and** splits LHS using addition law. If one of the terms is taken to the other side it can be awarded for taking logs of both sides **and** using the subtraction law. M1 Taking both powers down using power rule. It is not wholly dependent upon the first M1 but logs of both sides must have been taken. Below is an example of M0M1 $\ln 2^x \times \ln e^{3x+1} = \ln 10 \Rightarrow x\ln 2 \times (3x+1)\ln e = \ln 10$ dM1 This is dependent upon both previous two M's being scored. It can be awarded for a full method to solve their linear equation in *x*. The terms in *x* must be collected on one side of the equation and factorised. You may condone slips in signs for this mark but the process must be correct and leading to *x* = … M1 Uses ln e = 1. This could appear in line 2, but it must be part of their equation and not just a statement. Another example where it could be awarded is $e^{3x+1} = \frac{10}{2^x} \Rightarrow 3x+1 = ...$ A1 Obtains answer $x = \frac{-1+\ln 10}{3+\ln 2} = \left(\frac{\ln 10 - 1}{3+\ln 2}\right) = \left(\frac{\log_e 10 - 1}{3+\log_e 2}\right)$ *oe*. **DO NOT ISW HERE** Note 1: If the candidate takes $\log_{10}$'s of both sides can score M1M1dM1M0A0 for 3 out of 5. Answer = $x = \frac{-\log e + \log 10}{3\log e + \log 2} = \left(\frac{-\log e + 1}{3\log e + \log 2}\right)$ Note 2: If the candidate writes $x = \frac{-1+\log 10}{3+\log 2}$ without reference to natural logs then award M4 but with hold the last A1 mark, scoring 4 out of 5.

Question Number	Scheme	Marks
Alt 1 to 6(b)	Writes lhs in e's $2^x e^{3x+1} = 10 \Rightarrow e^{x\ln 2} e^{3x+1} = 10$	1st M1
	$\Rightarrow e^{x\ln 2 + 3x+1} = 10, \quad x\ln 2 + 3x + 1 = \ln 10$	2nd M1, 4th M1
	$x(\ln 2 + 3) = \ln 10 - 1 \Rightarrow x = ..$	dM1
	$x = \dfrac{-1+\ln 10}{3+\ln 2}$	A1 **(5)**

Notes for Question 6 Alt 1

M1 Writes the lhs of the expression in e's. Seeing $2^x = e^{x\ln 2}$ in their equation is sufficient

M1 Uses the addition law on the lhs to produce a single exponential

dM1 Takes ln's of both sides to produce and attempt to solve a linear equation in *x*
You may condone slips in signs for this mark but the process must be correct leading to *x*= ..

M1 Uses ln e = 1. This could appear in line 2

Question Number	Scheme	Marks
7(a)	$0 \leqslant f(x) \leqslant 10$	B1
		(1)
(b)	ff(0) = f(5), = 3	B1,B1
		(2)
(c)	$y = \dfrac{4+3x}{5-x} \Rightarrow y(5-x) = 4+3x$	
	$\Rightarrow 5y - 4 = xy + 3x$	M1
	$\Rightarrow 5y-4 = x(y+3) \Rightarrow x = \dfrac{5y-4}{y+3}$	dM1
	$g^{-1}(x) = \dfrac{5x-4}{3+x}$	A1
		(3)
(d)	$gf(x) = 16 \Rightarrow f(x) = g^{-1}(16) = 4$ oe	M1A1
	$f(x) = 4 \Rightarrow x = 6$	B1
	$f(x) = 4 \Rightarrow 5 - 2.5x = 4 \Rightarrow x = 0.4$ oe	M1A1
		(5)
		(11 marks)
Alt 1 to 7(d)	$gf(x) = 16 \Rightarrow \dfrac{4+3(ax+b)}{5-(ax+b)} = 16$	M1
	$ax + b = x - 2$ *or* $5 - 2.5x$	A1
	$\Rightarrow x = 6$	B1
	$\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = \ldots$	M1
	$\Rightarrow x = 0.4$ oe	A1 **(5)**

Notes for Question 7

(a)

B1 Correct range. Allow $0 \leqslant \text{f}(x) \leqslant 10$, $0 \leqslant \text{f} \leqslant 10$, $0 \leqslant y \leqslant 10$, $0 \leqslant \text{range} \leqslant 10$, $[0,10]$

Allow $\text{f}(x) \geqslant 0$ and $\text{f}(x) \leqslant 10$ but not $\text{f}(x) \geqslant 0$ or $\text{f}(x) \leqslant 10$

Do Not Allow $0 \leqslant x \leqslant 10$. The inequality must include BOTH ends

(b)

B1 For correct one application of the function at x=0

Possible ways to score this mark are f(0)=5, f(5) $0 \rightarrow 5 \rightarrow \ldots$

B1: 3 ('3' can score both marks as long as no incorrect working is seen.)

