

1.

$$g(x) = \frac{6x+12}{x^2+3x+2} - 2, \quad x \geq 0$$

(a) Show that  $g(x) = \frac{4-2x}{x+1}, \quad x \geq 0$

(3)

(b)

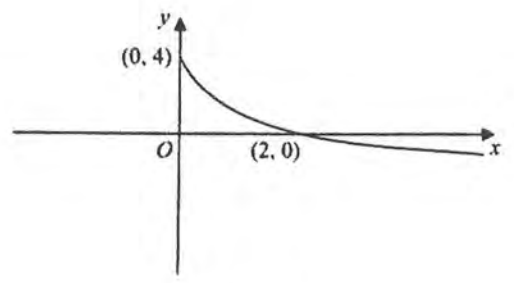


Figure 1

Figure 1 shows a sketch of the curve with equation  $y = g(x), \quad x \geq 0$   
 The curve meets the  $y$ -axis at  $(0, 4)$  and crosses the  $x$ -axis at  $(2, 0)$ .

On separate diagrams sketch the graph with equation

- (i)  $y = 2g(2x)$ ,
- (ii)  $y = g^{-1}(x)$ .

Show on each sketch the coordinates of each point at which the graph meets or crosses the axes.

(5)

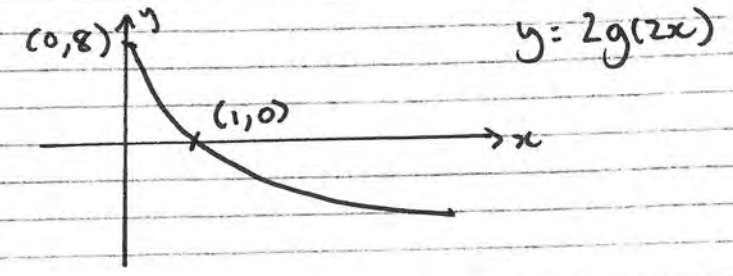
$$g(x) = \frac{6(x+2)}{(x+1)(x+2)} - \frac{2(x+1)(x+2)}{(x+1)(x+2)}$$

$$= \frac{6-2x-2}{x+1} = \frac{4-2x}{x+1} \quad \#$$

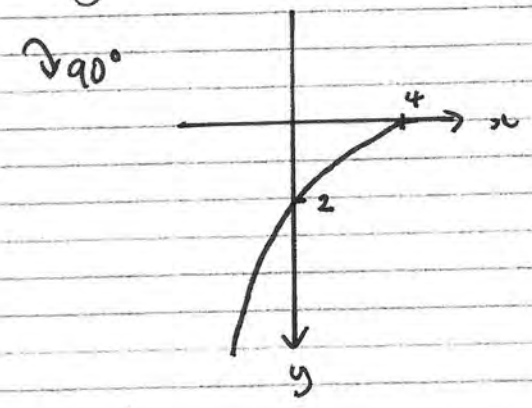
Question 1 continued

b)  $2g(2x)$

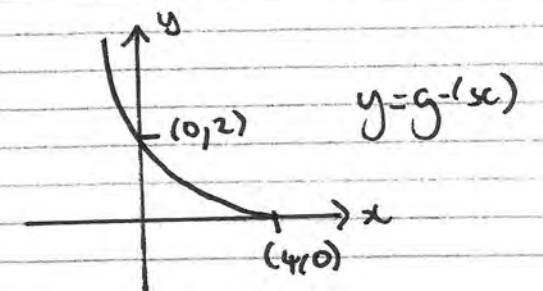
$\uparrow$   
 $\downarrow \rightarrow 2x$



ii)  $g^{-1}(x)$



reflect



2. Given that  $\tan 40^\circ = p$ , find in terms of  $p$

(a)  $\cot 40^\circ$

(1)

(b)  $\sec 40^\circ$

(2)

(c)  $\tan 85^\circ$

(2)

$$a) \cot 40 = \frac{1}{\tan 40} = \frac{1}{p}$$

$$b) \sec^2 40 = 1 + \tan^2 40$$

$$\therefore \sec 40 = \sqrt{1+p^2}$$

$$c) \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(45+40) = \frac{\tan 45 + \tan 40}{1 - \tan 45 \tan 40}$$

$$\therefore \tan 85 = \frac{1+p}{1-p}$$

3.

