

S14 C2(R)

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1. Find the first 4 terms, in ascending powers of x , of the binomial expansion of

$$\left(1 + \frac{3x}{2}\right)^8$$

giving each term in its simplest form.

(4)

$$(a+b)^8 = a^8 + 8a^7b + \binom{8}{2}a^6b^2 + \binom{8}{3}a^5b^3$$

$$\begin{aligned} \left(1 + \frac{3x}{2}\right)^8 &= 1 + 8\left(\frac{3x}{2}\right) + 28\left(\frac{3x}{2}\right)^2 + 56\left(\frac{3x}{2}\right)^3 \\ &= 1 + 12x + 63x^2 + 189x^3 \end{aligned}$$

2. A geometric series has first term a , where $a \neq 0$, and common ratio r . The sum to infinity of this series is 6 times the first term of the series.

(a) Show that $r = \frac{5}{6}$

(2)

Given that the fourth term of this series is 62.5

- (b) find the value of a .

(2)

- (c) find the difference between the sum to infinity and the sum of the first 30 terms, giving your answer to 3 significant figures.

(4)

$$a) S_{\infty} = \frac{a}{1-r} = 6a \Rightarrow \frac{a}{6a} = 1-r$$

$$\Rightarrow \frac{1}{6} = 1-r \quad (\times 6) \quad 1 = 6 - 6r \Rightarrow 6r = 5 \therefore r = \frac{5}{6}$$

$$b) u_4 = ar^3 = a\left(\frac{5}{6}\right)^3 = 62.5 \therefore a = 108.$$

$$c) S_{30} = 108 \frac{1 - \left(\frac{5}{6}\right)^{30}}{\frac{1}{6}} = 645.27...$$

$$\begin{aligned} \therefore \text{difference} &= \frac{108}{\left(\frac{1}{6}\right)} - 645.27... \\ &= 2.73 \end{aligned}$$

3.

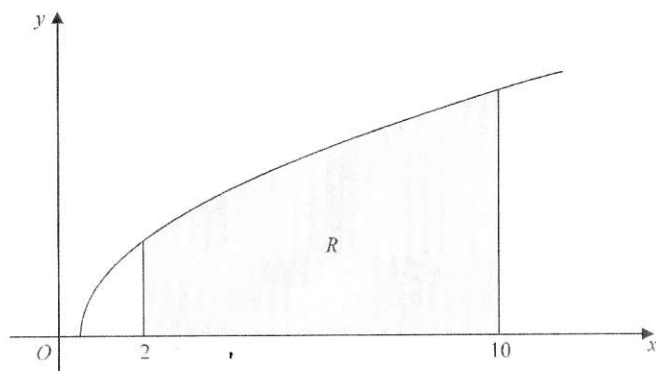


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{2x-1}$, $x \geq 0.5$

The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis and the lines with equations $x = 2$ and $x = 10$.

The table below shows corresponding values of x and y for $y = \sqrt{2x-1}$.

x	2	4	6	8	10
y	$\sqrt{3}$	$\sqrt{7}$	$\sqrt{11}$	$\sqrt{15}$	$\sqrt{19}$

(a) Complete the table with the values of y corresponding to $x = 4$ and $x = 8$.

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R , giving your answer to 2 decimal places.

(3)

(c) State whether your approximate value in part (b) is an overestimate or an underestimate for the area of R .

(1)

$$\text{Area} \approx \frac{1}{2}(2)[\sqrt{3} + 2(\sqrt{7} + \sqrt{11} + \sqrt{15}) + \sqrt{19}] = 25.76$$

c) under estimate, trapezia will be below curve.

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4.

$f(x) = -4x^3 + ax^2 + 9x - 18$, where a is a constant.

Given that $(x - 2)$ is a factor of $f(x)$,

(a) find the value of a ,

(2)

(b) factorise $f(x)$ completely,

(3)

(c) find the remainder when $f(x)$ is divided by $(2x - 1)$.

