

P1-P3 Mark Schemes

There aren't separate files for the mark schemes of relevant questions for C1-C4. This file contains all the mark schemes for the P1-P3 papers. They are in chronological order. You also use the bookmarks to navigate through or use the search function.

EDEXCEL FOUNDATION - LONDON EXAMINATIONS

Stewart House 32 Russell Square London WC1B 5DN

January 2001

Advanced Supplementary/Advanced Level

General Certificate of Education

Subject **PURE MATHEMATICS 6671**

Paper No. **P1**

Question number	Scheme	Marks
1. (a)	$8 + 4\sqrt{7} - 2\sqrt{7} - 7 = 1 + 2\sqrt{7}$	M1 A1 (2)
(b)	$\frac{2+\sqrt{7}}{4+\sqrt{7}} \times \frac{4-\sqrt{7}}{4-\sqrt{7}} = \frac{1+2\sqrt{7}}{16-7}$	M1 A1✓
	$c = \frac{1}{9} \quad d = \frac{2}{9}$	A1 (3) (5)
2. (a)	$(x+k)^2, -k^2 + c (=0)$	M1, A1
	$(x+k)^2 = k^2 - c \quad x = -k \pm \sqrt{(k^2 - c)} . *$	M1 A1 c.s.o. (4)
(b)	(Discriminant = 0, $k^2 = 81$) $k = 9, \text{ or } -9$	B1, B1 (2) (6)
3. (a)	$(\theta + 75 - 60), 300, 420$ $\theta = -15^\circ, \theta = 345^\circ$ One of these... $\theta + 75 = 360 - "60"$ <u>$\theta = 225, 345$</u>	B1 M1 A1 (3)
(b)	$(2\theta) = 44.4$	B1
	$(2\theta) = 135.6$ One more soln.	B1✓
	$(2\theta) = 404.4, 495.6$ Other 2 in range	B1✓
	<u>$\theta = 22.2, 67.8, 202.2, 247.8$</u> ($\div 2$)	M1 A1 (5) (8)

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Question number	Scheme	Marks
4. (a)	$5 + 2x - x^2 = 2 \quad \text{OR} \quad x^2 - 2x - 3 = 0$ $(x - 3)(x + 1) = 0 \quad x = -1, x = 3$	M1
	$\int (5 + 2x - x^2) dx = [5x + x^2 - \frac{1}{3}x^3]$ Using limits: $(15 + 9 - 9) - (-5 + 1 + \frac{1}{3}) = 18\frac{2}{3}$ Shaded area = $18\frac{2}{3} - 8 = 10\frac{2}{3}$.	S.C. One correct answer: 31 M1 A1 (3) M1 A1 M1 A1 M1 A1 (6) (9)
	Or: Consider $(5 + 2x - x^2) - 2$ $\int (5 + 2x - x^2) - 2 dx = 3x + x^2 - \frac{1}{3}x^3$ $= (9 + 9 - 9) - (-3 + 1 + \frac{1}{3}) = 10\frac{2}{3}$	M1 M1A2,1,0 M1 A1
5. (a)	$r = 5.12 \div 6.4 = \underline{0.8}$	M1 A1 (2)
(b)	$a = 6.4 \div 0.64 = \underline{10}$ (✓: 3 s.f. or better reqd.)	M1 A1 ✓ (2)
(c)	Sum to $\infty = a \div (1 - r) = 10 \div (1 - 0.8) = \underline{50}$	M1 A1 (2)
(d)	$S_{25} = 10 (1 - 0.825) \div (1 - 0.8) (= 49.8111)$ $50 - 49.8111 = \underline{0.189}$ a.w.r.t 0.19	M1 A1 ✓ M1 A1 (4) (10)

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Question number	Scheme	Marks
6. (a)	$AB : m = -\frac{4}{3}$, $BC : m = \frac{3}{4}$ (s.c. $AB : \frac{4}{3}$, $BC : \frac{3}{4}$ B1)	B1, M1 A1✓ (3)
(b)	$BC = \sqrt{(8^2 + (k-4)^2)}$ ($= \sqrt{(k^2 - 8k + 80)}$) $BC^2 = \dots$ M1	M1 A1 (2)
(c)	$(k^2 - 8k + 80) = 100$ (Their $BC^2 = 100$)	M1
	$k^2 - 8k - 20 = 0$ ($(k-10)(k+2) = 0$)	M1 A1
	$k = 10$, $k = -2$ (rejected)	A1, (4)
(d)	(11, 6)	B1 B1 (2)
		(11)
7. (a)	$100 = 81 + 25 - (2 \times 9 \times 5 \cos BAC)$	M1 A1
	$\cos BAC = \frac{81 + 25 - 100}{90}$ ($= \frac{1}{15}$), $BAC = \underline{1.504 \text{ radians.}}$ *	A1 (3)
(b)	$\frac{1}{2} r^2 \theta = \frac{1}{2} \times 9 \times 1.504 = \underline{6.768 \text{ cm}^2}$ (6.77)	M1 A1 (2)
(c)	Area of triangle $= \frac{1}{2} \times 45 \times \sin 1.504$ ($= 22.450 \text{ cm}^2$)	M1 A1
	Shaded area $= 22.450 - 6.768 = \underline{15.682 \text{ cm}^2}$ (15.68, 15.7)	A1 (3)
(d)	Arc length $= r\theta = 3 \times 1.504$ ($= 4.512 \text{ cm}$)	M1 A1
	Perimeter $= 10 + 6 + 2 + 4.512 = \underline{22.512 \text{ cm}}$ (22.51, 22.5)	M1 A1✓ (4)
		(12)

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8. (a)	$2x^2 h = 1030$, $h = \frac{515}{x^2}$	M1, A1 (2)
(b)	$A = 4x^2 + 6xh$ (Or unsimplified version)	B1
	$A = 4x^2 + \frac{3090}{x}$ *	M1 A1 (3)
(c)	$\frac{dA}{dx} = 8x - 3090x^{-2}$	M1 A1
	$8x - 3090x^{-2} = 0$	M1
	$x^3 = (386.25)$	M1
	$x = 7.283$ (7.28, 7.3)	A1 (5)
(d)	$A = 4 \times 7.28^2 + \frac{3090}{7.28}$	M1
	$= 636.4 \text{ cm}^2$ (636, 640)	A1 (2)
(e)	Second derivative $= 8 + 6180x^{-3}$ <i>Attempt to diff.</i>	M1
	Correct deriv, > 0 , $\therefore$ Min.	A1 (2)
	(Or equivalent method).	(14)

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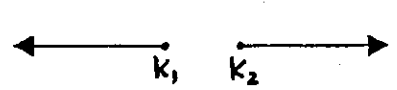
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Paper No. P1

Question number	Scheme	Marks
1. (a)	$k = 3$	B1 (1)
(b)	$(2^2)^x = (2^3)^{2-x}$ (A1 for $2x$ and $3(2-x)$) $2x = 3(2-x)$ $5x = 6$ <u>$x = 1.2$</u>	M1 A1 M1 A1 (4)
2. (a)	$\sin 2\theta \div \cos 2\theta = \tan 2\theta$, $\tan 2\theta = 0.5$ *	M1 (1)
(b)	$\tan 2\theta = 0.5$, $2\theta = 26.6^\circ$ $2\theta = 206.6$, One more soln. 386.6 , 566.6 Other 2 solns in range <u>$\theta = 13.3, 103.3, 193.3, 283.3$</u> (M: dividing by 2)	B1 B1ft B1ft M1 A1 (5)
3. (a)	$b^2 - 4ac \geq 0$ $(5k)^2 - 8k \geq 0$, $k(25k - 8) \geq 0$ *	M1, A1 (2)
(b)	Critical values: $k = 0$, $k = \frac{8}{25}$ $k \leq 0$, $k \geq \frac{8}{25}$ 	B1 B1 M1 A1ft (4)
(c)	$k = 0$ $k = \frac{8}{25}$ (Clearly seen as a soln. for (c))	B1 (1)

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Question number	Scheme	Marks
4. (a)	$a + (n-1)d = 500 + 39 \times 50 = \underline{\pounds 2450}$	M1 A1 (2)
(b)	$\frac{1}{2}n(a + l) = 20(500 + 2450) = \underline{\pounds 59000}$	M1 A1✓ (2)
(c)	Brian: $20(1780 + 39d) = (b)$ Solve: $\underline{d = 30}$	M1 A1✓ M1 A1 (4)
5. (a)	$f''(x) = 2x - 2x^{-3}$ $= 8 - \frac{2}{64} = 7\frac{31}{32} \quad (7.97)$	M1 A1 A1 (3)
(b)	$f(x) = \frac{1}{3}x^3 - 2x - \frac{1}{x} \quad (+C)$ $0 = 9 - 6 - \frac{1}{3} + C \quad C = -\frac{8}{3} \quad (\text{or } -2.67)$	M1 A1 M1 A1 (4)
(c)	$f'(x) > 0$ needed, or $f'(x) \geq 0$, or "as x increases, $f(x)$ increases." $f'(x) = (x - \frac{1}{x})^2$, > 0 always, or ≥ 0 always.	B1 M1, A1 (3)
	s.c. for last 2 marks in (c): B1. Noting that $f'(1) = 0$. B1. Convincing argument, e.g. $f(x)$ is <u>not</u> increasing because $f'(x)$ is not always positive in its domain.	

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6. (a)	Area of $X = 2d^2 + \frac{1}{2}\pi d^2$, Area of $Y = \frac{1}{2}(4d^2)\theta$	B1, M1 A1
(b)	Equate and divide by d^2 : $2 + \frac{1}{2}\pi = 2\theta$, $\theta = 1 + \frac{1}{4}\pi$ *	M1 A1 (5)
(c)	$12 + 3\pi$	B1 B1 (2)
(d)	$4d + r\theta = 12 + 6(1 + \frac{1}{4}\pi)$, $= 18 + \frac{3}{2}\pi$	M1, A1, A1 (3)
(d)	$X: 12 + 3\pi = 21.425 \text{ cm}$, $Y: 18 + \frac{3}{2}\pi = 22.712 \text{ cm}$	
	Difference = <u>13 mm</u> (or 12.9mm) or 12.88 mm	M1 A1 (2) (12)
7. (a)	$y = x(x^2 - 6x + 9) = x(x - 3)^2$, * <u>A(3,0)</u>	B1, B1 (2)
(b)	$\frac{dy}{dx} = 3x^2 - 12x + 9$	M1 A1
	$3(x^2 - 4x + 3) = 0$ $3(x - 1)(x - 3) = 0$	M1 A1
	At B, $x = 1$ $y = 4$ <u>(1,4)</u>	A1 (5)
(c)	$\int (x^3 - 6x^2 + 9x) dx = \frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2$	M1 A2,1,0
	$\left[\frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2\right]_0^3 = \frac{81}{4} - 54 + \frac{81}{2} = 6\frac{3}{4}$	M1 A1 (5) (12)

