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1. Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(3 - \frac{1}{3}x\right)^5$$

giving each term in its simplest form.

(4)

$3^5 - 5(3)^4\left(\frac{1}{3}x\right) + 10(3)^3\left(\frac{1}{3}x\right)^2 - 10(3)^2\left(\frac{1}{3}x\right)^3$

$= 243 - 135x + 30x^2 - \frac{10}{3}x^3$

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2. In the triangle ABC, $AB = 16$ cm, $AC = 13$ cm, angle $ABC = 50°$ and angle $BCA = x°$

Find the two possible values for x, giving your answers to one decimal place.

(4)

$$\frac{\sin x}{16} = \frac{\sin 50°}{13}$$

$$\sin x = \frac{16}{13} \sin 50°$$

$$= 0.9428$$

$$x = 70.5°, \ 109.5°$$

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3. (a) $$y = 5^x + \log_2(x + 1), \qquad 0 \leqslant x \leqslant 2$$

Complete the table below, by giving the value of y when $x = 1$

x	0	0.5	1	1.5	2
y	1	2.821	6	12.502	26.585

(1)

(b) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for

$$\int_0^2 (5^x + \log_2(x + 1))\,dx$$

giving your answer to 2 decimal places.

(4)

(c) Use your answer to part (b) to find an approximate value for

$$\int_0^2 (5 + 5^x + \log_2(x + 1))\,dx$$

giving your answer to 2 decimal places.

(1)

(b) $A = \frac{1}{2} \times 0.5(1 + 26.585 + 2(2.821 + 6 + 12.502))$

$= 17.56$

(c) $17.56 + 5 \times 2 = 27.56$

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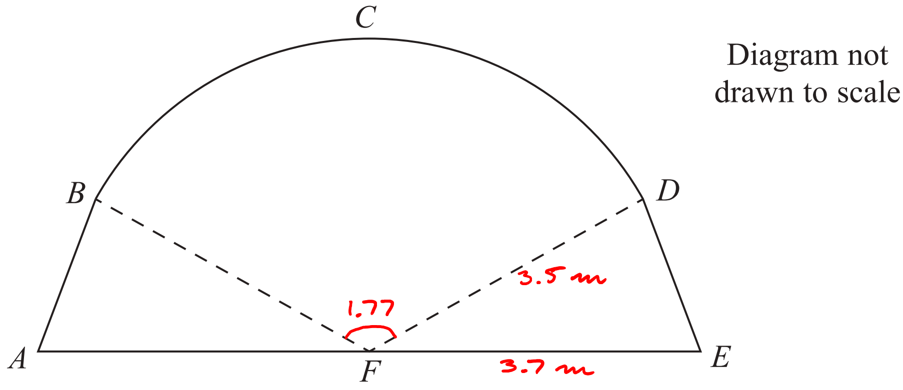

Figure 1

Figure 1 is a sketch representing the cross-section of a large tent *ABCDEF*.
AB and *DE* are line segments of equal length.
Angle *FAB* and angle *DEF* are equal.
F is the midpoint of the straight line *AE* and *FC* is perpendicular to *AE*.
BCD is an arc of a circle of radius 3.5 m with centre at *F*.
It is given that

$$AF = FE = 3.7\text{m}$$

$$BF = FD = 3.5\text{m}$$

$$\text{angle } BFD = 1.77 \text{ radians}$$

Find

(a) the length of the arc *BCD* in metres to 2 decimal places,
(2)

(b) the area of the sector *FBCD* in m^2 to 2 decimal places,
(2)

(c) the total area of the cross-section of the tent in m^2 to 2 decimal places.
(4)

(a) $l = r\theta$

$= 3.5 \times 1.77$

$= 6.20$ m

(b) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 3.5^2 \times 1.77$

$= 10.84$ m^2

(c) $\angle DFE = \frac{1}{2}(\pi - 1.77)$

$= 0.6858$

$\therefore A = 2 \times \frac{1}{2} \times 3.5 \times 3.7 \times \sin 0.6858 + 10.84$

$= 8.201 + 10.84$

$= 19.04$ m^2

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5. The circle C has equation

$$x^2 + y^2 - 10x + 6y + 30 = 0$$

Find

(a) the coordinates of the centre of C,

(2)

(b) the radius of C,

(2)

(c) the y coordinates of the points where the circle C crosses the line with equation $x = 4$, giving your answers as simplified surds.

