07 May 2011
22:33

M1 November 03 Model Solutions

① a)

$\uparrow a = -9.8$

$u = u$, $v = 0$, $a = -9.8$, $s = 40$, $t = \backslash$

$v^2 = u^2 + 2as$

$0 = u^2 + 2 \times -9.8 \times 40$

$\Rightarrow u^2 = 784$

$u = 28\ ms^{-1}$

b) From start to finish:

$u = 28$, $v = \backslash$, $a = -9.8$, $s = 0$, $t = t$

$s = ut + \frac{1}{2}at^2$

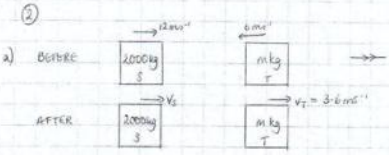
$0 = 28t - 4.9t^2$

$t(4.9t - 28) = 0$

$t = 0$ or $t = \frac{28}{4.9} = 5.71\ s$

$t = 0$ at start; $t = 5.71\ s$ at finish

②



Impulse of T on S is in negative direction:

$I = 2000v_S - 2000 \times 12 = -28800$

$2000v_S - 24000 = -28800$

$2000v_S = -4800$

$v_S = -2.4ms^{-1}$

Speed of S = $2.4ms^{-1}$

b) Direction of S is WEST.

c) Conservation of momentum: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

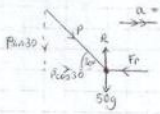
$\Rightarrow 2000 \times 12 - 6m = -2.4 \times 2000 + 3.6m$

$\Rightarrow 24000 - 6m = 3.6m - 4800$

$\Rightarrow 28800 = 9.6m$

$\Rightarrow m = 3000\ kg$

③



$\mu = \frac{3}{5}$

$Fr = \mu R$

$Fr = \frac{3}{5}R$

Vertically: $R = P\sin 30 + 50g$

$Fr = \frac{3}{5}R = \frac{3}{5}(P\sin 30 + 50g)$

$\therefore Fr = \frac{3P}{10} + 30g$ —①

Horizontally: $RF = ma$

$P\cos 30 - Fr = 50 \times 0$

$\Rightarrow P\cos 30 = Fr$ —②

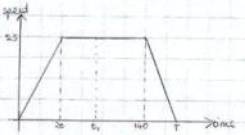
Let ① = ②: $P\cos 30 = \frac{3P}{10} + 30g$

$(\times 10)$ $10P\cos 30 = 3P + 300g$

$P(10\cos 30 - 3) = 300g$

$P = \frac{300g}{10\cos 30 - 3} = 519.4 \approx 519\ N$

④

a) 

b) Distance = area under graph = 4 km.

$\frac{1}{2} \times 25 \times (T + 120) = 4000$

$T + 120 = 320$

$T = 200\ s$

c) When the car has travelled 1.5 km (point t_1 on the graph):

$\frac{1}{2}(25)(t_1 + (t_1 - 20)) = \frac{1}{2} \times 25 \times (2t_1 - 20) = 1500$

$\Rightarrow 2t_1 - 20 = 120$

$t_1 = 70s$.

The motorbike set of 10s later so has been travelling $70 - 10 = 60\ s$

d) $u = 0$, $v = v$, $a = \backslash$, $s = 1500$, $t = 60$

$s = \left(\frac{u+v}{2}\right)t$

$1500 = \left(\frac{0+v}{2}\right) \times 60$

$25 = \frac{v}{2} \Rightarrow v = 50\ ms^{-1}$

⑤

a) $\underset{\sim}{u} = 3\underset{\sim}{i} - 5\underset{\sim}{j}$, $\underset{\sim}{v} = -5\underset{\sim}{i} + 11\underset{\sim}{j}$, $\underset{\sim}{a} = \underset{\sim}{a}$, $\underset{\sim}{s} = \backslash$, $t = 4$

$\underset{\sim}{v} = \underset{\sim}{u} + \underset{\sim}{a}t \Rightarrow \underset{\sim}{a} = \frac{\underset{\sim}{v} - \underset{\sim}{u}}{t}$

$= \frac{(-5\underset{\sim}{i} + 11\underset{\sim}{j}) - (3\underset{\sim}{i} - 5\underset{\sim}{j})}{4}$

$= \frac{-8\underset{\sim}{i} + 16\underset{\sim}{j}}{4} = -2\underset{\sim}{i} + 4\underset{\sim}{j}\ ms^{-2}$

b) $\underset{\sim}{F} = m\underset{\sim}{a}$

$= 3 \times (-2\underset{\sim}{i} + 4\underset{\sim}{j}) = -6\underset{\sim}{i} + 12\underset{\sim}{j}\ N$

$|F| = \sqrt{6^2 + 12^2} = \sqrt{180} = 13.4\ N$

c) When $t = 6$, what is $\underset{\sim}{r}$?