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Question number	Scheme	Marks
8. (a)	Gradient of $AB = \frac{4}{8} = \frac{1}{2}$	M1 A1 (2)
(b)	Gradient of $BC = -2$, $\frac{4-2}{k-7} = -2$ (or full Pythag. method)	M1
	$k = 6$	A1 (2)
(c)	$AB = \sqrt{4^2 + 8^2}$	M1 A1
	$= \sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$	A1 (3)
(d)	$BC = \sqrt{1^2 + 2^2} = \sqrt{5}$ (or $AC = \sqrt{7^2 + 6^2} = \sqrt{85}$)	B1ft
	Area of $ABC = \frac{1}{2} (4\sqrt{5} \times \sqrt{5}) = 10$	M1 A1 (3)
	Other <u>exact</u> methods can score M1 A2.	
	Non-exact methods score M1 A0 (but may gain the B1).	
(e)	$y - 2 = -2(x - 7)$	M1 B1
	$2x + y - 16 = 0$	M1 B1 (2)
(f)	When $y = 0$, $x = 8$ $D(8, 0)$	
	When $x = 0$, $y = 16$ $E(0, 16)$ (both)	B1✓
	Mid-point $(4, 8)$	M1 A1ft (3)

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Paper No. P2

Question number	Scheme	Marks
1.	$y = 2e^x + 3x^2 + 2 \quad \frac{dy}{dx} = 2e^x + 6x$ Evidence of differentiation M1 correct $\frac{dy}{dx}$ A1 At (0, 4) $\frac{dy}{dx} = 2$ Tangent at (0, 4) $y - 4 = 2x$ (any <u>correct</u> linear form A1)	M1 A1 A1 ft. M1 A1 cso [5]
2.	$f(x) = x + \ln 2x - 4$; $x_{n+1} = 4 - \ln 2x_n$, $x_0 = 2.4$ (a) $x_1 = 2.431$... A single small application of iteration $x_2 = 2.418$... $x_3 = 2.423$... At least x_3 reached Root = 2.422 (A2) 2.42 or "correct" unrounded to 3 d.p. answer A1	M1 M1 A2, 1, 0 (4)
	(b) Choosing an appropriate interval e.g. [2.4215, 2.4225] Establishing change of sign + conclusion	M1 A1 (2)
3.	Estimate for $M^2 = \frac{0.25}{2} [(48^2 + 29^2) + 2(207^2 + 37^2 + 161^2)]$ For squaring values M1, outside multiplier $\frac{0.25}{2}$ B1, inside bracket M1. Evaluating this estimate to 17900 (A, WRT) $M \approx 134$ (133.9), (130) If no attempt made to present final answer for M to nearest integer or 1 d.p. withhold this final mark.	M1 B1 M1 M1 A1 A1 (6)

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Paper No. P2

Question number	Scheme	Marks
4(i)	Choosing values of A and B and attempting to evaluate LHS and RHS of statement Showing that LHS $\neq$ RHS + conclusion	M ₁ A ₁ (2)
(ii)	Using $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ to obtain $\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$ Using $\cos^2 \theta + \sin^2 \theta \equiv 1$ Using $2 \sin \theta \cos \theta \equiv \sin 2\theta$ leading without any error or fudge to $2 \csc 2\theta$	M ₁ A ₁ M ₁ M ₁ A ₁ also [5]
5	Realising that $(x^2+3)^2$ required (i.e. y^2 terms for) $(x^2+3)^2 = x^4 + 6x^2 + 9$ $\int y^2 dx \rightarrow \frac{x^5}{5} + \frac{6x^3}{3} + 9x$ (two terms ✓ B ₁) [] ₁ ³ Using limits top-bottom or bottom-top Volume $\pi \int_1^3 y^2 dx \rightarrow 118.4\pi$ [M ₁ method complete] <u>Notes on special cases:</u> π omitted throughout - loses 1st M ₁ A ₁ $\rightarrow \frac{5}{7}$ trivialised to $\int \pi y dx$ or just $\int y dx$ - only M ₁ for limits $\rightarrow \frac{1}{7}$ $(x^2+3)^2$ taken as $x^4 + 9$ sums M ₁ B ₀ B ₁ M ₁ M ₁ A ₀ $\rightarrow \frac{4}{7}$ or $\frac{3}{7}$ if π omitted	M ₁ B ₁ B ₂ , 1, 0 M ₁ M ₁ A ₁ (7)

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Question number	Scheme	Marks
6 (a)	Attempting to get to $a^6 =$ from $800 = \frac{2000a^6}{4+a^6}$ $a^6 = \frac{3200}{1200}$ $a = \left(\frac{3200}{1200}\right)^{\frac{1}{6}} \rightarrow 1.1776 \text{ (4 d.p.)}$	M ₁ A ₁
(b)	Substituting $p = 1800$ into formula with a^t as unknown $a^t = 36 \rightarrow t = 22$ Number of years needed for p from 800 to 1800 $= 16$ years	M ₁ A ₁ , M ₁ A ₁ f.t. (4)
(c)	$p = \frac{2000}{1+4a^{-t}}$, $4a^{-t} \rightarrow 0 \rightarrow t \rightarrow \infty$ so $p \rightarrow 2000$ but does not exceed it	B ₁ (1)
7 (a)	Using $x^2 - 1 \equiv (x-1)(x+1)$ somewhere in solution Using a common denominator e.g. $\frac{x - (x-1)}{(x-1)(x+1)}$ clear, sound, complete proof of $f(x) = \frac{1}{(x-1)(x+1)}$	M ₁ M ₁ A ₁ (3)
(b)	Range of f is y , where $y > 0$ If $y \geq 0$ given allow B ₁ .	B ₂ (3)
(c)	$g(f(x)) = g\left(\frac{1}{(x-1)(x+1)}\right) = 2(x-1)(x+1)$ M ₁ requires correct order and $g(x) = \frac{2}{x}$ used $2(x-1)(x+1) = 70$ M ₁ is independent of previous work $x = 6$ (treat -6 extra as 15w)	M ₁ A ₁ M ₁ A ₁ (4)

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Question number	Scheme	Marks
8 (a)	EITHER Expanding Using coefficients 1, 5, 10, 10, 5, 1 as necessary Using powers x^5, $2x^4$, $2x^3$ etc as necessary Completing the method $A = 64$ $B = 160$, $C = 20$ OR Substituting values for x $x = \rightarrow A = 64$ Forming a first equation in B and C Forming a second equation in B and C Solving to complete the method down to either $B =$ or $C =$ $B = 160$, $C = 20$	M_1 M_1 M_1 B_1 $A2, 1, 0 (6)$
(b)	Candidate's values of A, B, C used to form $20x^4 + 160x^2 + 64 = 349$ $4y^2 + 32y - 57 = 0$ [3 term quadratic needed] Solving for y Replacing by x^2 and completion to obtain all relevant values of x $\pm \sqrt{\frac{3}{2}}$ or <u>AWRT ± 1.22</u>	M_1 $A1, f.t.$ M_1 M_1 $A1cao (5)$

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Question number	Scheme	Marks
9	(a) $R = \sqrt{29} = 5.39$ $\tan \alpha = \frac{5}{2} \rightarrow \alpha = 1.19, 0.379\pi, 68.2^\circ$ (b) $\text{Max} = \sqrt{29}$ (or as in (a)) at $\theta = 1.19$ (or as in (a) above) (c) $T = 15 + \sqrt{29} \cos\left(\frac{\pi t}{12} - 1.19\right)$ $\text{Max. } T = 15 + \sqrt{29}$ 20.4°C (accept 20°AWRT) occurs when $t = \frac{12 \times 1.19}{\pi}$ $= 4.5$ hours (accept AWRT 4.5 or 4.6) (d) $12 = 15 + \sqrt{29} \cos\left(\frac{\pi t}{12} - 1.19\right)$ $\cos\left(\frac{\pi t}{12} - 1.19\right) = -\frac{3}{\sqrt{29}}$ (using cond.'s values of R & α) $\frac{\pi t}{12} - 1.19 = 2.16(2)$ or $4.12(2)$ $t = 12.8(0)$ or $20.2(9)$ (either) i.e. 0100 or 0830 (both)	B_1 $M_1 A_1$ (3) B_1 f.t. B_1 f.t. (2) M_1 A_1 M_1 A_1 (4) M_1 A_1 f.t. $M_1 + M_1$ A_1 A_1 (6)

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Question number	Scheme	Marks
1. (a) (b)	centre is $(5, -3)$ radius is 7 [m1 if sign errors] [m1 attempts $\sqrt{g^2 + f^2 - c}$]	m1 A1 (2) m1 A1 (2)
2. (a) (b)	uses $f(1) = 9, \Rightarrow a + b = 2$ (o.e.) uses $f(-2) = 0, \Rightarrow -8a + 4b = -28$ (o.e.) $\therefore \underline{a = 3}, \underline{b = -1}$ (solves to find both values - m1)	m1, A1 (2) m1, A1 (2) m1 A1 cao. (2)
3. (a) (b) (c)	$x \ln a \equiv kx \ln e$ $\therefore k \ln e = \ln a \Rightarrow k = \ln a$ (*) $y = e^{kx} \Rightarrow \frac{dy}{dx} = k e^{kx}$ and $k = \ln 2$ $\therefore y = 2^x \Rightarrow \frac{dy}{dx} = \ln 2 \cdot 2^x$ as $e^{\ln 2} = 2$ (*) when $x = 2, \frac{dy}{dx} = 4 \ln 2, = \ln 2^4 = \ln 16$ (*)	m1 A1 (2) m1 A1 (2). m1, A1 (2).

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4 (a)	$1 - 3x + 9x^2 - 27x^3 + \dots$	B 1; B 1 (3)
(b)	$(1+x) \left(\begin{aligned} &1 - 3x + 9x^2 - 27x^3 + \dots \\ &= 1 + (x-3x) + (9x^2-3x^2) + (9x^3-27x^3) + \dots \\ &= 1 - 2x + 6x^2 - 18x^3 + \dots \end{aligned} \right) \quad \text{uses product.} \quad (*)$	M 1 A 1 (2)
(c)	$x = 0.01$ $1 - 0.02 + 0.0006 - 0.000018, = 0.98058$	substitutes. B 1 M 1, A 1 calc (3)
5 (a)	$\begin{aligned} &x \tan x, - \int \tan x \, dx \\ &= x \tan x + [\ln \cos x] \quad (\text{or equivalent}) \\ &= \frac{\pi}{4} + \ln \cos \frac{\pi}{4} \quad \text{use of limits} \\ &= \frac{\pi}{4} + \ln \frac{1}{\sqrt{2}} = \frac{\pi}{4} - \frac{1}{2} \ln 2 \quad (*) \end{aligned}$	M 1, A 1 [M 1 A 1] M 1 A 1. (6)
(b)	$\begin{aligned} V &= \pi \int_0^{\pi/4} x \sec^2 x \, dx \quad \text{uses } \pi \times \int \\ &= \frac{\pi^2}{4} - \frac{\pi}{2} \ln 2 = 1.38 \end{aligned}$	M 1 A 1 (2)
(c)	$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2} x^{-1/2} \sec x + x^{1/2} \sec x \tan x \quad \text{uses product rule.} \\ &= 2.05 \quad \left(\approx \frac{\sqrt{2\pi}}{2\pi} (2 + \pi) \right) \end{aligned}$	M 1 A 1 A 1 (3)

