



2.

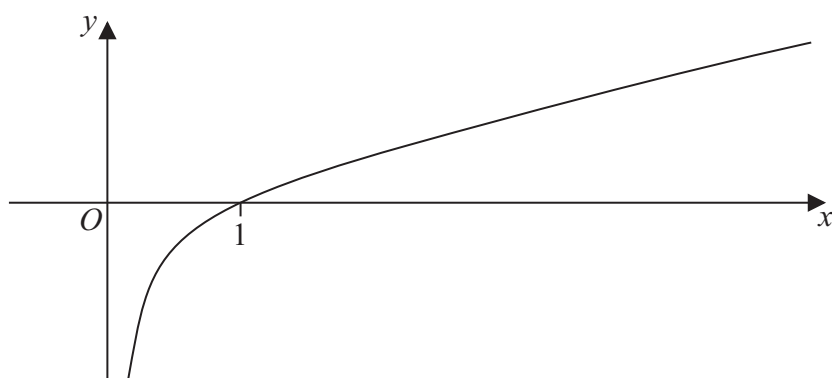


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$, $x > 0$, where f is an increasing function of x . The curve crosses the x -axis at the point $(1, 0)$ and the line $x = 0$ is an asymptote to the curve.

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$, $x > 0$ (2)

(b) $y = |f(x)|$, $x > 0$ (3)

Indicate clearly on each sketch the coordinates of the point at which the curve crosses or meets the x -axis.

3.

$$f(x) = 7\cos x + \sin x$$

Given that $f(x) = R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$,

- (a) find the exact value of R and the value of α to one decimal place.

(3)

- (b) Hence solve the equation

$$7\cos x + \sin x = 5$$

for $0 \leq x < 360^\circ$, giving your answers to one decimal place.

(5)

- (c) State the values of k for which the equation

$$7\cos x + \sin x = k$$

has only one solution in the interval $0 \leq x < 360^\circ$

(2)













7.

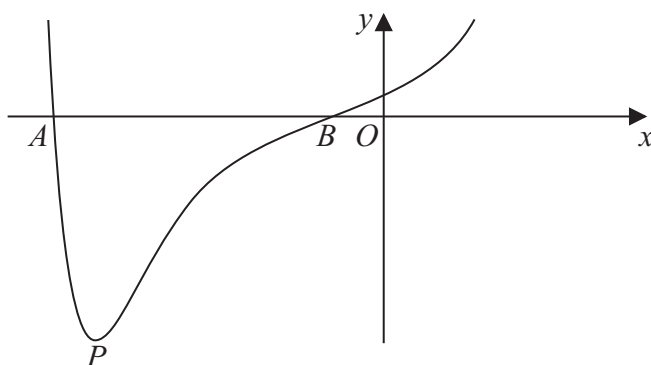


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = (x^2 + 3x + 1)e^{x^2}$$

The curve cuts the x -axis at points A and B as shown in Figure 2.

- (a) Calculate the x coordinate of A and the x coordinate of B , giving your answers to 3 decimal places. (2)

- (b) Find $f'(x)$. (3)

The curve has a minimum turning point at the point P as shown in Figure 2.

- (c) Show that the x coordinate of P is the solution of

$$x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)}$$
(3)

- (d) Use the iteration formula

$$x_{n+1} = -\frac{3(2x_n^2 + 1)}{2(x_n^2 + 2)}, \quad \text{with } x_0 = -2.4,$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 3 decimal places. (3)

The x coordinate of P is α .

- (e) By choosing a suitable interval, prove that $\alpha = -2.43$ to 2 decimal places. (2)





8.

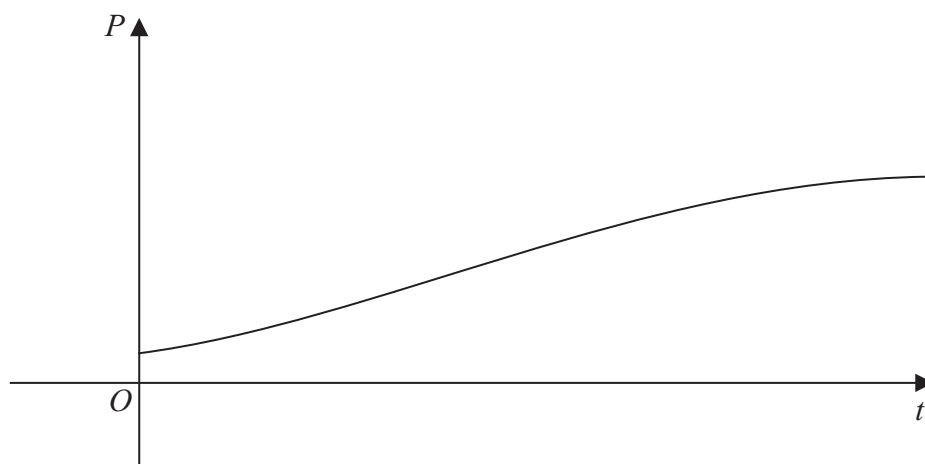


Figure 3

The population of a town is being studied. The population P , at time t years from the start of the study, is assumed to be

$$P = \frac{8000}{1 + 7e^{-kt}}, \quad t \geq 0,$$

where k is a positive constant.

The graph of P against t is shown in Figure 3.

Use the given equation to

(a) find the population at the start of the study, (2)

(b) find a value for the expected upper limit of the population. (1)

Given also that the population reaches 2500 at 3 years from the start of the study,

(c) calculate the value of k to 3 decimal places. (5)

Using this value for k ,

(d) find the population at 10 years from the start of the study, giving your answer to 3 significant figures. (2)

(e) Find, using $\frac{dP}{dt}$, the rate at which the population is growing at 10 years from the start of the study. (3)



Question 8 continued

(Total 13 marks)

Q8

TOTAL FOR PAPER: 75 MARKS

END

