

GCE

Mathematics (MEI)

Unit **4752**: Concepts for Advanced Mathematics

Advanced Subsidiary GCE

Mark Scheme for June 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.



All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

Annotation in scoris	Meaning
 and 	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
MO M1	Method mark awarded 0, 1
AO A1	Accuracy mark awarded 0, 1
B0 B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

- a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance	
1	(i)	$kx^{\frac{1}{2}-1}$ or $kx^{-\frac{1}{2}}$ seen $3x^{\frac{1}{2}}$ or $\frac{3}{\sqrt{x}}$ isw	M1 A1 [2]	$k > 0$ A0 for eg $3x^{-\frac{1}{2}} + c$	B2 for correct answer unsupported
1	(ii)	kx^{-2+1} or kx^{-1} oe seen $-12x^{-1}$ or $-\frac{12}{x}$ or $\frac{-12}{x}$ isw $+ c$	M1 A1 A1 [3]	for any non-zero k seen at least once following integration	SC0 for $\frac{12}{2x}$ or $\frac{6}{x}$ A0 for $\frac{12}{-x}$ or $\frac{12x^{-1}}{-1}$ do not allow MR for integration of $12x^2$
2	(i)	(i) [5], 10, 5, [10] [10 + 5 + 10 =] 25 (ii) 0	M1 A1 B1 [3]	ignore extra terms not from wrong working	condone wrongly attributed terms B2 for 25 unsupported
3		$1.5 + (4 - 1)d = 12$ or better $d = 3.5$ $r = 2$ $1.5 \times \text{their } 2^9 - (1.5 + 9 \times \text{their } 3.5)$ oe difference = 735	M1 A1 B1 M1 A1 [5]	or $1.5 \times r^{(4-1)} = 12$ or better $r = 2$ $d = 3.5$ M0 for use of their S_{10} in either term	if first M0 B0 allow B3 for $d = 3.5$ and $r = 2$; B2 for one of these; may be embedded in calculation of difference NB 768 – 33 allow –735

Question	Answer	Marks	Guidance
4	$5.6^2 + 7.2^2 - 2 \times 5.6 \times 7.2 \times \cos 68$ seen 53 or 53.0 [BC =] 7.3 or 7.27 to 7.28 $\sin C = \frac{7.2 \times \sin 68}{\text{their } BC}$ 66 or awrt 66.5 <i>Alternatively</i> eg if the perpendicular from B to AC, BX, is used $7.2 \times \cos 68$ seen 2.7 or 2.697 to 2.70 $XC = 5.6 - \text{their } AX$ $\tan C = \left[\frac{BX}{XC} \right] = \frac{7.2 \times \sin 68}{\text{their } XC}$ 66 or awrt 66.5	M1 M1 A1 M1 A1 [5] M1* A1 M1dep* M1 A1 [5]	may be implied by 53 or BC in range may be implied by BC in range NB 7.27954... or $[\cos C] = \frac{\text{their } BC^2 + 5.6^2 - 7.2^2}{2 \times 5.6 \times \text{their } BC}$ allow 1.2 or awrt 1.16 (radians); A0 for eg 1.2 degrees [5] if unsupported, B2 for 2.70 or better NB 2.902832527 allow 1.2 or awrt 1.19 (radians); A0 for eg 1.2 degrees NB 52.9917243; (allow 47.7 to 47.71 from calculator in radian mode; may be implied by 6.90 to 6.91) NB $\sin C = 0.917053...$ $\cos C = 0.398766...$ eg if perpendicular from C to AB, CY, is used, mark as follows $5.6 \times \cos 68$ seen 2.1 or 2.097 to 2.10 $BY = 7.2 - \text{their } AY$ $\tan B = \left[\frac{CY}{BY} \right] = \frac{5.6 \times \sin 68}{\text{their } BY}$ $C [= 90 - B] = 66$ or awrt 66.5

Question		Answer	Marks	Guidance
5	(i)	$\sin kx$ $y = \sin 2x$	M1 A1 [2]	$k > 0$ and $k \neq 1$ must see “y =” at some stage for A1 condone use of other variable condone $f(x) = \sin 2x$
5	(ii)	sketch of sine curve with period 360° and amplitude 1 sine curve centred on $y = -3$ and starting at $(0, -3)$	B1 B1 [2]	for $0 \leq x \leq 450$; ignore curve outside this range; do not allow sketch of $y = \cos x$ or $y = -\sin x$ for either mark amplitude, period and centring on $y = -3$ must be clear from correct numerical scale, numerical labelling or comment; strokes on axes insufficient to imply scale: mark intent allow full marks if $y = \sin x$ and $y = \sin x - 3$ seen on same diagram
6	(i)	$\frac{1}{2}r^2\theta$ or $\frac{1}{2}a^2 \sin \theta$ or $a^2 \sin \frac{1}{2}\theta \cos \frac{1}{2}\theta$ seen $\frac{1}{2}r^2\theta - \frac{1}{2}a^2 \sin \theta$ isw oe	M1 A1 [2]	do not allow use of variable other than θ allow eg $\frac{\theta}{2\pi} \times \pi r^2$ or $\frac{1}{2}a^2 \sin\left(\frac{180\theta}{\pi}\right)$ seen oe
6	(ii)	$\frac{1}{2}a^2 \sin 0.8 = \frac{1}{2} \times 12^2 \times 0.8 - \frac{1}{2}a^2 \sin 0.8$ oe $[a =] 8.96$ cao; mark the final answer	B1 B1 [2]	or eg $\frac{1}{2}a^2 \sin 0.8 = \frac{1}{4} \times 12^2 \times 0.8 [= 28.8]$ or equivalent in degrees NB $\theta = 45.8366236\dots^\circ$ if unsupported, allow B2 for 8.96 or allow B1 for 9.0 or 8.96074...to 4 sf or more NB $a^2 = \frac{57.6}{0.717356} = 80.29485$ NB $\theta = 45.83662361\dots^\circ$ NB $\frac{1}{2} \sin 0.8 = 0.35867\dots$

