

1.

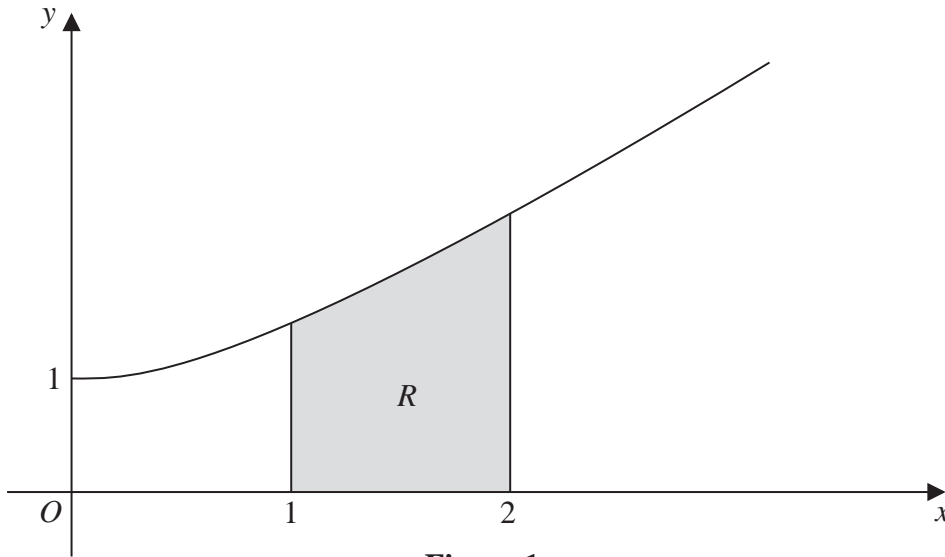


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{(x^2 + 1)}$, $x \geq 0$

The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis and the lines $x = 1$ and $x = 2$

The table below shows corresponding values for x and y for $y = \sqrt{(x^2 + 1)}$.

x	1	1.25	1.5	1.75	2
y	1.414		1.803	2.016	2.236

- (a) Complete the table above, giving the missing value of y to 3 decimal places. (1)
- (b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R , giving your answer to 2 decimal places. (4)



4. Use integration to find

$$\int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined. (5)



5.

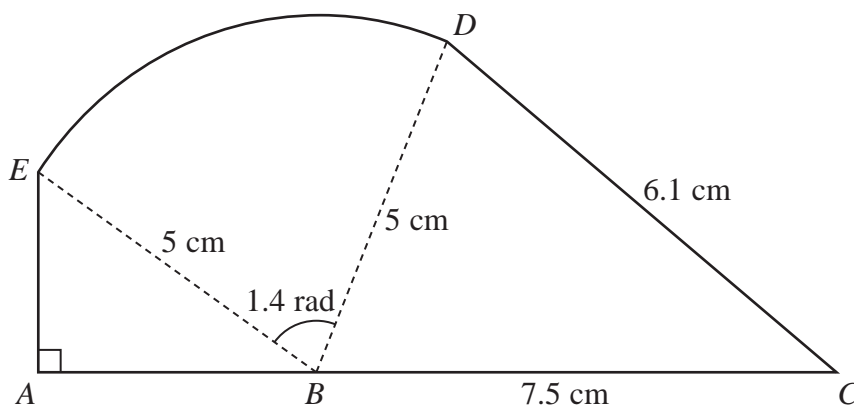


Figure 2

The shape $ABCDEA$, as shown in Figure 2, consists of a right-angled triangle EAB and a triangle DBC joined to a sector BDE of a circle with radius 5 cm and centre B .

The points A , B and C lie on a straight line with $BC = 7.5$ cm.

Angle $EAB = \frac{\pi}{2}$ radians, angle $EBD = 1.4$ radians and $CD = 6.1$ cm.

- (a) Find, in cm^2 , the area of the sector BDE . (2)
- (b) Find the size of the angle DBC , giving your answer in radians to 3 decimal places. (2)
- (c) Find, in cm^2 , the area of the shape $ABCDEA$, giving your answer to 3 significant figures. (5)



Question 6 continued

Blank lined area for writing the answer to Question 6.



7. (i) Solve, for $0 \leq \theta < 360^\circ$, the equation

$$9 \sin(\theta + 60^\circ) = 4$$

giving your answers to 1 decimal place.
You must show each step of your working.

(4)

(ii) Solve, for $-\pi \leq x < \pi$, the equation

$$2 \tan x - 3 \sin x = 0$$

giving your answers to 2 decimal places where appropriate.

[Solutions based entirely on graphical or numerical methods are not acceptable.]

(5)



8. (a) Sketch the graph of

$$y = 3^x, \quad x \in \mathbb{R}$$

showing the coordinates of any points at which the graph crosses the axes.