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Question number	Scheme	Marks
b(a)	$1 \times 4 - 2 \times 1 - 2 \times 1 = 0$ i.e. $\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} = 0 \therefore \perp^e$	M1 A1 (2)
(b)	$3 + \lambda = 9 + 4\mu$ and either $4 - 2\lambda = 1 + \mu$ or $-5 + 2\lambda = -2 - \mu$ Eliminate to obtain, $\mu = -1$ or $\lambda = 2$ point is $(5, 0, -1)$ (no check needed) vector is $\begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$	M1 A1 M1, A1 B1 (5)
(c)	$\lambda = -3 \Rightarrow$ point lies on 1st line l_1 show contradiction for $\mu \Rightarrow$ point not on l_2	M1 A1 B1 (3)
(d)	$\sqrt{5^2 + 10^2 + 10^2} = 15, \Rightarrow 1.5 \text{ km}$	M1, A1 (2)
7.(a)	$\int \frac{dx}{(1-2x)(1-4x)} = \int k dt$ $\int \frac{-1}{1-2x} + \frac{2}{1-4x} dx = \int k dt$ $\frac{1}{2} \ln(1-2x) - \frac{1}{2} \ln(1-4x) = kt + \frac{c}{2}$ $\ln \frac{1-2x}{1-4x} = 2kt + c$ (4)	B1 M1 A1, M1 A1, A1 A1 csa (7)
(b)	use $x=0$ when $t=0 \Rightarrow c=0$ $\therefore \frac{1-2x}{1-4x} = e^{2kt}$ $\therefore x(4e^{2kt} - 2) = e^{2kt} - 1, \therefore x = \frac{e^{2kt} - 1}{4e^{2kt} - 2}$	B1 M1 M1, A1 oae. (4)
(c)	As $t \rightarrow \infty, x \rightarrow \frac{1}{4}$	M1 A1 (2)

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Advanced Supplementary/Advanced Level

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Subject PURE MATHEMATICS 6673

Paper No. P3

Question number	Scheme	Marks
8(a)	$y^2 = 81 \sin^2 2t$ $= 81 \times 4 \sin^2 t \cos^2 t$ $= 4 \times 9 (1 - \cos^2 t) \times 9 \cos^2 t$ $= 4 (9 - x^2) x^2$ squares + Substitutes use of double angle use of $\sin t = 1 - \cos t$	M1 M1 M1 A1 (4)
(b)	$\int y dx = - \int_0^{\frac{\pi}{2}} 9 \sin 2t \cdot 3 \sin t dt$ $= \int_0^{\frac{\pi}{2}} 27 \sin 2t \sin t dt \quad (\text{ie: } A=27)$ used $\int y dx$ form.	M1, A1 B1 (3)
(c)	$27 \int_0^{\frac{\pi}{2}} \sin 2t \sin t dt = 27 \int 2 \sin^2 t \cos t dt$ $= 27 \left[\frac{2}{3} \sin^3 t \right], = 18$	M1 M1A1, A1
	OR $-\frac{27}{2} \int (\cos 3t - \cos t) dt =$ $= -\frac{27}{2} \left[\frac{1}{3} \sin 3t - \sin t \right], = 18.$	M1 M1A1, A1 (4)
(d)	Rectangular area = $18 \times 6 = 108$ Red area = Rectangular - $4 \times \text{Blue} = 108 - 72 = 36$	M1A1 M1A1 (4)

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Paper No. P1

Question number	Scheme	Marks
1.	(a) $x = -\frac{1}{2}$ $4 = 2^2$ and $\sqrt{2} = 2^{\frac{1}{2}}$, $y = 2^{\frac{1}{2}}$ (b) $y - x = 3$ $2^3 = 8$ (Or: $4\sqrt{2} \div \frac{1}{\sqrt{2}} = 8$)	B1 M1, A1 (3) M1 A1 (2)
2.	(a) $64 - 46 - 28 + c = 0$ $c = -20$ (b) $(x - 4)(x^2 + 3x + 5)$ (B1 for $(x - 4)$) (c) For $x^2 + 3x + 5$, $b^2 - 4ac = -11 < 0$, $\therefore$ No real roots.	M1 A1 (2) B1 M1 A1 (3) M1 A1ft (2)
3.	$2\sin^2 \theta - 2\sin \theta = 1 - \sin^2 \theta$ $3\sin^2 \theta - 2\sin \theta - 1 = 0$ $(3\sin \theta + 1)(\sin \theta - 1) = 0$ (or attempt by formula) $\sin \theta = -\frac{1}{3}$ $\sin \theta = 1$ <u>$\theta = -19.5^\circ, -160.5^\circ, 90^\circ$</u> Final 3 marks: subtract 1 for each extra soln in range. Ignore extra solutions outside range. Special case, if the 2nd M mark has not been earned: Noting that $\sin \theta = 1$ (B1) so $\theta = 90^\circ$ (B1)	M1 A1 M1 A1ft A1 (5) A1 1ft A1 (3)

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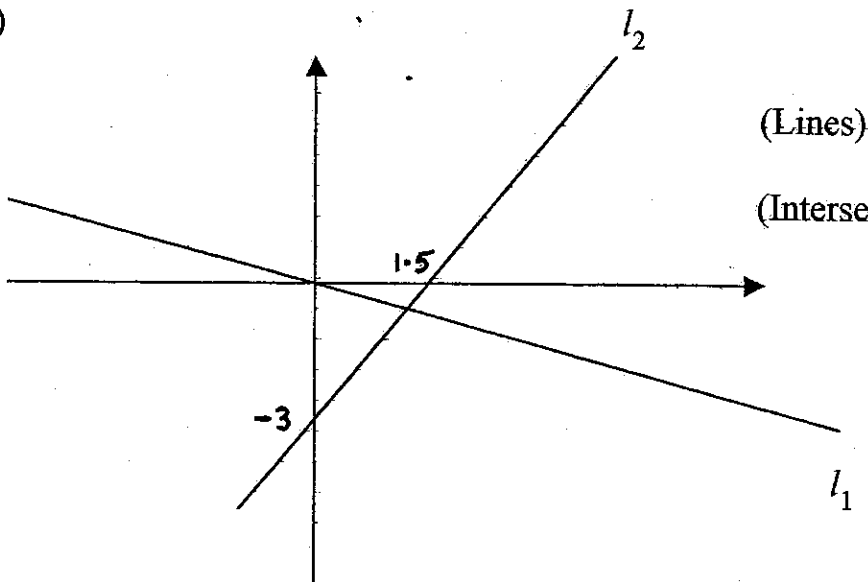
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Subject **PURE MATHEMATICS 6671**

Paper No. **P1**

Question number	Scheme	Marks
4.	(a)  (Lines) (Intersections) (b) $-\frac{1}{4}x = 2x - 3$ $\frac{9}{4}x = 3$ $x = \frac{4}{3}$ $y = -\frac{1}{3}$ (c) Perp. to l_1 : $m = 4$ $y + \frac{1}{3} = 4(x - \frac{4}{3})$ $12x - 3y - 17 = 0$	B1 B1 B1 (3) M1 A1 A1 (3) B1 M1 A1 (3) ⑨

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Subject PURE MATHEMATICS 6671

Paper No. P1

Question number	Scheme	Marks
5.	(a) $\frac{dy}{dx} = 3x^2 - 10x + 5$ (b) $3x^2 - 10x + 5 = 2$ $3x^2 - 10x + 3 = 0$ $(3x - 1)(x - 3) = 0$ $x = \frac{1}{3}$ (c) When $x = 3$, $y = 27 - 45 + 15 + 2 = -1$ $y + 1 = 2(x - 3)$ $y = 2x - 7$ (d) $R: x = 0$ $y = -7$ $S: y = 0$ $x = 3.5$ (Both for M1) $RS = \sqrt{(7^2 + (\frac{7}{2})^2)} = \frac{7}{2}\sqrt{5}$ (or equivalent)	M1 A1 (2) M1 A1 (2) B1 M1 A1 (3) M1 A1ft M1 A1 (4)
6.	(a) $\frac{1}{2}R^2\theta = \frac{49}{2}\theta$ or $\frac{1}{2}r^2\theta = \frac{25}{2}\theta$ $\frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta = \frac{49}{2}\theta - \frac{25}{2}\theta = 12\theta$ (b) $12\theta = 15$ $\theta = 1.25$ (*) (c) $R\theta = 7 \times 1.25$ (or $r\theta = 5 \times 1.25$) $R\theta + r\theta + 4 = 8.75 + 6.25 + 4 = 19 \text{ m}$ (d) $\sin 0.625 = \frac{x}{5}$ $AD = 2x$ (= 5.851 m) $6.25 - 5.85 = 0.399$ 40 cm (M dep. on previous M)	B1 M1 A1 (3) M1 A1 (2) B1 M1 A1 (3) M1 M1 A1 (3)

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Paper No. P1

Question number	Scheme	Marks
7.	(a) $S = a + ar + ar^2 + \dots + ar^{n-1}$ $rS = ar + ar^2 + \dots + ar^n$ Subtract: $S(1-r) = a(1-r^n)$ $S = \frac{a(1-r^n)}{1-r}$ (b) $ar = 3$ $ar^3 = 1.08$ Divide: $r^2 = 0.36$ $r = 0.6$ $a = 6 \div 1.2 = 5$ (c) $S = \frac{5}{1-0.6}$ $= \underline{12.5}$	B1 M1 M1 A1 (4) B1 B1 M1 A1 A1 (5) M1 A1ft A1 (3) (12)

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Paper No. P1

Question number	Scheme	Marks
8.	(a) $x + 1 = 6x - x^2 - 3$ $x^2 - 5x + 4 = 0$ $(x - 1)(x - 4)$ (or use of formula) $x = \dots$ $x = 1$ $x = 4$ $y = 2$ $y = 5$ (b) $\int (6x - x^2 - 3) dx = 3x^2 - \frac{x^3}{3} - 3x$ Limits x_A and x_B: $(48 - \frac{64}{3} - 12) - (3 - \frac{1}{3} - 3)$ (= 15) Trapezium: $\frac{1}{2}(2 + 5) \times 3 = 10.5$ Area of $R = 15 - 10.5 = 4.5$ <u>Alternative for (b)</u> $(6x - x^2 - 3) - (x + 1) = 5x - x^2 - 4$ $\int (5x - x^2 - 4) dx = \frac{5x^2}{2} - \frac{x^3}{3} - 4x$ Limits x_A and x_B: $(40 - \frac{64}{3} - 16) - (\frac{5}{2} - \frac{1}{3} - 4)$, = 4.5	M1 M1 A1 M1 A1 (5) M1 A1 M1 A1 B1ft M1 A1 (7) M1 A1 M1 A1ft M1 A1, A1

(12)