(2)

$$a) f(2) = 0 \Rightarrow -32 + 4a + 18 - 18 = 0 \therefore a = 8$$

$$b) x \quad -4x^2 + 9$$

x	$-4x^3$	$9x$	/
-2	$8x^2$	-18	/

 $r=0$

$$= -(x-2)(4x^2-9)$$

$$= -(x-2)(2x-3)(2x+3)$$

2

$$c) f\left(\frac{1}{2}\right) = -4\left(\frac{1}{8}\right) + 8\left(\frac{1}{4}\right) + 9\left(\frac{1}{2}\right) - 18 = -12$$

$$\therefore \text{remainder is } -12$$

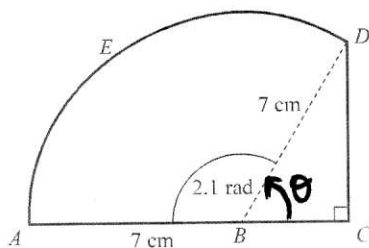


Figure 2

Figure 2 shows the shape $ABCDEA$ which consists of a right-angled triangle BCD joined to a sector $ABDEA$ of a circle with radius 7 cm and centre B .

A , B and C lie on a straight line with $AB = 7$ cm.

Given that the size of angle ABD is exactly 2.1 radians,

(a) find, in cm, the length of the arc DEA ,

(2)

(b) find, in cm, the perimeter of the shape $ABCDEA$, giving your answer to 1 decimal place.

(4)

$$a) \text{ arc} = r\theta = 7 \times 2.1 = \underline{14.7}$$

$$b) \theta = \pi - 2.1$$

$$BC = 7 \cos(\pi - 2.1) = 3.53$$

$$CD = 7 \sin(\pi - 2.1) = 6.04$$

$$P = \underline{31.3}$$

6.

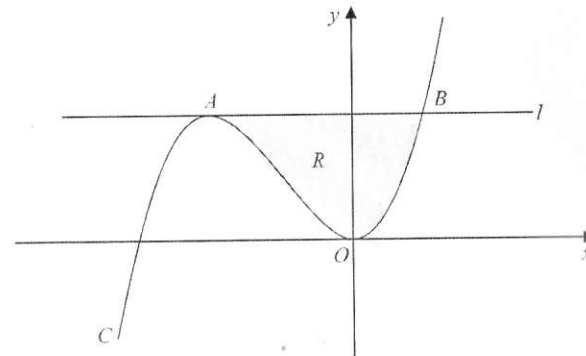


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{1}{8}x^3 + \frac{3}{4}x^2, \quad x \in \mathbb{R}$$

The curve C has a maximum turning point at the point A and a minimum turning point at the origin O .

The line l touches the curve C at the point A and cuts the curve C at the point B .

The x coordinate of A is -4 and the x coordinate of B is 2.

The finite region R , shown shaded in Figure 3, is bounded by the curve C and the line l .

Use integration to find the area of the finite region R .

$$x=2 \quad y = \frac{1}{8}(8) + \frac{3}{4}(4) = 4 \quad \boxed{24} \begin{matrix} 4 \\ -4 \quad 6 \quad 2 \end{matrix}$$

$$\begin{aligned} \therefore R &= 24 - \int_{-4}^2 \left(\frac{1}{8}x^3 + \frac{3}{4}x^2 \right) dx \\ &= 24 - \left[\frac{1}{32}x^4 + \frac{3}{12}x^3 \right]_{-4}^2 \\ &= 24 - \left[\left(\frac{5}{2} \right) - (-8) \right] = \underline{13.5} \end{aligned}$$

7. (i) Solve, for $0 \leq \theta < 180^\circ$, the equation

$$\frac{\sin 2\theta}{(4\sin 2\theta - 1)} = 1$$

giving your answers to 1 decimal place.