(2)

- (b) Use algebra to solve the equation

$$3^{2x} - 9(3^x) + 18 = 0$$

giving your answers to 2 decimal places where appropriate.

(5)



9.

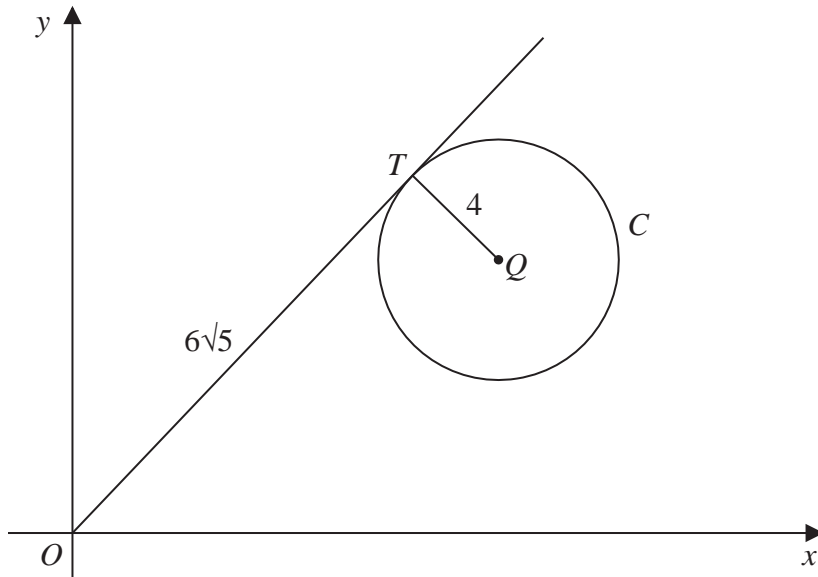


Figure 3

Figure 3 shows a circle C with centre Q and radius 4 and the point T which lies on C .

The tangent to C at the point T passes through the origin O and $OT = 6\sqrt{5}$

Given that the coordinates of Q are $(11, k)$, where k is a positive constant,

(a) find the exact value of k , (3)

(b) find an equation for C . (2)



Question 9 continued

A series of horizontal lines for writing.



10.

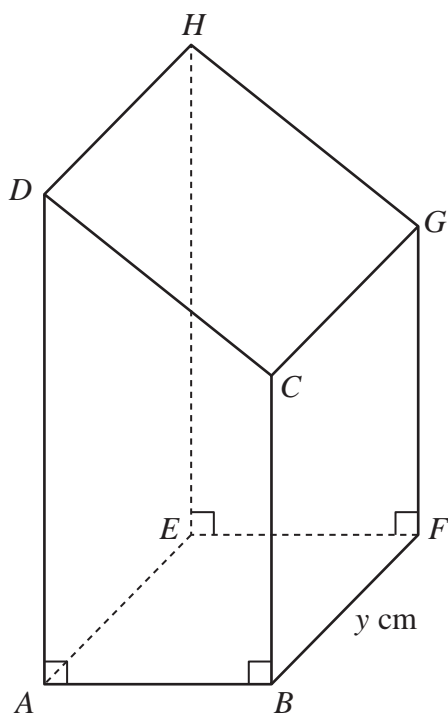


Figure 4

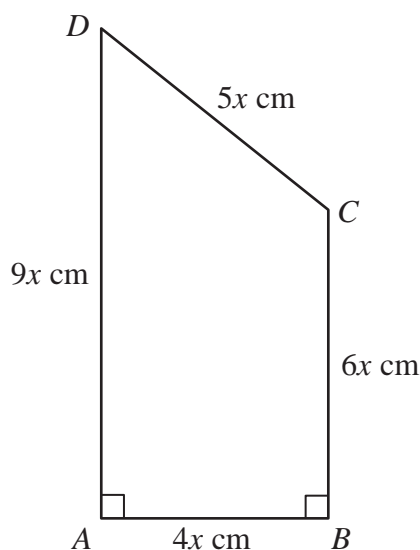


Figure 5

Figure 4 shows a closed letter box $ABFEHGC D$, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base $ABFE$ of the prism is a rectangle. The total surface area of the six faces of the prism is S cm².

The cross section $ABCD$ of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5. The angle $DAB = 90^\circ$ and the angle $ABC = 90^\circ$.

The volume of the letter box is 9600 cm³.

(a) Show that

$$y = \frac{320}{x^2} \tag{2}$$

(b) Hence show that the surface area of the letter box, S cm², is given by

$$S = 60x^2 + \frac{7680}{x} \tag{4}$$

(c) Use calculus to find the minimum value of S .

(6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)