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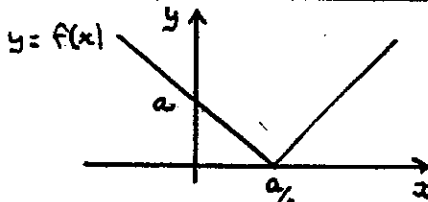
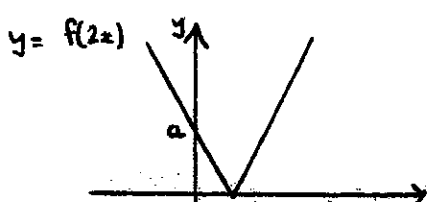
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Paper No. P2

Question number	Scheme	Marks
1. (a)	$p = \underline{1.357}$; $q = \underline{1.382}$ AWRT	B1 ; B1 (2)
(b)	$I \approx \frac{0.5}{2} , [1 + 2(1.216 + \sqrt{1.357} + 1.413) + \sqrt{1.382}]$ $= \underline{2.589}$ or $\underline{2.59}$ only	B1, [M1 A1] (4) A1 (4) (6)
2. (a)	$\log_2 16 = \log_2 2^4$, $\therefore p = 4\log_2 2$ i.e. $\log_2 2 = \underline{\frac{p}{4}}$	M1, A1 (2)
(b)	$\log_2(8q) = \log_2 8 + \log_2 q$ $= \dots + 1$ $= 3\log_2 2 + \dots$ $\therefore \log_2(8q) = \underline{\frac{3}{4}p + 1}$ $\log_2 q = 1$ $\log_2 8$ in terms of $\log_2 16$ or $\log_2 2$	M1 B1 M1 A1 (4) (6)
3. (a)	$y = f(x)$ 	fairly even V, vertex on the x axis ONLY $(\frac{a}{2}, 0)$ and $(0, a)$ on graph or in text [Clearly read off graph paper work]
(b)	$y = f(2x)$ 	Steeper, even V and 1 correct intersection ONLY both $(\frac{a}{4}, 0)$ and $(0, a)$ on graph or in text
(c)	$-(2x - a) = \frac{1}{2}x$ when $x = 4$, $\Rightarrow a - 8 = 2 \therefore \underline{a = 10}$ $2x - a = \frac{1}{2}x$ when $x = 4$, $\Rightarrow 8 - a = 2 \therefore \underline{a = 6}$	M1, A1 M1, A1 (4) (8)

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Paper No. P2

Question number	Scheme	Marks
4.	$[f(x)]^2 = x^2 + \frac{4}{x^4} + \frac{4}{x}$ > 2 terms correct $\int [f(x)]^2 dx = \left[\frac{x^3}{3} - \frac{4}{3x^3} + 4\ln x \right]$ $\int_1^2 [f(x)]^2 dx = \left(\frac{8}{3} - \frac{4}{24} + 4\ln 2 \right) - \left(\frac{1}{3} - \frac{4}{3} + 4\ln 1 \right)$ $= \left(\frac{7}{2} + 4\ln 2 \right)$ $V = \pi \int_1^2 [f(x)]^2 dx, \Rightarrow V = \pi \left(\frac{7}{2} + \ln 16 \right) \text{ or } \frac{a=7/2}{b=16}$	M1 M1 A1, B1 M1 M1, A1, A1 (8)
S.C.	Sydc can score M1 for integration and M1 for Limits only	i.e. max of $\frac{2}{8}$
5. (a)	$u_1 = 1.05 \times 500\,000 - 15\,000 = 510\,000$ $u_2 = 520\,500$ $u_3 = 531\,525$ the population is <u>increasing</u>	M1 A1 (all 3) B1 (3)
(b)	$\begin{pmatrix} u_1 = 425\,000 \\ u_2 = 346\,250 \\ u_3 = 263\,562.5 \\ u_4 = 176\,740.625 \end{pmatrix}$ $u_5 = 85\,577.64\dots$ $u_6 = -10\,143.46\dots$ $u_5 > 0, u_6 < 0$ so population died out during 6th year	Attempt up to u_5 and u_6 M1 A1 B1 (3)
(c)	Require $u_1 = u_0$ i.e. $1.05 \times 500\,000 - d = 500\,000$ i.e. $d = 0.05 \times 500\,000$ i.e. $d = \underline{\underline{25\,000}}$	M1 A1 (2) (8)

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Paper No. P2

Question number	Scheme	Marks
6 (a)	$\text{LHS} = \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$ $= \frac{\sin\theta}{\cos\theta} = \tan\theta = \text{RHS}$	Attempt $\cos 2\theta$ or $\sin 2\theta$ both correct M1 A1 All c.s.o. (3)
(b)	$\text{From (a)} \quad \frac{1 - \cos 2\theta}{\tan\theta} = \frac{1}{2} \quad \text{or} \quad 4\sin^2\theta = \frac{\sin\theta}{\cos\theta}$ $\Rightarrow \sin 2\theta = \frac{1}{2}$ $\Rightarrow 2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $\therefore \theta = \frac{\pi}{12}, \frac{5\pi}{12}$	$(\sin\theta)(4\sin\theta\cos\theta - 1) = 0$ $\Rightarrow \sin 2\theta = \frac{1}{2}$ one sol ⁿ for 2θ (β) $\frac{\pi - \beta}{2}$ both M1 A1 Steps to show $\sin 2\theta = \frac{1}{2}$ $\alpha = \frac{1}{2}$ B1 (accept degrees) M1 A1 c.s.o. (6) (9)
7 (a)	$f'(x) = 0.5e^x - 2x$ $f'(0) = 0.5$ $\text{Equation of tangent at A is: } y = f'(0)x + f(0), \text{ i.e. } \underline{y = 0.5x + 0.5}$	diff. M1 A1 c.s.o. M1, A1 (4)
(b)	$f'(x) = 0 \Rightarrow 2x = \frac{1}{2}e^x$ $\text{i.e. } 4x = e^x$ $\Rightarrow x = \ln(4x) \quad (*)$	$f'(x) = 0$ M1 (dependent) M1 A1 c.s.o. (3)
(c)	$x_1 = \ln 8.6 = 2.1517622$ $x_2 = 2.1525814$ $x_3 = 2.152962... = \underline{2.1530} \text{ (4dp) ONLY}$	3 iterations M1 A1 c.s.o. (2) (9)

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Paper No. P2

Question number	Scheme	Marks
8. (a)	$f(x) = \frac{2(2x+1) - 6}{(x-1)(2x+1)}, = \frac{4x-4}{(x-1)(2x+1)} \quad \left[\text{M for Attempt same denominator} \right]$	M1, A1
	$\text{i.e. } f(x) = \frac{4(x-1)}{(x-1)(2x+1)}, = \frac{4}{(2x+1)} \quad (*) \quad \left[\text{M for factoring} \right]$	M1, A1 c.s.o. (4)
(b)	$0 < f < 4/3 \quad \text{or} \quad 0 < y < 4/3$	$\alpha < f < \beta, \alpha=0, \beta=4/3$ B1 both B1 (2)
(c)	$y = \frac{4}{2x+1} \Rightarrow y(2x+1) = 4$	M1
	$\text{i.e. } x = \frac{4-y}{2y}$	M1
	$\therefore f^{-1}(x) = \frac{4-x}{2x} \quad (\text{o.e.})$	$\text{must be } f^{-1}(x)$ A1 (3)
(d)	$\text{Range of } f^{-1} = \text{domain of } f \quad \therefore \quad \underline{f^{-1} > 1} \quad \text{or } y > 1 \text{ or } x > 1$	B1
	$[\text{BO for } f > 1 \text{ or } x > 1 \text{ or } f^{-1} > 1]$	(10) (1)
9. (a)	$f(x) = \dots + \binom{n}{2} \frac{x^2}{k^2} + \binom{n}{3} \frac{x^3}{k^3} \dots$	Attempt both terms M1
	$\frac{2 \times n(n-1)}{2 k^2} = \frac{n(n-1)(n-2)}{6 k^3}$	Correct eqn. No (A) (factorials OK.) M1
	$\Rightarrow 6k = n-2 \quad \text{or} \quad \underline{n = 6k+2} \quad (*)$	A1 c.s.o. (3)
(b)	$\frac{n(n-1)(n-2)(n-3)}{4! k^4} = \frac{n(n-1)(n-2)(n-3)(n-4)}{5! k^5}, \Rightarrow 5k = n-4 \quad (\text{o.e.})$	M1, A1
	$\text{Solving: } 5k = 6k+2-4, \Rightarrow \underline{k=2 \text{ and } n=14} \quad (*)$	M1, A1 c.s.o. (4)
(c)	$(1 + \frac{x}{2})^{14} = 1 + 7x + \binom{14}{2} \frac{x^2}{4} + \binom{14}{3} \frac{x^3}{8} + \binom{14}{4} \frac{x^4}{16} + \binom{14}{5} \frac{x^5}{32} \dots$	M1 (≥ 3 correct)
	$= 1 + 7x + \frac{91}{4} x^2 + \frac{91}{2} x^3 + \frac{1001}{16} x^4 + \frac{1001}{16} x^5 \dots$	B1, A1, A1 (4)

FINAL
MARK
SCHEME

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Subject PURE MATHEMATICS 6673

Paper no. P3

Question number	Scheme	Marks
1.	Try to use remainder theorem i.e. evaluate $f(-\frac{1}{2})$ or $f(+\frac{1}{2})$ Uses correct substitution to give $4(-\frac{1}{2})^3 + 3(-\frac{1}{2})^2 - 2(-\frac{1}{2}) - 6 = -4\frac{3}{4}$ Alternative : Uses long division dividing by $(2x+1)$, obtaining $2x^2 + \dots$ Continues as far as remainder, to get $2x^2 + \frac{1}{2}x - \frac{5}{4} \text{ rem } -4\frac{3}{4}$	M1 M1 A1 (3) M1 M1, A1 (3)
2.	$y = \tan x = \frac{\sin x}{\cos x}$ $\frac{dy}{dx} = \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$ (use of quotient rule) (or of appropriate product rule) $= \frac{1}{\cos^2 x} = \sec^2 x.$ *	M1 M1 A1 A1 (4)
3.	(a) $A = 2, B = 16$ (complete method to find A and B) (b) $A(1-2x)^{-1} + B(2+x)^{-1}$ and attempt at expansion $A(1+2x+4x^2+8x^3+\dots)$ $+ \frac{B}{2}(1-\frac{x}{2}+\frac{x^2}{4}-\frac{x^3}{8}+\dots)$ $= 10+10x^2+15x^3+\dots$ (final M mark needs correct powers of 2)	M1 A1 A1 (3) M1 A1 M1 A1 A1 (5)

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Subject **PURE MATHEMATICS 6673**

