Name _____ **Class** _____

_____ **Date** _____

Materials

For this paper you must have:

- The booklet of formulae and statistical tables
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen.
  Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use a supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily have to use all the space provided.

Question	Mark
1	
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16	
Total	

Answer ALL questions. Write your answer in the spaces provided.

1 Find the equation of the line perpendicular to $2x - y = 4$ which passes through the point (4, 7) **[3 marks]**

2 Fully describe the transformation that maps the curve with equation $y = x^2 - 3$ onto the curve with equation $y = 2x^2 - 6$ **[3 marks]**

3 a Given that $2^x = 8^{3x-1}$, work out the value of x **[3 marks]**

b Write $\log x + \log x^2 - \log \sqrt{x}$ as a single logarithm in terms of $\log x$ **[3 marks]**

4 An engineer is making a circular disc.

A model of the disc is drawn as a circle on a set of axes such that it passes through coordinate points $P(-1,\ -1)$, $Q(5,\ k)$ and $R(7,\ 3)$

Given that angle *PRQ* is 90°, find the value of *k* and the area of the disc. **[5 marks]**

Turn over for the next question

5 The curve C has equation $y = x^2 - 3x + 2$

Find the equation of the tangent to C at $x = 2$ **[5 marks]**

Turn over for the next question

6 Solve the equation

$$\sin^2 x - 3\cos x + 2 = 0$$

Give **all** solutions in degrees, correct to 1 decimal place, in the interval $0 \leq x \leq 360°$

Fully justify your answer. **[6 marks]**

Turn over for the next question

7 a Find the first four terms of the binomial expansion of $(1 + 2x)^8$ in ascending powers of x
Give each term in its simplest form. **[4 marks]**

b Hence calculate an approximate value for 1.02^8 **[2 marks]**

8 Point A has position vector $6\mathbf{i} + 5\mathbf{j}$. Point B has position vector $p\mathbf{i} + q\mathbf{j}$

a Given that vector $\overrightarrow{AB} = 14\mathbf{i} - 7\mathbf{j}$, find the values of p and q **[2 marks]**

b Work out the magnitude and direction of $\overrightarrow{AB}$ **[4 marks]**

9 Prove that $\frac{1-\cos\theta}{\sin\theta} \equiv \frac{\sin\theta}{1+\cos\theta}$ **[4 marks]**

10 Determine the range of values of k for the equation $x^2 - 4kx + 2k = 0$ to have two distinct real roots. **[4 marks]**

11 **a** Sketch, in the interval $-180° \leq x \leq 180°$, the graph of $f(x) = 0.5 + \sin x$

Show clearly the coordinates of the maximum and minimum points, and the points where the graph crosses the axes. **[2 marks]**

b Hence state, with a reason, the number of solutions of the equation $0.5 + \sin x = 1$ in the interval $-180° \leq x \leq 180°$. **[2 marks]**

Turn over for the next question

12 $f(x) = x^3 - 2x^2 - 5x + 6$

a Show that $(x - 1)$ is a factor of f(x) and find the other two roots of the equation f(x) = 0 **[5 marks]**

b Sketch the curve $y =$ f(x), showing clearly the points where the curve cuts the axes. **[2 marks]**

c Show that the x-coordinates of the turning points on the curve $y = f(x)$ can be written in the form $\frac{a \pm \sqrt{b}}{c}$, where a and b and c are integers to be found and b is the smallest possible integer.

[5 marks]