(3)

(a) $x^2 - 10x + 25 + y^2 + 6y + 9 = -30 + 25 + 9$

$(x-5)^2 + (y+3)^2 = 4$

$\therefore C(5, -3)$

(b) $r = 2$

(c) $(4-5)^2 + (y+3)^2 = 4$

$(y+3)^2 = 3$

$y = -3 \pm \sqrt{3}$

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6. $$f(x) = -6x^3 - 7x^2 + 40x + 21$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$ **(2)**

(b) Factorise $f(x)$ completely. **(4)**

(c) Hence solve the equation

$$6(2^{3y}) + 7(2^{2y}) = 40(2^y) + 21$$

giving your answer to 2 decimal places. **(3)**

(a) $f(-3) = -6(-3)^3 - 7(-3)^2 + 40(-3) + 21$

$= 0$

$\therefore x + 3$ is a factor

(b) $-6x^3 - 7x^2 + 40x + 21 = (x+3)(-6x^2 + 11x + 7)$

$= -(x+3)(6x^2 - 11x - 7)$

$= -(x+3)(6x^2 - 14x + 3x - 7)$

$= -(x+3)(2x(3x-7) + 3x - 7)$

$= -(x+3)(3x-7)(2x+1)$

$= (x+3)(7-3x)(2x+1)$

(c) $2^y = \frac{7}{3}$ $\quad$ (-3 & $-\frac{1}{2}$ are extraneous)

$y = \log_2 \frac{7}{3}$

$= 1.22$

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7. (i) $$2\log(x + a) = \log(16a^6), \text{ where } a \text{ is a positive constant}$$

Find x in terms of a, giving your answer in its simplest form.

(3)

(ii) $$\log_3(9y + b) - \log_3(2y - b) = 2, \text{ where } b \text{ is a positive constant}$$

Find y in terms of b, giving your answer in its simplest form.

(4)

(i) $2\log(x+a) = 2\log(4a^3)$

$x + a = 4a^3$

$x = 4a^3 - a$

$= a(2a+1)(2a-1)$

(ii) $\log_3(9y+b) - \log_3(2y-b) = 2$

$\frac{9y+b}{2y-b} = 3^2$

$9y + b = 9(2y-b)$

$= 18y - 9b$

$10b = 9y$

$y = \frac{10}{9}b$

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8. (a) Show that the equation

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

can be written in the form

$$(3\sin x - 1)^2 = 2$$

(3)

(b) Hence solve, for $0 \leqslant x < 360°$,

$$\cos^2 x = 8\sin^2 x - 6\sin x$$

giving your answers to 2 decimal places.

(5)

(a) $1 - \sin^2 x = 8\sin^2 x - 6\sin x$

$2 = 9\sin^2 x - 6\sin x + 1$

$2 = (3\sin x - 1)^2$, QED.

(b) $3\sin x - 1 = \pm\sqrt{2}$

$\sin x = \frac{1 \pm \sqrt{2}}{3}$

$x = 53.58°, 126.42°, 187.94°, 352.06°$

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9. The first three terms of a geometric sequence are

$$7k - 5, \; 5k - 7, \; 2k + 10$$

where k is a constant.

(a) Show that $11k^2 - 130k + 99 = 0$

(4)

Given that k is not an integer,

(b) show that $k = \dfrac{9}{11}$

(2)

For this value of k,

(c) (i) evaluate the fourth term of the sequence, giving your answer as an exact fraction,

(ii) evaluate the sum of the first ten terms of the sequence.

(6)

(a) $\dfrac{5k-7}{7k-5} = \dfrac{2k+10}{5k-7}$

$(5k-7)^2 = (2k+10)(7k-5)$

$25k^2 - 70k + 49 = 14k^2 + 60k - 50$

$11k^2 - 130k + 99 = 0$, QED.