(3)

- (ii) Solve, for $0 \leq x < 2\pi$, the equation

$$5\sin^2 x - 2\cos x - 5 = 0$$

giving your answers to 2 decimal places.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(5)

$$\begin{aligned} \text{i) } \sin 2\theta &= 4\sin 2\theta - 1 \Rightarrow 3\sin 2\theta = 1 \\ \Rightarrow \sin 2\theta &= \frac{1}{3} \Rightarrow 2\theta = \sin^{-1}\left(\frac{1}{3}\right) = 0.3398 \\ 2\theta &= 0.3398, 2.8018 \dots \\ \theta &= 0.2^\circ, 1.4^\circ \quad \therefore \theta = \underline{9.7^\circ}, \underline{80.3^\circ} \end{aligned}$$

$$\text{ii) } 5(1 - \cos^2 x) - 2\cos x - 5 = 0$$

$$\Rightarrow 5\cos^2 x + 2\cos x = 0$$

$$\Rightarrow \cos x (5\cos x + 2) = 0$$

$$\cos x = 0$$

$$x = \underline{\frac{\pi}{2}}, \underline{\frac{3\pi}{2}}$$

$$\cos x = -\frac{2}{5}$$

$$x = \underline{1.98^\circ}, \underline{4.30^\circ}$$

$2\pi -$

8. (i) Solve

$$5^y = 8$$

giving your answer to 3 significant figures.

(2)

- (ii) Use algebra to find the values of x for which

$$\log_2(x+15) - 4 = \frac{1}{2}\log_2 x$$

(6)

$$\text{i) } y = \log_5 8 = \underline{1.29}$$

$$\text{ii) } \log_2(x+15) - \log_2(\sqrt{x}) = 4$$

$$\log_2\left(\frac{x+15}{\sqrt{x}}\right) = 4 \Rightarrow \frac{x+15}{\sqrt{x}} = 2^4 = 16$$

$$x+15 = 16\sqrt{x}$$

$$x - 16\sqrt{x} + 15 = 0$$

$$(\sqrt{x} - 15)(\sqrt{x} - 1) = 0$$

$$\begin{aligned} \therefore \sqrt{x} &= 15 & \sqrt{x} &= 1 \\ x &= \underline{225} & x &= \underline{1} \end{aligned}$$

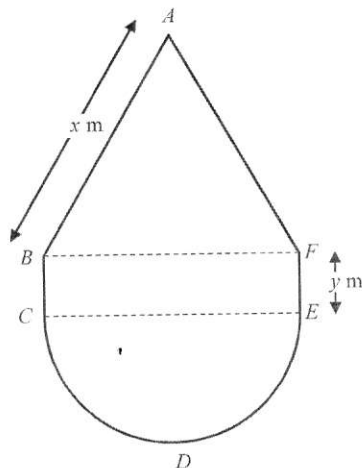


Figure 4

Figure 4 shows the plan of a pool.

The shape of the pool $ABCDEF$ consists of a rectangle $BCEF$ joined to an equilateral triangle BEA and a semi-circle CDE , as shown in Figure 4.

Given that $AB = x$ metres, $EF = y$ metres, and the area of the pool is 50 m^2 .

(a) show that

$$y = \frac{50}{x} - \frac{x}{8}(\pi + 2\sqrt{3}) \quad (3)$$

(b) Hence show that the perimeter, P metres, of the pool is given by

$$P = \frac{100}{x} + \frac{x}{4}(\pi + 8 - 2\sqrt{3}) \quad (3)$$

(c) Use calculus to find the minimum value of P , giving your answer to 3 significant figures. (5)