Question	Answer	Marks	Guidance
7 (i)	substitution of $\tan x = \frac{\sin x}{\cos x}$ or $\sqrt{1 - \sin^2 x} = \sqrt{\cos^2 x}$ or $\cos x$ in given LHS both substitutions seen and completion to $\sin x$ as final answer	M1 A1 [2]	if no substitution, statements must follow a logical order and the argument must be clear; if one substitution made correctly, condone error in other part of LHS NB AG ; answer must be stated allow consistent use of other variable eg θ for both marks condone omission of variable throughout for M1 only, but allow recovery from omission of variable at end M0 if first move is to square one or both sides Simply stating eg $\tan x = \frac{\sin x}{\cos x}$ is insufficient <i>Alternatively</i> SC2 for complete argument eg $\tan x = \frac{\sin x}{\cos x}$ $[\tan x \times \cos x = \sin x]$ $\sin^2 x + \cos^2 x = 1$ $\cos x = \sqrt{1 - \sin^2 x}$ $\tan x = \frac{\sin x}{\sqrt{1 - \sin^2 x}}$ $\tan x \times \sqrt{1 - \sin^2 x} = \sin x \quad \text{oe}$
7 (ii)	0, 180, 360 14 or 14.47 to 14.5 166 or awrt 165.5	B1 B1 B1 [3]	all 3 required radians: mark as scheme but deduct one from total 0, π , 2π ; 0.25 or 0.253 or awrt 0.2527; 2.89 or 2.889 or awrt 2.8889 NB $\sin y = 0$ or $\frac{1}{4}$ ignore extra values outside range if B3 , deduct 1 mark for extra values within range

Question		Answer	Marks	Guidance
8	(i)	$\log_a 1 = 0$ soi or $3m \log_a a$ or $\log_a a^{-3m}$ seen $-3m$ cao	M1 A1 [2]	do not condone $3m \log a$ do not allow MR for $(\log_a a^m)^3$
8	(ii)	$(2x+1) \log_3 3 = \log_3 1000$ or $2x+1 = \log_3 1000$ oe $[x =] \frac{\log_3 1000 - 1}{2}$ oe 2.64 cao; mark the final answer	M1 M1 A1 [3]	Or $(2x+1) \log_{10} 3 = \log_{10} 1000$ [= 3] $\frac{3}{\log_{10} 3} - 1$ or $[x =] \frac{\log_{10} 3}{2}$ oe not from wrong working condone omission of brackets; allow omission of base 10 or consistent use of other base allow one sign error and / or omission of brackets allow recovery from bracket error for A1 0 if unsupported or for answer obtained by trial and error on $3^{2x+1} = 1000$
9	(i)	$\frac{h}{2} \times (0 + 0 + 2[4 + 4.9 + 5 + 4.9 + 4])$ oe all non-zero y-values correctly placed $h = 1$ used in formula or consistently with two triangles and four trapezia area = 22.8 and volume = 1140 isw cao	M1 M1 B1 A1 [4]	correct formula used with 4, 5 or 6 strips and numerical value for h ; condone omission of zeros or omission of outer brackets for both M marks M0M0 if 1, 2, 3 or 6 used as y-values (these are x-values) if M0M0 allow B1 for $h = 1$ and B2 for 22.8 from area of 4 trapezia and 2 triangles and B1 for 1140 ignore units allow eg $\frac{1}{2} \times 1 \times (4 + 4 + 2[4.9 + 5 + 4.9])$ $\frac{1}{2} \times 1 \times (4 + 0 + 2[4 + 4.9 + 5 + 4.9])$ (NB may be implied by 18.8 & 20.8 respectively) if M0M0B0 allow SC4 for 22.8 and 1140 obtained correctly by other method