(c)

M1 For an attempt to make x or a replaced y the subject of the formula. This can be scored for putting y = g(x), multiplying across, expanding and collecting x terms on one side of the equation. Condone slips on the signs

dM1 Take out a common factor of x (or a replaced y) and divide, to make x subject of formula. Only allow **one sign error** for this mark

A1 Correct answer. No need to state the domain. Allow $\text{g}^{-1}(x) = \dfrac{5x-4}{3+x}$ $y = \dfrac{5x-4}{3+x}$

Accept alternatives such as $y = \dfrac{4-5x}{-3-x}$ and $y = \dfrac{5-\frac{4}{x}}{1+\frac{3}{x}}$

(d)

M1 Stating or implying that $\text{f}(x) = \text{g}^{-1}(16)$. For example accept $\dfrac{4+3\text{f}(x)}{5-\text{f}(x)} = 16 \Rightarrow \text{f}(x) = ..$

A1 Stating $\text{f}(x) = 4$ or implying that solutions are where $\text{f}(x) = 4$

B1 x = 6 and may be given if there is no working

M1 Full method to obtain other value from line $y = 5 - 2.5x$

$5 - 2.5x = 4 \Rightarrow x = \ldots$.

Alternatively this could be done by similar triangles. Look for $\dfrac{2}{5} = \dfrac{2-x}{4}\,(oe) \Rightarrow x = ..$

A1 0.4 or 2/5

Alt 1 to (d)

M1 Writes $\text{gf}(x) = 16$ with a linear $\text{f}(x)$. The order of gf(x) must be correct

Condone invisible brackets. Even accept if there is a modulus sign.

A1 Uses $\text{f}(x) = x - 2$ or $\text{f}(x) = 5 - 2.5x$ in the equation $\text{gf}(x) = 16$

B1 x = 6 and may be given if there is no working

M1 Attempt at solving $\dfrac{4+3(5-2.5x)}{5-(5-2.5x)} = 16 \Rightarrow x = \ldots$. The bracketing must be correct and there must be no more than one error in their calculation

A1 $x = 0.4, \dfrac{2}{5}$ or equivalent

Question Number	Scheme	Marks
8(a)	$R = \sqrt{\left(7^2 + 24^2\right)} = 25$	B1
	$\tan\alpha = \frac{24}{7}, \quad \Rightarrow \quad \alpha = \text{awrt } 73.74°$	M1A1
		(3)
(b)	maximum value of 24sin$x$ + 7cos$x$ = 25 so $V_{\min} = \frac{21}{25} = (0.84)$	M1A1
		(2)
(c)	Distance $AB = \frac{7}{\sin\theta}$, with $\theta = \alpha$	M1, B1
	So distance = 7.29m $= \frac{175}{24}$ m	A1
		(3)
(d)	$R\cos(\theta - \alpha) = \frac{21}{1.68} \Rightarrow \cos(\theta - \alpha) = 0.5$	M1, A1
	$\theta - \alpha = 60 \Rightarrow \theta = .., \theta - \alpha = -60 \Rightarrow \theta = ..$	dM1, dM1
	$\theta = \text{awrt } 133.7, 13.7$	A1, A1
		(6) **(14 marks)**

Notes for Question 8

(a)

B1 25. Accept 25.0 but not $\sqrt{625}$ or answers that are not exactly 25. Eg 25.0001

M1 For $\tan\alpha = \pm\frac{24}{7}, \tan\alpha = \pm\frac{7}{24}$.

If the value of R is used only accept $\sin\alpha = \pm\frac{24}{R}, \cos\alpha = \pm\frac{7}{R}$

A1 Accept answers which round to 73.74 – must be in degrees for this mark

(b)

M1 Calculates $V = \frac{21}{their\ 'R'}$ NOT - R

A1 Obtains correct answer. $V = \frac{21}{25}$ Accept 0.84

Do not accept if you see incorrect working- ie from $\cos(\theta - \alpha) = -1$ or the minus just disappearing from a previous line.

Questions involving differentiation are acceptable. To score M1 the candidate would have to differentiate V by the quotient rule (or similar) , set V'=0 to find θ and then sub this back into V to find its value.

Notes for Question 8 Continued

(c)

M1 Uses the trig equation $\sin\theta = \dfrac{7}{AB}$ with a numerical θ to find $AB = ...$

B1 Uses $\theta =$ their value of α in a trig calculation involving sin. ($\sin\alpha = \dfrac{AB}{7}$ is condoned)

A1 Obtains answer $\dfrac{175}{24}$ or awrt 7.29

(d)

M1 Substitutes $V = 1.68$ and their answer to part (a) in $V = \dfrac{21}{24\sin\theta + 7\cos\theta}$ to get an equation of the form $R\cos(\theta \pm \alpha) = \dfrac{21}{1.68}$ or $1.68R\cos(\theta \pm \alpha) = 21$ or $\cos(\theta \pm \alpha) = \dfrac{21}{1.68R}$.

Follow through on their *R* and *α*

A1 Obtains $\cos(\theta \pm \alpha) = 0.5$ oe. Follow through on their *α*. It may be implied by later working.

dM1 Obtains one value of θ **in the range** $0 < \theta < 150$ from inverse cos +their α
It is dependent upon the first M being scored.

dM1 Obtains second angle of θ **in the range** $0 < \theta < 150$ from inverse cos +their α
It is dependent upon the first M being scored.

A1 one correct answer awrt $\theta = 133.7$ *or* 13.7 1dp

A1 both correct answers awrt $\theta = 133.7$ *and* 13.7 1dp.

Extra solutions in the range loses the last A1.

Answers in radians, lose the first time it occurs. Answers must be to 3dp

For your info $\alpha = 1.287, \theta_1 = 2.334, \theta_2 = 0.240$

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