Paper no. **P3**

Question number	Scheme	Marks
4.	(a) $(x-3)^2 + (y-4)^2 = 18$ ^{OR} EQUIVALENT (accept $(3\sqrt{2})^2$) (b) Use $y = x + 3$ to obtain $(x-3)^2 + (x-1)^2 = 18$ And thus $2x^2 - 8x = 8$ Solve quadratic, to obtain $x = 2 \pm \sqrt{8}$, $y = 5 \pm \sqrt{8}$ (c) Distance = $\sqrt{((2\sqrt{8})^2 + (2\sqrt{8})^2)} = 8$	M1 A1 (2) M1 A1 M1, A1, A1 ✓ ✓ (5) M1A1 c50 (2)
5.	(a) $\frac{dN}{dt} = -kN$ -sign, or k negative needed for A1 (b) $\int \frac{dN}{N} = \int -k dt$ (✓ on sign error only) $\ln N = -kt + c$ (✓ on sign error only) $N = e^{-kt+c} = Ae^{-kt}$ * (c) $3 \times 10^{17} = 7 \times 10^{18} e^{-8k}$ $e^{-k} = \sqrt[8]{\frac{3}{70}} = .6745$ or $k = \frac{1}{8} \ln \frac{70}{3}$ or equivalent $k = .3937$ (d) $N = 7 \times 10^{18} e^{-0.3937 \times 16}$ or $\frac{3}{70} \times 3 \times 10^{17}$ $= 1.286 \times 10^{16}$	M1 A1 (2) B1 ✓ M1 A1 ✓ M1 A1 (5) M1 M1 A1 (3) M1 A1 (2)

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Subject **PURE MATHEMATICS 6673**

Paper no. **P3**

Question number	Scheme	Marks
6.	(a) $\vec{AB} = 3\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$ (b) $\cos A = \frac{-12 - 48 + 6}{\sqrt{81}\sqrt{81}} = -\frac{2}{3}$ OR $\cos A = \frac{81 + 81 - 270}{162} = -\frac{2}{3}$ (c) $\lambda = 4$ at point A and $\lambda = 7$ at point B. $\mathbf{r} = -9\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ represents a line. (d) $(2\lambda + 2\lambda\mathbf{j} + (2\lambda - 9)\mathbf{k}) \cdot (\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 0$ $\lambda + 4\lambda + 4\lambda - 18 = 0$. Therefore $\lambda = 2$ (e) The point is (2, 4, -5)	B1 (1) M1 A1 A1 (3) B1 B1 B1 (3) M1 M1 A1 (3) M1 A1 (2)
7.	(i) $\int_1^3 x^2 \ln x \, dx = \left[\frac{x^3}{3} \ln x \right] - \int \frac{x^3}{3} \frac{1}{x} \, dx$ $= \left[\frac{x^3}{3} \ln x \right] - \int \frac{x^2}{3} \, dx$ $= \left[\frac{x^3}{3} \ln x \right] - \frac{1}{9} x^3$ $= 9 \ln 3 - 3 + 1/9 = 9 \ln 3 - 2\frac{8}{9}$ or any equivalent (ii) $\int \frac{\cos \theta}{\cos^3 \theta} \, d\theta = \int \sec^2 \theta \, d\theta$ $= \tan \theta (+c)$ $= \frac{\sin \theta}{\cos \theta} (+c) = \frac{x}{\sqrt{1-x^2}} (+c)$ *	M1 A1 M1 A1 M1 A1 (6) M1 A1, M1 B1 M1 A1 (6)

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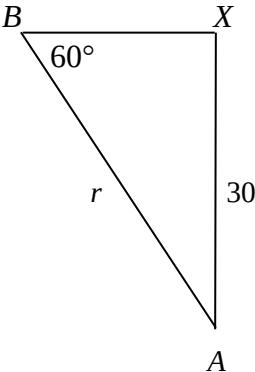
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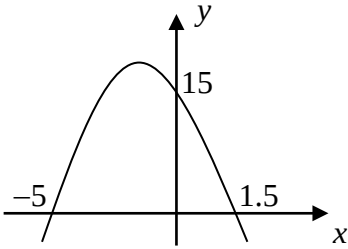
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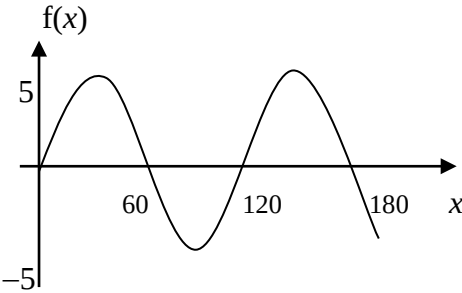
Subject **PURE MATHEMATICS 6673**

Paper no. **P3**

Question number	Scheme	Marks
8.		
(a)	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{4\cos\theta}{-5\sin\theta}$ Equation of tangent is $y - 4\sin\alpha = \frac{4\cos\alpha}{-5\sin\alpha}(x - 5\cos\alpha)$ $\therefore 5y\sin\alpha + 4x\cos\alpha = 20(\cos^2\alpha + \sin^2\alpha) = 20 \quad *$	M1 A1
(b)	$\int y \frac{dx}{d\theta} d\theta = -\int 4\sin\theta 5\sin\theta d\theta$ $= 10 \int (\cos 2\theta - 1) d\theta$ $= [5\sin 2\theta - 10\theta]$ Area = 20π (appropriate limits used correctly)	M1 M1 M1 A1 c50 (4)
(c)	When $x = 0, y = \frac{4}{\sin\alpha}$, OR when $y = 0, x = \frac{5}{\cos\alpha}$. Area of parallelogram = $4 \times \frac{10}{\sin\alpha \cos\alpha} = \frac{80}{\sin 2\alpha}$ $\therefore A = \frac{80}{\sin 2\alpha} - 20\pi \quad *$	B1 M1A1 A1 (4)
(d)	$\frac{80}{\sin 2\alpha} - 20\pi = 20\pi$ $\sin 2\alpha = \frac{2}{\pi}$ $\alpha = 0.345$	M1 A1 A1 (3)

Question Number	Scheme	Marks
1.	(a) $1 \times 7 + 2 \times 7 + \dots$ $a = 7, d = 7, n = 142$ $n = 142$ $S_n = \frac{1}{2}n(a + b)$ or $\frac{1}{2}n(2a + (n - 1)d)$ or $7 \times \frac{n(n + 1)}{2}$ $= \frac{142}{2}(7 + 994)$ or $\frac{142}{2}(14 + 141 \times 7)$ or $7 \times \frac{142 \times 143}{2} = \mathbf{71\,071}$ (b) $\sum_{r=1}^{142} (7r + 2) = \sum_{r=1}^{142} 7r + \sum_{r=1}^{142} 2$ split $\sum_{r=1}^{142} 2 = 2 \times 142$ $\therefore \sum_{r=1}^{142} (7r + 2) = 71\,071 + 2 \times 142 = \mathbf{71\,355}$	B1 M1 (use of correct formula) A1 (3) M1 A1 (3) (6 marks)
2.	(a)  $\sin 60^\circ = \frac{3}{r}$ or $r = 2x, 4x^2 = x^2 + 3^2, x = \sqrt{3}$ $r = \frac{6}{\sqrt{3}}$ or $r = 2\sqrt{3}$ (b) Area = $\frac{1}{2}r^2\theta^c$ or $\frac{\theta^\circ}{360^\circ} \times \pi r^2 =, \frac{1}{6} \times \pi \times 12 = \mathbf{2\pi}$ (cm²) (c) Arc = $r^2\theta^c$ or $\frac{\theta^\circ}{360^\circ} \times 2\pi r =, \frac{1}{6} \times 2\pi \times 2\sqrt{3}$ Perimeter = Arc + $2r =, \frac{2\sqrt{3}}{3}\pi + 2 \times 2\sqrt{3} = \frac{2\sqrt{3}}{3}(\pi + 6)$ (cm) (*)	M1 A1 (2) M1, A1 (2) M1 M1, A1 cso (3) (7 marks)

Question Number	Scheme	Marks
3.	(a) $f(x) = 0 \Rightarrow 2x^2 + 7x - 15 = 0$ $(2x - 3)(x + 5) = 0$ attempt to solve $f(x) = 0$ $\therefore$ points are $(\frac{3}{2}, 0), (-5, 0); (0, 15)$ (b)  (c) Symmetry: $x = \frac{1}{2}(-5 + 1.5)$ or Calculus: $-7 - 4x = 0$ or Algebra: $-2[(x + \frac{7}{4})^2 - k]$ $\Rightarrow x = -\frac{7}{4}, y = 21\frac{1}{8}$	M1 A1 (both); B1 (3) shape B1 vertex in correct quadrant B1 ft (2) M1 A1, A1 (3) (8 marks)
4.	(a) $(x + k)^2 - 7 - k^2 = 0$ $(x + k)^2$ $\Rightarrow (x + k)^2 = 7 + k^2 = 0 \therefore x + k = (\pm) \sqrt{7 + k^2}$ (LHS) $\therefore x = -k \pm \sqrt{7 + k^2}$ (b) $7 + k^2 > 0$ (or discriminant > 0) $\therefore$ roots are real and distinct (c) $k = \sqrt{2} \Rightarrow x = -\sqrt{2} \pm \sqrt{7 + 2}$ $x = -\sqrt{2} + 3$ or $-\sqrt{2} - 3$	M1 A1 M1 (no need for $\pm$) A1 (both) (4) M1 A1 (2) M1 A1 (both) (2) (8 marks)

Question Number	Scheme	Marks
5.	(a)  shape 60, 120, 180 on x-axis 5, -5 on y-axis (may be implied by part (b)) (b) $(30^\circ, 5); (150^\circ, 5); (90^\circ, -5)$ one x-coordinate all x-coordinates all correct (c) $f(x) = 2.5 \Rightarrow \sin 3x^\circ = \frac{1}{2}$ $3x = 30 \quad (150, 390, 510)$ $3x = (\alpha), 180 - \alpha, 360 + \alpha, (540 - \alpha)$ $x = 10, 50, 130, 170$ one correct value	B1 B1 B1 (3) B1 B1 B1 (3) B1 M1, M1 A1 (ignore extras out of range) (4) (10 marks)
6.	(a) $2x^{\frac{3}{2}} - 3x^{-\frac{3}{2}} = 0$ $x^3 = \frac{3}{2}$ $x = \sqrt[3]{\frac{3}{2}}$ $= 1.1447... = 1.14 \text{ (3 sf)}$ $x = \sqrt[3]{\alpha}$ (b) $f(x) = 4x^3 + 9x^{-3} - 12 + 5$ $= 4x^3 + \frac{9}{x^3} - 7$ $A = 4$ $B = 9, C = -7$ (c) $\int_1^2 f(x) \, dx = \left[x^4 - \frac{9}{2}x^{-2} - 7x \right]_1^2$ $x^n \rightarrow x^{n+1}$ $= \left(2^4 - \frac{9}{2} \times 2^{-2} - 14 \right) - \left(1 - \frac{9}{2} - 7 \right)$ $= 11\frac{3}{8} \text{ or } 11.375$ $[2] - [1]$	M1 M1 A1 cao (3) B1 B1, B1 (3) M1 A2 ft (candidate's A B, C) (-1 eeo) M1 (use of limits) A1 (5) (11 marks)