Turn over for the next question

13 **a** The derivative, $f'(x)$, of a function $f(x)$ is given by

$$f'(x) = x^{\frac{1}{2}} - 3x^2 + 1, \quad x \geq 0$$

Find the function $f(x)$ **[4 marks]**

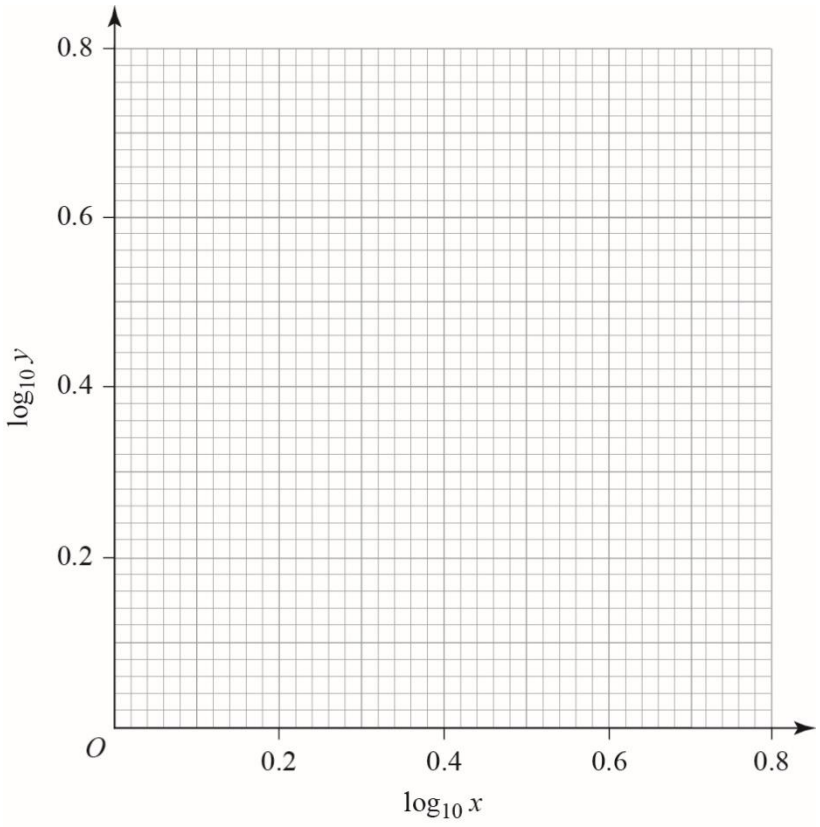
b Given that the points $(1,\ 2)$ and $(4,\ q)$ lie on the curve $y = f(x)$, work out the value of q **[4 marks]**

14 An engineer conducts a test and records the following data for two variables, x and y

x	1	2	3	4	5
y	3	3.346	3.737	3.959	4.139
$\log_{10} x$					
$\log_{10} y$					

The engineer deduces that the relationship between x and y can be modelled by an equation of the form $y = ax^b$

a By plotting values of $\log_{10} y$ against $\log_{10} x$, state, with a reason, which value of y is likely to have been recorded incorrectly. **[3 marks]**



b Draw an appropriate straight line on your graph in part **a** to find the values of a and b, giving your answers to two significant figures. Hence express y in terms of x **[4 marks]**

Turn over for the next question

15 A botanical garden is modelled as a quadrilateral *ABCD* shown below.

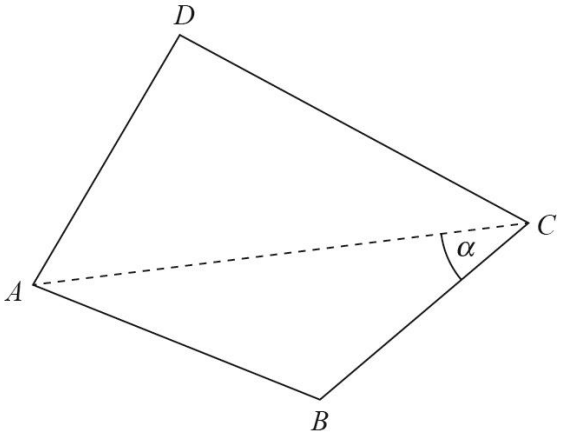


Diagram not to scale

Angle *ADC* = 80°
AD = 125 m, *BC* = 110 m and *CD* = 158 m.
Angle *BAC* = 31°

Find, correct to the nearest square metre, the area of the garden. **[8 marks]**

16 A kite has sides of length 5 cm and 9 cm

Given that the area of the kite is 20 cm^2, and that the largest interior angle is α, find the exact value of $\cos\alpha$

[5 marks]

Turn over for the next question

17 Given that n is an odd number, prove that $n^2 + n$ is even. **[3 marks]**

18 Philomena is solving a mathematics problem set by her teacher. Her working is shown below:

Find all the solutions to the equation $x^2 - 3x = 0$

Solution:

Add $3x$ to both sides: $x^2 = 3x$

Cancel x from both sides: $x = 3$

Philomena has made a mistake and has not found all of the solutions. Identify her mistake and find all the solutions to the equation. **[2 marks]**

19 Prove that the function $f(x) = 2 - 5x + 2x^2 - x^3$ is a decreasing function.

[6 marks]

End of questions