$\underset{\sim}{r} = (3\underset{\sim}{i} - 5\underset{\sim}{j}) + (-2\underset{\sim}{i} + 4\underset{\sim}{j}) \times 6$

$= -9\underset{\sim}{i} + 19\underset{\sim}{j}$

3s after F is removed: displacement = velocity x time

$= (-9\underset{\sim}{i} + 19\underset{\sim}{j}) \times 3$

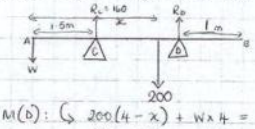
$= -21\underset{\sim}{i} + 57\underset{\sim}{j}$

Position vector of P at this point $= (6\underset{\sim}{i} - 29\underset{\sim}{j}) + (-27\underset{\sim}{i} + 57\underset{\sim}{j})$

$\underset{\sim}{k} = -21\underset{\sim}{i} + 28\underset{\sim}{j}$

Distance OB $= |\underset{\sim}{k}| = \sqrt{21^2 + 28^2} = 35\ m$

⑥



a) $M(D)$: $\circlearrowleft 200(4 - x) + W \times 4 = 160 \times 2.5$ ②

$800 - 200x + 4W = 400$

$200x - 4W = 400$

$50x - W = 100$ QED

b) W moved to B: $M(D)$ $\circlearrowleft 200(4 - x) = 50 \times 2.5 + W \times 1$ ②

$(R_C = 50)$ $800 - 200x = 125 + W$

$200x + W = 675$

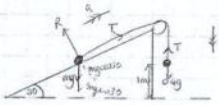
c) $50x - W = 100 \Rightarrow W = 50x - 100$

$200x + W = 675 \Rightarrow 200x + 50x - 100 = 675$

$\Rightarrow x = \frac{775}{250} = 3.1\ m$

$W = 50x - 100 = 55N$

⑦



$\uparrow a$

a) $a = \frac{1}{5}g$ $RF = ma \Rightarrow 0.4g - T = 0.4a$

$\Rightarrow 0.4g - T = 0.4 \times \frac{1}{5}g$

$\Rightarrow 0.4g - T = 0.08g$

b) $T = 0.4g - 0.08g = 0.32g = 3.136\ N$

c) (A): $T - mg\sin 30 = m \times \frac{1}{5}g$

$3.136 - mg\sin 30 = \frac{mg}{5}$

$\Rightarrow 3.136 = m\left(g\sin 30 + \frac{g}{5}\right)$

$\Rightarrow m = \frac{3.136}{g\sin 30 + \frac{g}{5}} = \frac{1}{3.5}$ QED

d) No friction on the pulley.
Tension is equal throughout the string.

e) $u = 0$, $v = v$, $a = \frac{1}{5}g$, $s = 1$, $t = \backslash$

$v^2 = u^2 + 2as$

$= 0^2 + 2 \times \frac{1}{5}g \times 1$

$= \frac{2}{5}g$

$\Rightarrow v = 1.98\ ms^{-1}$

f) Tension is removed after 1m of movement.
To calculate the new acceleration: $-mg\sin 30 = ma$ $(m = \frac{1}{3.5})$ $a = -\frac{1}{2}g$

How far until particle A comes to rest?

$u = 1.98$, $v = 0$, $a = -\frac{1}{2}g$, $s = s$, $t = \backslash$

$v^2 = u^2 + 2as$

$0 = 1.98^2 + 2 \times -\frac{1}{2}gs$

$\Rightarrow gs = 1.98^2$

$\Rightarrow s = \frac{1.98^2}{g} = \frac{2}{5} = 0.4\ m.$

So after a further 0.4m A comes to rest. This is at the pulley, 1.4m from the starting position.