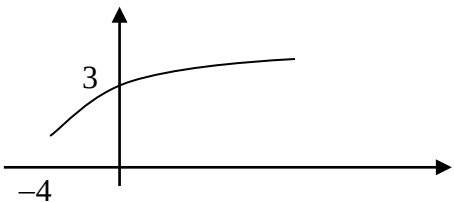
Question Number	Scheme	Marks
7.	(a) $l = (50 - 2x)$ $w = (40 - 2x)$ $V = x(50 - 2x)(40 - 2x)$ $V = xlw$ $V = x(2000 - 80x - 100x + 4x^2) = 4x(x^2 - 45x + 500)$ (*) (b) $0 < x < 20$ (accept $\leq$) (c) $\frac{dV}{dx} = 12x^2 - 360x + 2000$ (accept $\div 4$) $\frac{dV}{dx} = 0 \Rightarrow 3x^2 - 90x + 500 = 0 \Rightarrow x = \frac{90 \pm \sqrt{8100 - 6000}}{6}$ $x = (22.6),$ required $x = 7.36$ or 7.4 or 7.362 (d) $V_{\max} = 4 \times 7.36(7.36^2 \dots), = 6564 \text{ or } 6560 \text{ or } 6600$ (e) e.g. $V'' = 24x - 360 \big _{x=7.36} (= -183 \dots) < 0, \therefore \text{maximum}$	B1 M1 A1 cso (3) B1 (1) M1, A1 M1 ($dV/dx = 0$ & attempt to solve) A1 (4) M1, A1 (2) M1 full method A1 full accuracy (2) (12 marks)
8.	(a) Mid-point of $AB = [\frac{1}{2}(-3 + 8), \frac{1}{2}(-2 + 4)], = (\frac{5}{2}, 1)$ (b) $M_{AB} = \frac{4 - (-2)}{8 - (-3)}, = \frac{6}{11}$ Equation of $AB: y - 4 = \frac{6}{11}(x - 8)$ $\Rightarrow 11y - 44 = 6x - 48, \Rightarrow 6x - 11y - 4 = 0$ (or equivalent) (c) Gradient of tangent $= -\frac{11}{6}$ Equation: $y - 4 = -\frac{11}{6}(x - 8)$ (or $6y + 11x - 112 = 0$) (d) Equation of $l: y = \frac{2}{3}x$ Substitute into part (c): $\frac{2}{3}x - 4 = -\frac{11}{6}x + \frac{88}{6}$ $\Rightarrow x = 7\frac{7}{15}, y = 4\frac{44}{45}$	M1, A1 (2) M1, A1 M1 A1 (4) B1 ft M1 A1 (3) B1 M1 A1, A1 (4) (13 marks)

Question Number	Scheme	Marks
1.	(a) $1 + n(3x) + \frac{n(n-1)}{2!}(3x)^2 + \frac{n(n-1)(n-2)}{3!}(3x)^3$ (b) $\frac{n(n-1)(n-2)}{6} \times 27 = 10 \times \frac{n(n-1)}{2} \times 9$ $n = 12$ (c) $\frac{n(n-1)(n-2)(n-3)}{4!}(3x)^4$ coefficient: 40 095	B1, B1 (2) M1 A1 (2) M1 A1 (2) (6 marks)
2.	(a) $\frac{3}{x(x+2)} + \frac{x-4}{(x+2)(x-2)}$ $= \frac{3(x-2) + x(x-4)}{x(x+2)(x-2)}$ $= \frac{(x-3)(x+2)}{x(x+2)(x-2)}$	B1 B1 M1 A1 M1 A1 A1 (7) (7 marks)
3.	(a) 0, 29.05, 33.46 (b) When $t = 24.5$, $v = 33.76$. Slower at $t = 25$ (c) $s = \frac{1}{2}(5)[2(1.56 + 7.23 + 17.36 + 29.05) + 33.46]$ $= 359.65$ (359.7, 360)	B1 B1 B1 (3) B1 (1) M1 A1 A1ft A1 (4) (8 marks)
4.	(a) Adding: $\sin(A+B) + \sin(A-B) = 2 \sin A \cos B$ $\left. \begin{matrix} A+B=X \\ A-B=Y \end{matrix} \right\} 2A = X+Y$ $A = \frac{1}{2}(X+Y), B = \frac{1}{2}(X-Y)$ $\sin X + \sin Y = 2 \sin \left(\frac{X+Y}{2} \right) \cos \left(\frac{X-Y}{2} \right) (*)$ (b) $\sin 4\theta + \sin 2\theta = 2 \sin 3\theta \cos \theta$ $\sin 3\theta = 0$ (or $\cos \theta = 0$) $\theta = 0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ$ $90^\circ, 270^\circ$	M1 M1 A1 A1 (4) M1 M1 4 correct: A1 6 correct: A1 8 correct: A1 (5) (9 marks)

(*) indicates final line given in the question paper; ft = follow-through mark

Question Number	Scheme	Marks
5.	(a) $2 \log x = \log x^2$	B1
	Combine logs, e.g. $\log_2 \left(\frac{y}{x^2} \right) = 3$	M1
	$\frac{y}{x^2} = 2^3, \quad y = 8x^2 \quad (*)$	A1 (3)
	(b) $14x - 3 = 8x^2$	M1
	$(4x - 1)(2x - 3) = 0$ Roots $\frac{1}{4}$ and $\frac{3}{2}$	M1 A1 (3)
	(c) $\log_2 \alpha = \log_2 \frac{1}{4} = \log_2 (2^{-2}) = -2 \quad (*)$	B1 (1)
	(d) $\log_2 1.5 = k \quad 2^k = 1.5$	M1
	$k = \frac{\log 1.5}{\log 2} = 0.585$	M1 A1 (3)
		(10 marks)
6.	(a) $f(3.1) = 10 + \ln 9.3 - \frac{1}{2} e^{3.1} = 1.131$	
	$f(3.2) = 10 + \ln 9.6 - \frac{1}{2} e^{3.2} = -0.0045$	M1
	Sign change, so $3.1 < k < 3.2$	A1 (2)
	(b) $f'(x) = \frac{1}{x} - \frac{1}{2} e^x$	M1 A2 (1, 0) (3)
	(c) $f(1) = 10 + \ln 3 - \frac{1}{2} e$	B1
	$f'(x) = 1 - \frac{1}{2} e$	B1
	(i) $y - (10 + \ln 3 - \frac{1}{2} e) = (1 - \frac{1}{2} e)(x - 1)$	M1
	(ii) $x = 0: y = 10 + \ln 3 - \frac{1}{2} e - 1 + \frac{1}{2} e$	M1
	$= 9 + \ln 3$	A1 (5)
		(10 marks)

(*) indicates final line given in the question paper

Question Number	Scheme	Marks
7.	(a) $A: y = 16, B: y = 2$ (b) $y(x - 3) = 4, yx - 3y = 4$ $x = \frac{3y + 4}{y} \quad (*)$ (c) $x^2 = \left(3 + \frac{4}{y}\right)^2 = 9 + \frac{24}{y} + \frac{16}{y^2}$ $\int x^2 dy = \int (9 + 24y^{-1} + 16y^{-2}) dy$ $= 9y - \frac{16}{y} + 24 \ln y$ $\left[9y + 24 \ln y - 16y^{-1} \right]_2^{16} = (144 + 24 \ln 16 - 1) - (18 + 24 \ln 2 - 8)$ $V = \pi(133 + 24 \ln 8)$ (d) $V \times 27 \approx 15\,500 \quad (*)$	B1 (1) M1 (1) M1 A1 M1 A1ft, A1ft M1 A1 (7) M1 A1 (2) (11 marks)
8.	(a) $f(x) \geq -4$ (b) Domain: $x \geq -4$, range: $f^{-1}(x) \geq 1$ (c)  (d) $gf(x) = (x^2 - 2x - 3) - 4$ (e) $x^2 - 2x - 7 = 8: x^2 - 2x - 15 = 0$ $(x - 5)(x + 3) = 0$ $x = 5, x = -3 \text{ (reject)}$ $x^2 - 2x - 7 = -8: x^2 - 2x + 1 = 0$ $x = 1$	B1 (1) B1, B1 (2) Shape: B1 Above x-axis, right way round: B1 x-scale: -4 B1 y-intercept: 3 B1 (4) M1 A1 (2) M1 A1 A1ft M1 A1 (5) (14 marks)

(*) indicates final line given in the question paper; ft = follow-through mark

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June 2002

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General Certificate of Education

Subject **PURE MATHEMATICS 6673**

Paper no. **P3**

Question number	Scheme	Marks
1.	(a) <i>Complete</i> attempt at remainder theorem, or long division Either $f(3) = 27 + 9a + 3b - 10 = 14$, Or complete attempt at long division by $(x-3)$ leading to equation. Either $f(-1) = -1 + a - b - 10 = -18$ or long division by $(x+1)$ leading to equation. Equation equivalent to $9a + 3b = -3$ $(3a + b = -1)$ Equation equivalent to $a - b = -7$ Solve two equations to get $a = -2, b = 5$ (b) Either $f(2) = 8 - 8 + 10 - 10 = 0$, Or complete division with no remainder. $\therefore (x - 2)$ is a factor. Or $f(x) = (x - 2)(x^2 + 5)$	M1 A1 A1 M1, A1 (5) M1, A1 (M1 A1) (2)
2.	(a) $\int x \cos 2x dx = \frac{x \sin 2x}{2} - \int \frac{\sin 2x}{2} dx$ (integration in correct direction) $= \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} (+k)$ (second integration) (b) $x \frac{2 \sin x \cos x}{2} + \frac{1 - 2 \sin^2 x}{4} (+k)$ (use of appropriate double angle formulae) $= \frac{1}{2} \sin x (2x \cos x - \sin x) + \frac{1}{4} + k$ for $\frac{1}{4} + k$ $= \frac{1}{2} \sin x (2x \cos x - \sin x) + C$ ★	M1 A1 M1 A1 (4) M1 A1 A1 c.a.o. (3)

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Subject **PURE MATHEMATICS 6673**

Paper no. **P3**

Question number	Scheme	Marks
3.	(a) Either completion of square, $(x-4)^2 + (y-8)^2 = 209 + 16 + 64$ Or use of formulae, $(-f, -g)$, $r = \sqrt{(f^2 + g^2 - c)}$ Centre is (4,8), radius 17. (b) <i>Either</i> $2x + 2y \frac{dy}{dx} - 8 - 16 \frac{dy}{dx} = 0$ $\frac{dy}{dx}(2y-16) = 8-2x$, so $\frac{dy}{dx} = \frac{4-x}{y-8}$. Or gradient of CP is $\frac{y-8}{x-4}$. Tangent is perpendicular to CP and so has gradient $\frac{4-x}{y-8}$. (c) At (21,8) the tangent is vertical, so equation is $x = 21$.	M1 A1,A1 (3) M1, A1 A1 (3) B1 M1, A1 (3) M1, A1 (2)
4.	(a) $x^2 + 1 \equiv A(1+x)(3-x) + B(3-x) + C(1+x)$; $A = -1$ $2=4B$, $B=\frac{1}{2}$; $10 = 4C$, $C=\frac{5}{2}$ (b) $\int (-1 + \frac{1}{2(1+x)} + \frac{5}{2(3-x)}) dx$ $= -x + \frac{1}{2} \ln(1+x) - \frac{5}{2} \ln(3-x)$ $\int_0^2 f(x) = (-2 + \frac{1}{2} \ln 3) - (-\frac{5}{2} \ln 3) = -2 + 3 \ln 3$	B1 M1 A1; A1 (4) M1A1✓A1✓ M1A1 (5)