Question	Answer	Marks	Guidance
9 (ii)	A substitution of $x = 1.2$ or 4.8 to find y $y = 4.35$ or 4.352 and correct comparison with 4.4 isw	M1 A1 [2]	allow substitution of $1.2 \leq x \leq 1.234$ or $4.766 \leq x \leq 4.8$ or M1 for $y = 4.4, x = 1.234$ [or 4.766] and A1 for comparison of 1.234 with 1.2 or 4.766 with 4.8 [so gap less than 3.6]
9 (ii)	B $F[x] = \frac{5}{81} \left(\frac{108}{2}x^2 - \frac{54}{3}x^3 + \frac{12}{4}x^4 - \frac{x^5}{5} \right)$ oe eg $\frac{10}{3}x^2 - \frac{10}{9}x^3 + \frac{5}{27}x^4 - \frac{1}{81}x^5$ $F[6] - F[0]$ or $2 \times (F[3] - F[0])$ 24 1200	M2 M1 A1 B1 [5]	M1 for 3 correct terms; ignore $+c$ allow coefficients $3.333333\dots, 1.11111\dots, 0.185185\dots, 0.01234567\dots$ r.o.t to 2 sf or better or decimal equivalents in numerator: $6.6666\dots, 3.333333\dots, 0.74074\dots, 0.061728\dots$ r.o.t to 2 sf or better dependent on at least two terms correctly integrated in bracket; condone omission of $-F(0)$ condone omission of $\frac{5}{81}$; M0 if $\frac{5}{81}x$ seen outside bracket but next M1 is still available; ignore subsequent attempt to evaluate c for first M2 M0 for non-zero lower limit 24 unsupported does not score ignore units
10 (i)	$\frac{(5.1^2 - 10.2) - (5^2 - 10)}{5.1 - 5}$ oe 8.1	M1 A1 [2]	condone omission of brackets 0 for 8.1 unsupported

Question	Answer	Marks	Guidance	
10 (ii)	$\frac{(5+h)^2 - 2(5+h) - \text{their } 15}{h}$ oe $25 + 10h + h^2 - 10 - 2h$ oe seen numerator is $8h + h^2$ $8 + h$ isw	M1 M1 A1 A1 [4]	condone omission of brackets allow one sign error	
10 (iii)	$h \rightarrow 0$ their 8	M1 A1 [2]	may be embedded; allow eg “tends to 0” FT their $k + h$ from part (ii)	M0 for differentiation of $x^2 - 2x$ M0 for following from part (i) M0 for $h = 0$
10 (iv)	$y = 8x - 25$ isw non-zero numerical value for x -intercept on their straight line found $[x =] 3.125$ oe $\frac{1}{2} \times$ their non-zero y -intercept \times their $\frac{25}{8}$ $\frac{625}{16}$ or $39\frac{1}{16}$ or 39.0625 isw	B1 M1 A1 M1 A1 [5]	or $y - 15 = 8(x - 5)$ isw or $y = 8x + c$ and $c = -25$ stated isw may be embedded in calculation for area condone arithmetic slips in finding values of intercepts accept rounded to 1 dp or better for A1 ; but A0 if final answer negative	or integration and evaluation of their $\int_0^{\frac{25}{8}} (8x - 25) dx$; lower limit must be 0

Question		Answer	Marks	Guidance	
11	(i)	$\log_{10} y = \log_{10} a + bt$ www gradient is b , intercept is $\log_{10} a$ cao	B1 B2 [3]	B0 for just $\log_{10} y = \log_{10} a + bt \log_{10} 10$ allow omission of base throughout question ignore t -intercept is $\frac{-\log_{10} a}{b}$ B0 for gradient is bt	
11	(ii)	1.58, 1.8[0], 1.98, 2.37, 2.68 all values correct and all plotted accurately ruled line of best fit for at least $1 \leq t \leq 10$ evaluation of $\frac{\log y_2 - \log y_1}{t_2 - t_1}$ or substitution of $(t_1, \log y_1)$ and $(t_2, \log y_2)$ in $\log y = bt + \log a$ to obtain a numerical value for the gradient $0.14 \leq b \leq 0.24$ $2.5 \leq a \leq 6.3$ $y = \text{their } a \times 10^{\text{their } b \times t}$ or $y = 10^{\text{their } bt + \text{their } \log a}$ or $10^{\text{their } \log a} \times 10^{\text{their } b \times t}$ oe a and b or $\log a$ and b both in acceptable range	B1 B1 B1 M1 A1 B1 M1 A1 [8]	allow values which round to these numbers to 2 dp; within tolerance on overlay; within tolerance on overlay: must not cut red or green line; line between (1, 0.6) and (1, 1.05) at lower limit and between (10, 2.3) and (10, 2.75) at upper limit; $(t_1, \log y_1)$ and $(t_2, \log y_2)$ are points on their line gradient must be identified as b for A1 must be identified as a ; not from wrong working $0.4 \leq \log a \leq 0.8$	all values must be correct use ruler tool to check if line is ruled where necessary; tolerance: one small square horizontally at each end; not dependent on correct plots condone use of values from table if M0A0B0M0 allow SC3 for substitution directly into given formula to obtain $y = a10^{bt}$ with a and b in acceptable range
11	(iii)	260 or 261	B1 [1]	B0 for non-integer answer	

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning

Telephone: 01223 553998

Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

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Head office
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Facsimile: 01223 552553

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