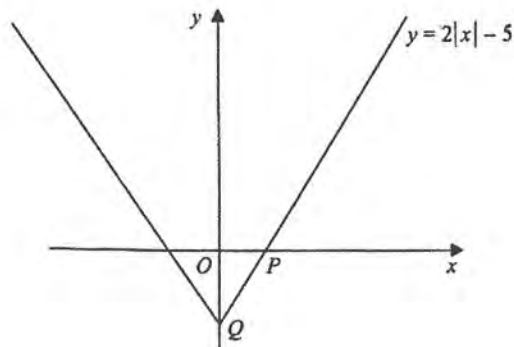


Figure 2

Figure 2 shows a sketch of the graph with equation  $y = 2|x| - 5$ .

The graph intersects the positive  $x$ -axis at the point  $P$  and the negative  $y$ -axis at the point  $Q$ .

(a) State the coordinates of  $P$  and the coordinates of  $Q$ .

(2)

(b) Solve the equation

$$2|x| - 5 = 3 - x$$

(3)

$$a) P(2.5, 0) \quad Q(0, -5)$$

$$b) 2|x| = 8 - x \Rightarrow |x| = 4 - \frac{1}{2}x$$

$$\therefore x = 4 - \frac{1}{2}x \quad x = \frac{1}{2}x - 4$$

$$\frac{3}{2}x = 4$$

$$\frac{1}{2}x = -4$$

$$x = \frac{8}{3}$$

$$x = -8$$

4. (a) On the same diagram, sketch and clearly label the graphs with equations

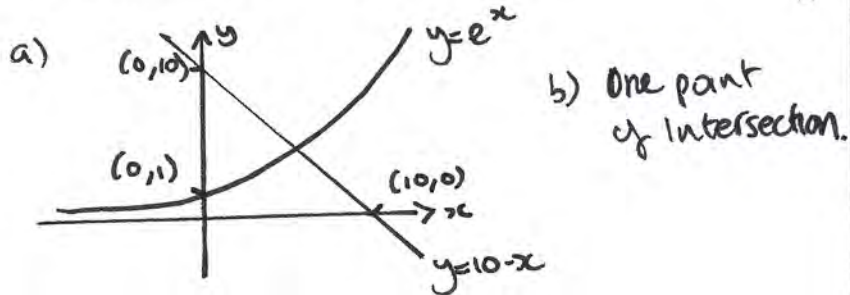
$$y = e^x \quad \text{and} \quad y = 10 - x$$

Show on your sketch the coordinates of each point at which the graphs cut the axes. (3)

(b) Explain why the equation  $e^x - 10 + x = 0$  has only one solution. (1)

(c) Show that the solution of the equation  $e^x - 10 + x = 0$  lies between  $x = 2$  and  $x = 3$  (2)

(d) Use the iterative formula  $x_{n+1} = \ln(10 - x_n)$ ,  $x_1 = 2$  to calculate the values of  $x_2$ ,  $x_3$  and  $x_4$ . Give your answers to 4 decimal places. (3)



c)  $f(2) = -0.6 < 0$   
 $f(3) = 13 > 0$   
 $\therefore$  by sign change rule solution lies between 2 and 3.

- d)
- $x_1 = 2$
  - $x_2 = 2.0794$
  - $x_3 = 2.0695$
  - $x_4 = 2.0707$

5. (i) (a) Show that  $\frac{d}{dx} \left( x^{\frac{1}{2}} \ln x \right) = \frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}$  (3)

The curve with equation  $y = x^{\frac{1}{2}} \ln x$ ,  $x > 0$  has one turning point at the point P.