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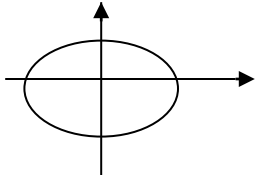
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Subject **PURE MATHEMATICS 6673**

Paper no. **P3**

Question number	Scheme	Marks
5.	(a) $(1 + \frac{5}{15})^{-\frac{1}{2}} = (\frac{4}{3})^{-\frac{1}{2}}, \quad = (\frac{3}{4})^{\frac{1}{2}} = \frac{\sqrt{3}}{2}; = \sin 60^\circ$ (b) $1 + 5x(-\frac{1}{2}) + \frac{(5x)^2}{2}(-\frac{1}{2})(-\frac{3}{2}) + \frac{(5x)^3}{6}(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2}) \dots$ $= 1 - \frac{5}{2}x + \frac{75}{8}x^2 - \frac{625}{16}x^3 \dots$ (c) $= 1 - \frac{5}{2}(\frac{1}{15}) + \frac{75}{8}(\frac{1}{15})^2 - \frac{625}{16}(\frac{1}{15})^3 \dots = 0.863(4259..)$ (d) $\sin 60^\circ - (\text{Ans}) \approx 0.0026$	M1,A1;A1 (3) M1, B1, A1, A1 (4) M1 A1 (2) A1 (1)
6.	(a) $5 \cos t = 0 : t = \frac{\pi}{2}, y = 2 \quad (0, 2)$ $t = \frac{3\pi}{2}, y = -6 \quad (0, -6)$ $4 \sin t = 2 : t = \frac{\pi}{6}, x = \frac{5\sqrt{3}}{2} \quad t = \frac{5\pi}{6}, x = -\frac{5\sqrt{3}}{2} \quad (\pm \frac{5\sqrt{3}}{2}, 0)$ (b)  shape position (c) $\frac{dx}{dt} = -5 \sin t, \quad \frac{dy}{dt} = 4 \cos t, \quad \frac{dy}{dx} = \frac{-4 \cos t}{5 \sin t}$ $y = \frac{5 \tan \frac{\pi}{6}}{4} (x - \frac{5\sqrt{3}}{2}), \quad \text{i.e.} \quad 8\sqrt{3}y = 10x - 25\sqrt{3} \quad \star$	M1A1 M1A1 (4) B1 B1 (2) M1A1 M1, A1 (4)

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Subject **PURE MATHEMATICS 6673**

Paper no. **P3**

Question number	Scheme	Marks
7.	(a) $\frac{dV}{dt} = -kV$ $\int \frac{1}{V} dV = -k \int dt, \ln V = -kt$ $\ln V = -kt + C \quad V = Ae^{-kt} \quad *$	B1 M1 A1 (3)
	(b) $t = 0, V = 20\,000: \quad 20\,000 = A$ $t = 3, V = 11\,000: \quad 11\,000 = Ae^{-3k}$ $e^{-3k} = 0.55$ $-3k = \ln 0.55$ $k \approx 0.199(3) \quad (\text{allow } 0.2)$ $t = 10 \quad V = 20\,000e^{-10k}; \quad = \pounds 2700$	B1 M1, A1 M1;A1 (5)
	(c) $500 = 20\,000e^{-kt} \quad e^{-kt} = 0.025$ $-0.199t = \ln 0.025$ $t \approx 18.5 \text{ (18.44) accept 18 or 19 yrs}$	M1 A1✓ A1 (3)

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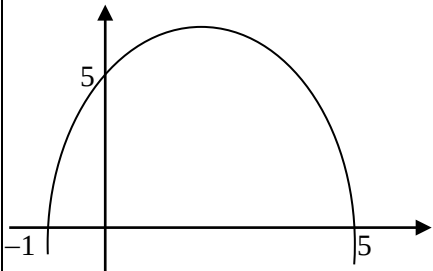
Paper no. **P3**

Question number	Scheme	Marks
8.	(a) $\mathbf{r} = (9\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + \lambda(-3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})$ (or any correct alternative) (b) Uses their line equation, or recognises B is mid point of AC or merely writes down p=6, q=11 (c) Calculates $\overrightarrow{OC} \bullet \overrightarrow{AB}$ Uses $\cos \alpha = \frac{OC \bullet AB}{ OC AB }$ to obtain α. $\cos \alpha = \frac{70}{\sqrt{166}\sqrt{50}}$, $\alpha = 39.8^\circ$ (accept 39.79 or 40) (d) Let OD be $(9\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + t(-3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})$ Use scalar product OD. AB=0 Obtains equation in t and solves to obtain $t = 0.6$ (or equivalent) Uses their t, to obtain $7.2\mathbf{i} + 0.4\mathbf{j} + 4\mathbf{k}$.	M1,A1 (2) M1, A1 (2) M1 M1 A1 (3) M1 M1 M1A1 M1, A1 (6)

EDEXCEL PURE MATHEMATICS P1 (6671) - NOVEMBER 2002 PROVISIONAL MARK SCHEME

Question Number	Scheme	Marks
1.	(a) $3x - x > 13 + 8$ $x > \frac{21}{2}$ (b) $x^2 - 5x - 14 > 0$ $(x - 7)(x + 2) > 0$ $x = 7, -2$ $x < -2$ or $x > 7$	M1, A1 (2) B1 M1, A1 ft (3) (5 marks)
2.	(a) $f(-3) = -27 - 27 + 30 + 24 = 0 \Rightarrow (x + 3)$ is factor (b) $(x + 3)(x^2 - 6x + 8)$ $(x + 3)(x - 2)(x - 4)$	M1 A1 (2) M1 A1 M1 A1 (4) (6 marks)
3.	(i) Divide: $1 + 2x^{-1}$ Differentiate: $6x^2 + \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-2}$ (ii) $\frac{x^2}{4} + \frac{x^{-1}}{-1}$ $[]^4 - []_1 = \left(4 - \frac{1}{4}\right) - \left(\frac{1}{4} - 1\right) = 4\frac{1}{2}$	M1 A1 M1 A2 (1,0) (5) M1 A1A1 M1 A1 (5) (10 marks)
4.	(a) $S = a + (a + d) + (a + 2d) + \dots + [a + (n - 1)d]$ $S = [a + (n - 1)d] + [a + (n - 2)d] + \dots + a$ Add: $2S = n[2a + (n - 1)d] \Rightarrow S = \frac{1}{2}n[2a + (n - 1)d]$ (b) $a = 54000$ and $n = 9$ $619200 = \frac{1}{2} \times 9 \times (2 \times 54000 + 8d)$ $d = 3700$ (c) $a + (n - 1)d = a + 10d = 54000 + 10d = \text{£}91000$ (d) $ar^{n-1} = 54000 \times 1.06^{10}$ (ft their n) $= \text{£}96700$ (or $\text{£}97000$)	B1 M1 M1 A1 (4) B1 M1 A1ft A1 (4) M1 A1 (2) M1 A1ft A1 (3) (13 marks)

EDEXCEL PURE MATHEMATICS P1 (6671) - NOVEMBER 2002 PROVISIONAL MARK SCHEME

Question Number	Scheme	Marks
5.	(i) $\arcsin 0.6 = 36.9^\circ$ (awrt) α $2x + 50 = 36.87, \quad 2x = -13.13^\circ + 360^\circ = 346.87^\circ$ $2x + 50 + 180 - 36.87, \quad 2x = 143.13^\circ - 50^\circ = 93.13^\circ$ $x = 46.6, \quad 173.4$ (ii) (a) $\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \frac{BC}{\left(\frac{1}{3}\right)} = \frac{18}{\sin 60^\circ}$ $BC = 6 \div \frac{\sqrt{3}}{2} \quad BC = \frac{12}{\sqrt{3}} = 4\sqrt{3} \quad (*)$ (b) $\cos^2 \theta = 1 - \sin^2 \theta = 1 - \frac{1}{9}$ $\sin \theta = \sqrt{\frac{8}{9}} \quad \left(= \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3} \right)$	B1 M1 M1 M1 M1 A1 A1 (7) B1, M1 M1 A1 (4) M1 A1 (2) (13 marks)
6.	(a) 9 (b)  Shape Position of max. 5 on y-axis -1 and 5 on x-axis (c) Gradient: $\frac{8 - (-7)}{3 - (-2)}$ $y - 8 = \text{"gradient"} (x - 3) \quad y = 3x - 1$ (d) Where $y = 0, \quad x = \frac{1}{3}$ (e) Mid point: $\left(\frac{-7+8}{2}, \frac{-2+3}{2} \right) = \left(\frac{1}{2}, \frac{1}{2} \right) \quad k = 1$	B1 (1) B1 B1 B1 M1 A1 (5) M1 A1 M1 A1 (4) M1 A1ft (2) M1 A1 (2) (14 marks)

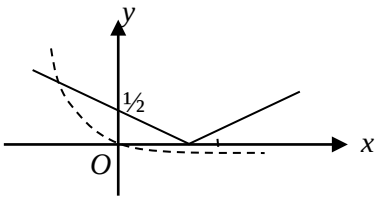
EDEXCEL PURE MATHEMATICS P1 (6671) - NOVEMBER 2002 PROVISIONAL MARK SCHEME

Question Number	Scheme	Marks
7.	(a) Integrate: $y = x^3 - 10x^2 + 29x$ (+C) $6 = 8 - 40 + 58 + C \Rightarrow C = -20$ ($y = x^3 - 10x^2 + 29x - 20$) (b) Substitute $x = 4$: $64 - 160 + 116 - 20 = 0$ (c) At $x = 2$, $\frac{dy}{dx} = 12 - 40 + 29 = 1$ Tangent: $y - 6 = x - 2$ ($y = x + 4$) (d) $\frac{dy}{dx} = 1$ $3x^2 - 20x + 28 = 0$ $(3x - 14)(x - 2) = 0$ $x = \frac{14}{3}$	M1 A1 M1 A1 (4) M1 A1 (2) B1 M1 A1 (3) M1 M1 M1 A1 A1 (5) (14 marks)

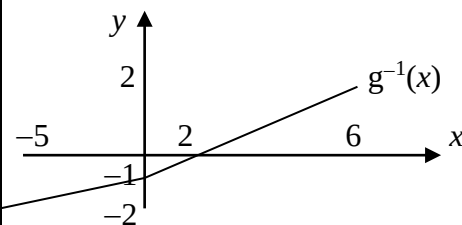
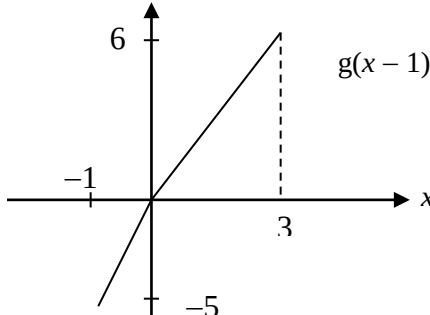
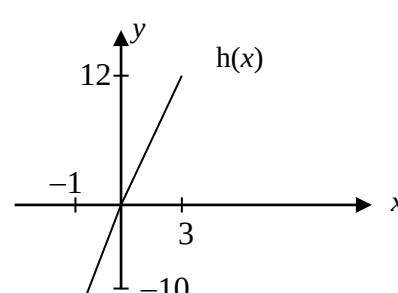
EDEXCEL PURE MATHEMATICS P2 (6672) - NOVEMBER 2002 PROVISIONAL MARK SCHEME