(d) Justify, by further differentiation, that the value of P that you have found is a minimum. (2)

a)

$$\text{area} = \frac{1}{2}(x)(x) \sin 60 = \frac{\sqrt{3}}{4}x^2$$

$$\text{area} = xy$$

$$\text{Area} = \frac{\pi(\frac{1}{2}x)^2}{2} = \frac{1}{8}\pi x^2$$

$$xy + \frac{1}{8}\pi x^2 + \frac{\sqrt{3}}{4}x^2 = 50$$

$$xy = 50 - \frac{1}{8}x^2(\pi + 2\sqrt{3})$$

$$\therefore y = \frac{50}{x} - \frac{1}{8}x(\pi + 2\sqrt{3})$$

b) $P = x + x + y + \frac{1}{2}\pi x + y$

$$P = 2y + (2 + \frac{1}{2}\pi)x$$

$$P = \frac{100}{x} - \frac{x}{4}(\pi + 2\sqrt{3}) + (2 + \frac{1}{2}\pi)x$$

$$P = \frac{100}{x} + \frac{x}{4}(8 + 2\pi - \pi - 2\sqrt{3})$$

$$P = \frac{100}{x} + \frac{x}{4}(\pi + 8 - 2\sqrt{3})$$

$$P = 100x^{-1} + \left(\frac{\pi + 8 - 2\sqrt{3}}{4} \right) x$$

$$P' = -100x^{-2} + \left(\frac{\pi + 8 - 2\sqrt{3}}{4} \right)$$

$$P'' = 200x^{-3}$$

$$\text{at Min } P' = 0 \Rightarrow \frac{100}{x^2} = \left(\frac{\pi + 8 - 2\sqrt{3}}{4} \right)$$

$$\Rightarrow x^2 = \frac{400}{\pi + 8 - 2\sqrt{3}} \quad \therefore x = 7.22$$

$$\therefore \text{Min } P = \frac{100}{7.22\dots} + \left(\frac{\pi + 8 - 2\sqrt{3}}{4} \right) (7.22\dots)$$

$$\text{Min } P = 27.7$$

$$d) \text{ at } x = 7.22 \quad P'' = \frac{200}{x^3} > 0$$


 $\therefore P$ is a minimum

10. The circle C , with centre A , passes through the point P with coordinates $(-9, 8)$ and the point Q with coordinates $(15, -10)$.

Given that PQ is a diameter of the circle C ,

- (a) find the coordinates of A .

(2)

- (b) find an equation for C .

(3)

A point R also lies on the circle C .

Given that the length of the chord PR is 20 units,

- (c) find the length of the shortest distance from A to the chord PR .

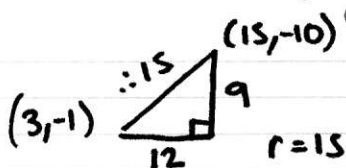
Give your answer as a surd in its simplest form.

(2)

- (d) Find the size of the angle ARQ , giving your answer to the nearest 0.1 of a degree.

(2)

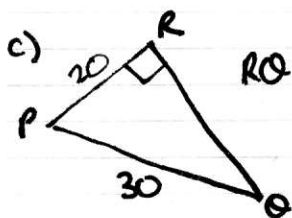
a) centre $A(3, -1)$



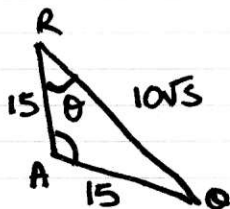
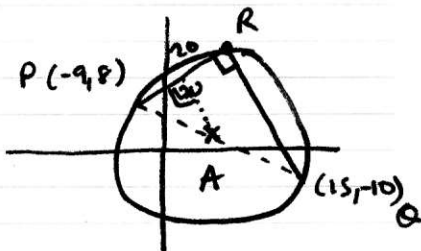
b) $(x-3)^2 + (y+1)^2 = 225$



$$\therefore \sqrt{15^2 - 10^2} = 5\sqrt{5}$$



$$RQ = \sqrt{30^2 - 20^2} = 10\sqrt{5}$$



$$\cos \theta = \frac{(10\sqrt{5})^2 + 15^2 - 30^2}{2 \times 15 \times 10\sqrt{5}} \quad \theta = 41.8$$