(b) Find the exact coordinates of P. Give your answer in its simplest form. (4)

(ii) A curve C has equation  $y = \frac{x-k}{x+k}$ , where k is a positive constant

Find  $\frac{dy}{dx}$ , and show that C has no turning points. (4)

a)  $u = x^{\frac{1}{2}} \quad v = \ln x$   
 $u' = \frac{1}{2}x^{-\frac{1}{2}} \quad v' = \frac{1}{x}$

$$\therefore \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} \ln x + x^{\frac{1}{2}} \frac{1}{x}$$

$$= \frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}} \neq$$

b) TP when  $\frac{dy}{dx} = 0 \quad \therefore \frac{\ln x}{2\sqrt{x}} = -\frac{1}{\sqrt{x}}$   
 $\Rightarrow \ln x = -2 \quad \Rightarrow x = e^{-2}$

$$y = (e^{-2})^{\frac{1}{2}} \ln e^{-2} = e^{-1} \times -2 = -2e^{-1} \quad (e^{-1}, -2e^{-1})$$

(ii)  $u = x-k \quad v = x+k \quad \Rightarrow \frac{dy}{dx} = \frac{(x+k) - (x-k)}{(x+k)^2}$   
 $u' = 1 \quad v' = 1$

$\therefore \frac{dy}{dx} = \frac{2k}{(x+k)^2} = \frac{dy}{dx} = 0$  if  $k=0$  but  $k$  is a positive constant  $\therefore$  no TP!



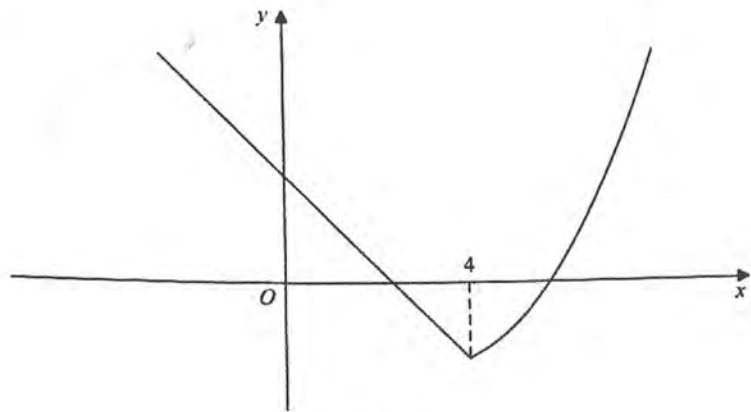


Figure 3

Figure 3 shows a sketch of the graph of  $y = f(x)$  where

$$f(x) = \begin{cases} 5 - 2x, & x \leq 4 \\ e^{2x-8} - 4, & x > 4 \end{cases}$$

(a) State the range of  $f(x)$ .

(1)

(b) Determine the exact value of  $ff(0)$ .

(2)

(c) Solve  $f(x) = 21$

(5)

Give each answer as an exact answer.

(d) Explain why the function  $f$  does not have an inverse.

(1)

a)  $x=4$   $f(4) = -3 \therefore$  range  $y=f(x) \geq -3$ .

b)  $f(0) = 5 \therefore ff(0) = f(5) = e^2 - 4$ .

c)  $5 - 2x = 21 \therefore 2x = -16 \therefore x = -8$   
 $e^{2x-8} - 4 = 21 \Rightarrow e^{2x-8} = 25 \Rightarrow 2x-8 = \ln 25$   
 $\therefore x = 4 + \frac{1}{2} \ln 25$

a)  $f$  is not a one to one function.

7. (a) Prove that

$$\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = 2 \sec x, \quad x \neq (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z}$$

(4)

(b) Hence find, for  $0 < x < \frac{\pi}{4}$ , the exact solution of

$$\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = 8 \sin x$$

(4)

$$\frac{\cos x (\cos x)}{(1 - \sin x)(\cos x)} + \frac{(1 - \sin x)(1 - \sin x)}{(1 - \sin x)(\cos x)}$$

$$= \frac{\cos^2 x + 1 - 2 \sin x + \sin^2 x}{(1 - \sin x) \cos x}$$

$$\sin^2 + \cos^2 = 1 \Rightarrow \frac{2 - 2 \sin x}{(1 - \sin x)(\cos x)} = \frac{2(1 - \sin x)}{(1 - \sin x)(\cos x)}$$

$$= \frac{2}{\cos x} = 2 \sec x \neq$$

b)  $2 \sec x = 8 \sin x \Rightarrow \frac{2}{\cos x} = 8 \sin x$

$$\Rightarrow 2 = 8 \sin x \cos x \Rightarrow 2 = 4(2 \sin x \cos x)$$

$$= \sin 2x = \frac{1}{2} \Rightarrow 2x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\therefore 2x = \frac{\pi}{6}, \frac{5\pi}{6} \therefore x = \frac{\pi}{12}, \frac{5\pi}{12}$$

8. (a) Express  $9\cos\theta - 2\sin\theta$  in the form  $R\cos(\theta + \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{\pi}{2}$ .