(b) $11k^2 - 121k - 9k + 99 = 0$

$11k(k-11) - 9(k-11) = 0$

$(k-11)(11k-9) = 0$

$\therefore k = \frac{9}{11}$, QED.

(c) First term $= 7\left(\frac{9}{11}\right) - 5 = \frac{8}{11}$ Second term $= 5\left(\frac{9}{11}\right) - 7 = -\frac{32}{11}$

$r = -\frac{32}{8} = -4$ Third term $= -\frac{32}{11} \times -4 = \frac{128}{11}$

$\therefore$ Fourth term $= \frac{128}{11} \times -4 = -\frac{512}{11}$

(ii) $S_n = \dfrac{a(r^n - 1)}{r - 1}$

$S_{10} = \dfrac{\frac{8}{11}((-4)^{10} - 1)}{-4 - 1}$

$= -152\,520$

10.

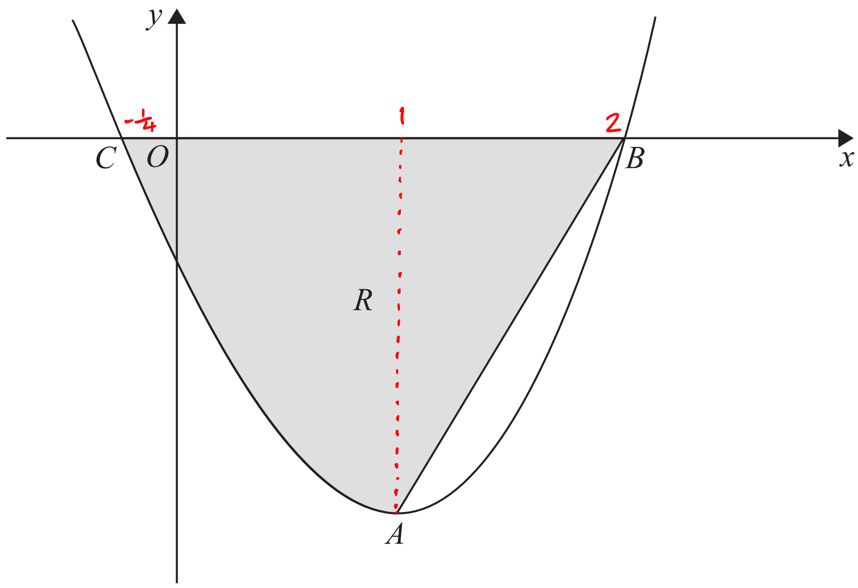

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \quad -0.5 \leqslant x \leqslant 2.2$$

The curve has a turning point at the point A.

(a) Using calculus, show that the x coordinate of A is 1

(3)

The curve crosses the x-axis at the points $B(2, 0)$ and $C\left(-\frac{1}{4}, 0\right)$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line AB, and the x-axis.

(b) Use integration to find the area of the finite region R, giving your answer to 2 decimal places.

(7)

(a) $\frac{dy}{dx} = 12x^2 + 18x - 30$

$0 = 2x^2 + 3x - 5$

$= 2x^2 - 2x + 5x - 5$

$= 2x(x-1) + 5(x-1)$

$= (x-1)(2x+5)$

$x = -\frac{5}{2}, 1$

$\therefore x = 1$, QED.

$y(1) = 4 + 9 - 30 - 8$

$= -25$ $\therefore A(1, -25)$

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Question 10 continued

(b) $\int_{-\frac{1}{4}}^{1} (4x^3 + 9x^2 - 30x - 8)\, dx$

$= \left[x^4 + 3x^3 - 15x^2 - 8x \right]_{-\frac{1}{4}}^{1}$

$= (1 + 3 - 15 - 8) - \left((-\tfrac{1}{4})^4 + 3(-\tfrac{1}{4})^3 - 15(-\tfrac{1}{4})^2 - 8(-\tfrac{1}{4}) \right)$

$= -19 - \frac{261}{256}$

$= -\frac{5125}{256}$

Area of triangle $= \frac{1}{2} \times 1 \times 25$

$= \frac{25}{2}$

$\therefore$ Area of $R = \frac{5125}{256} + \frac{25}{2}$

$= \frac{8325}{256}$

$= 32.52$