Give the exact value of  $R$  and give the value of  $\alpha$  to 4 decimal places.

(3)

- (b) (i) State the maximum value of  $9\cos\theta - 2\sin\theta$

(ii) Find the value of  $\theta$ , for  $0 < \theta < 2\pi$ , at which this maximum occurs.

(3)

Ruth models the height  $H$  above the ground of a passenger on a Ferris wheel by the equation

$$H = 10 - 9\cos\left(\frac{\pi t}{5}\right) + 2\sin\left(\frac{\pi t}{5}\right)$$



where  $H$  is measured in metres and  $t$  is the time in minutes after the wheel starts turning.

- (c) Calculate the maximum value of  $H$  predicted by this model, and the value of  $t$ , when this maximum first occurs. Give your answers to 2 decimal places.

(4)

- (d) Determine the time for the Ferris wheel to complete two revolutions.

(2)

$$a) R\cos(\theta + \alpha) = R(\cos\theta\cos\alpha - R\sin\theta\sin\alpha)$$

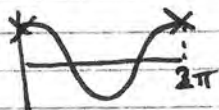
$$9\cos\theta - 2\sin\theta$$

$$\frac{R\sin\alpha = 2}{R\cos\alpha = 9}$$

$$\therefore \tan\alpha = \frac{2}{9} \quad \alpha = 0.2187$$

$$R^2 = 2^2 + 9^2 \quad \therefore R = \sqrt{85}$$

$$b) \sqrt{85}\cos(\theta + 0.2187) \quad \therefore \text{max} = \sqrt{85}$$



max at  $0, 2\pi$

$$\therefore \theta = 2\pi - 0.2187$$

$$\theta = 6.065$$

$$c) \text{max } H = 10 - (-\sqrt{85}) = 19.22$$

this occurs when  $\sqrt{85}\cos(\theta + 0.2187)$  is at its min value  $-\sqrt{85}$ . This is when  $\theta + 0.2187 = \pi$

$$\therefore \theta = 2.92$$

$$2.92 = \frac{\pi t}{5} \quad \therefore t = 4.65$$

d) two revolutions =  $4\pi$

$$\frac{\pi t}{5} = 4\pi \quad t = 20$$

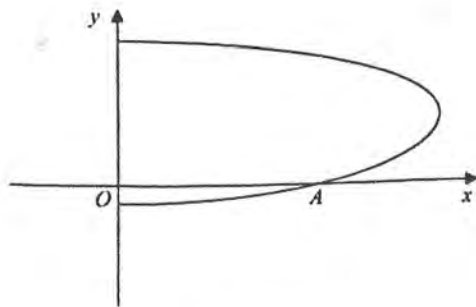


Figure 4

Figure 4 shows a sketch of the curve with equation  $x = (9 + 16y - 2y^2)^{\frac{1}{2}}$ .

The curve crosses the  $x$ -axis at the point  $A$ .

(a) State the coordinates of  $A$ .

(1)

(b) Find an expression for  $\frac{dx}{dy}$ , in terms of  $y$ .

(3)

(c) Find an equation of the tangent to the curve at  $A$ .

(4)

Question 9 continued

$$a) A(3,0)$$

$$b) \frac{dx}{dy} = \frac{1}{2}(9 + 16y - 2y^2)^{-\frac{1}{2}}(16 - 4y)$$

$$\therefore \frac{dx}{dy} = \frac{8 - 2y}{\sqrt{9 + 16y - 2y^2}}$$

$$c) \frac{dy}{dx} = \frac{\sqrt{9 + 16y - 2y^2}}{8 - 2y} \quad \therefore \text{M.T.} \Big|_{y=0} = \frac{\sqrt{9}}{8} = \frac{3}{8}$$

$$(3,0) \quad y - 0 = \frac{3}{8}(x - 3) \quad \therefore 8y = 3x - 9$$