Question Number	Scheme	Marks
1.	$\frac{y+3}{(y+1)(y+2)} - \frac{y+1}{(y+2)(y+3)} \equiv \frac{(y+3)^2 - (y+1)^2}{(y+1)(y+2)(y+3)}$ $\equiv \frac{(y^2 + 6y + 9) - (y^2 + 2y + 1)}{(y+1)(y+2)(y+3)} \equiv \frac{4y+8}{(y+1)(y+2)(y+3)}$ $\equiv \frac{4(y+2)}{(y+1)(y+2)(y+3)} \equiv \frac{4}{(y+1)(y+3)} \text{ or } \frac{4}{y^2 + 4y + 3}$	M1 M1 A1 M1, A1 (5 marks)
2.	(a) $4^x = (2^x)^2 = u^2$ or $2^{(x+1)} = 2 \cdot 2^x = 2u, \rightarrow u^2 - 2u - 15 (=0)$ (b) $u^2 - 2u - 15 = (u-5)(u+3)$ $u = 5 \Rightarrow 2^x = 5 \Rightarrow x = \frac{\log 5}{\log 2}, = 2.32$ [Ignore any other solution]	M1, A1 c.s.o (2) M1, A1 M1, A1 (4) (6 marks)
3.	(a) $1.5 \sin 2x + 2 \cos 2x = R \sin (2x + \alpha) = R[\sin 2x \cos \alpha + \cos 2x \sin \alpha]$ $R = \sqrt{1.5^2 + 2^2} = 2.5$ Full method for R or R² $\tan \alpha = \frac{2}{1.5}, \Rightarrow \alpha = 0.927$ Full method for tan α, sin α, cos α (b) $3 \sin x \cos x = 1.5 \sin 2x$ $4 \cos^2 x = 2[2 \cos^2 x] = 2(\cos 2x + 1)$ $\therefore 3 \sin x \cos x + 4 \cos^2 x = 1.5 \sin 2x + 2 \cos 2x + 2$ (c) Maximum value of $1.5 \sin 2x + 2 \cos 2x = R$ Maximum value of $3 \sin x \cos x + 4 \cos^2 x = R + 2$ or 4.5	M1, A1 M1, A1 (4) M1 A1 (2) M1 A1 ft (2) (8 marks)

Question Number	Scheme	Marks
4.	(a) $u_2 = 2p + 5$ $u_3 = p(2p + 5) + 5$ $8 = 2p^2 + 5p + 5$ or $2p^2 + 5p - 3 = 0$ $(2p - 1)(p + 3) = 0$ $p = -3$, or $\frac{1}{2}$ (b) $\log_2 \left(\frac{1}{2}\right) = \log_2 2^{-1} = -1$ (c) $\log_2 \left(\frac{p^3}{\sqrt{q}}\right) = \log_2 p^3 - \log_2 \sqrt{q}$ Use of $\log a - \log b$ b $= 3\log_2 p - \frac{1}{2} \log_2 q$ Use of $\log a^n$ $= -3 - \frac{1}{2} t$ accept $3 \log_2 p - \frac{1}{2} t$	B1 M1 M1 A1, B1 cso (5) B1 (1) M1 M1 A1 ft (3) (9 marks)
5.	(a) $(0, 2)$ on $C \Rightarrow 2 = p + q$ Use of $(0, 2)$ equation in p and q $\frac{dy}{dx} = qe^x$, at $p \Rightarrow 5 = 2q$ Solving $\Rightarrow q = 2.5, p = -0.5$ (or $2 - q$) (b) Gradient of normal at P is $-\frac{1}{5}$ Equation of normal at P is: $y - (p + 2q) = -\frac{1}{5}(x - \ln 2)$ at L $y = 0 \therefore x_L = 22.5 + \ln 2$ or $5(p + 2q) + \ln 2$ or $23.19\dots$ at M $x = 0 \therefore y_M = 4.5 + \frac{1}{5} \ln 2$ or $p + 2q + \frac{1}{5} \ln 2$ or $4.639\dots$ Area of triangle OLM is: $\frac{1}{2} x_L \times y_M = 53.792\dots \approx 53.8$	M1 M1, A1 (3) A1, A1 (5) B1 M1 M1 M1, A1 cso (5) (10 marks)

Question Number	Scheme	Marks
6. (a)	 Shape  with vertex on +ve x-axis (1, 0) and (0, 1/2)	B1
(b)	$x = \alpha$ given by: $e^{-x} - 1 = -\frac{1}{2}(x-1)$ Use of $-\frac{1}{2}(x-1)$ $\Rightarrow 2e^{-x} - 2 = -x + 1$, i.e. $x + 2e^{-x} - 3 = 0$	B1 (2) M1 A1
(c)	$f(x) = x + 2e^{-x} - 3$: $f(0) = 2 - 3 = -1$ 1 correct value to 1.s.f. $f(-1) = -4 + 2e^1 = 1.43\dots$ Change of sign $\therefore$ root in $-1 < \alpha < 0$ Both correct and comment	A1 cs0 (3) M1
(d)	$x_1 = -0.693(1\dots)$, $x_2 = -0.613(3\dots)$	A1 (2) B1, B1
(e)	$f(-0.575) = -0.0207\dots$ $f(-0.585) = 0.00498\dots$	Change of sign so root is -0.58 to 2dp. M1 A1 (2)
(11 marks)		
ALT (e)	$x_3 = 0.5914\dots$, $x_4 = -0.5854\dots$, $x_5 = -0.5837\dots$, $x_6 = 0.5832\dots$, $(x_7 = -0.5831\dots)$	M1 A1

Question Number	Scheme	Marks												
7. (a)	<table><tr><td>x:</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td></tr><tr><td>y:</td><td>2</td><td>2.25</td><td>3</td><td>4.25</td><td>6</td></tr></table> ≥ 2 correct ys	x:	0	0.5	1	1.5	2	y:	2	2.25	3	4.25	6	M1
	x:	0	0.5	1	1.5	2								
	y:	2	2.25	3	4.25	6								
	$R \approx \frac{1}{2} \times \frac{1}{2}, [2 + 2\{2.25 + 3 + 4.25\} + 6]$	B1, [M1 A1 ft]												
	$\frac{27}{4}$ or 6.75	A1												
	(b) Since curve bends under straight line $\rightarrow$ overestimate	B1 M1												
	(c) $V = \pi \int_0^2 y^2 dx = \pi \int_0^2 (x^4 + 4x^2 + 4) dx$	$\pi \int y^2, y^2 = (\quad)$ M1 M1												
	$= \pi \left[\frac{x^5}{5} + \frac{4}{3} x^3 + 4x \right]_0^2$	$x^n \rightarrow x^{n+1}$ M1 A1												
	$= \pi \left[\left(\frac{32}{5} + \frac{32}{3} + 8 \right) - (0) \right]$	Use of correct limits M1												
	$= \frac{\pi}{15} [96 + 160 + 120] = \frac{376}{15} \pi$ (or $25 \frac{1}{15}$ or 25.1π)	A1 (6)												
	(11 marks)													

Question Number	Scheme	Marks
8.	(a) $y = \frac{3x-1}{x-3} \Rightarrow y(x-3) = 3x-1$	M1
	$yx - 3x = 3y - 1$	
	$x(y-3) = 3y-1$	Collect x and factorise M1
	$x = \frac{3y-1}{y-3} \therefore f^{-1}(x) = \frac{3x-1}{x-3} = f(x)$	A1 cso (3)
	(b) $ff(k) = f^{-1}f(k) = k$	M1 A1 (2)
	(c) $g(-2) = -5$	B1
	$f(-5) = \frac{-15-1}{-8} = \frac{-16}{-8} = 2$	M1, A1 (3)
	(d) 	shape B1 (0, -1) and (2, 0) B1 Domain: $-5 \leq x \leq 6$ B1 (3)
		Translation +1 → B1 (lines join at (0,0))
		Stretch $\times 2 \uparrow$ B1 Range: $-10 \leq h(x) \leq 12$ B1 (3)
		(14 marks)

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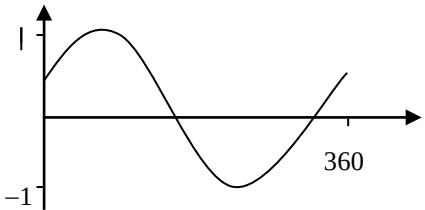
January 2003

Advanced Subsidiary / Advanced Level

General Certificate of Education

Subject **PURE MATHEMATICS 6671**

Paper No. **P1**

Question number	Scheme	Marks
1.	(a) $\frac{dy}{dx} = 10 \times \frac{3}{2} x^{\frac{1}{2}} \quad \left(= 15x^{\frac{1}{2}} \right)$ (b) $7x + 4x^{\frac{5}{2}} + C$	M1 A1 M1 A2(1,0)
2.	(a)  Scales (−1, 1 and 360) Shape, position (b) (0, 0.5) (150, 0) (330, 0) (c) $(x + 30 =) 210^\circ$ or 330° One of these $x = 180^\circ, 300^\circ$ M: Subtract 30, A: Both	B1 B1 B1 B1 B1 B1 M1 A1
3.	(a) $3^x = 3^{2(y-1)} \quad x = 2(y-1) \quad (*)$ (b) $(2y-2)^2 = y^2 + 7, \quad 3y^2 - 8y - 3 = 0$ $(3y+1)(y-3) = 0, y = \dots$ (or correct substitution in formula) $y = -\frac{1}{3}, \quad y = 3$ $x = -\frac{8}{3}, \quad x = 4$	M1 A1 M1, A1 M1 A1 M1 A1ft

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Subject **PURE MATHEMATICS 6671**

Paper No. **P1**

Question number	Scheme	Marks
4.	(a) $\frac{a}{1-r} = \frac{1200}{1-r} = 960$ $960(1-r) = 1200$ $r = -\frac{1}{4}$ (*) (b) $T_9 = 1200 \times (-0.25)^8$ (or T_{10}) Difference = $T_9 - T_{10} = 0.0183105... - (-0.0045776...)$ $= 0.023$ (or -0.023) (c) $S_n = \frac{1200(1 - (-0.25)^n)}{1 - (-0.25)}$ (d) Since n is odd, $(-0.25)^n$ is negative, so $S_n = 960(1 + 0.25^n)$ (*)	M1 A1 A1 M1 M1 A1 M1 A1 M1 A1

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Subject **PURE MATHEMATICS 6671**

Paper No. **P1**

Question number	Scheme	Marks
5.	(a) $\frac{dC}{dv} = -160v^{-2} + \frac{2v}{100}$ $-160v^{-2} + \frac{2v}{100} = 0$ $v^3 = 8\,000 \quad v = 20$ (b) $\frac{d^2C}{dv^2} = 320v^{-3} + \frac{1}{50}$ > 0, therefore minimum (c) $v = 20 : C = \frac{160}{20} + \frac{400}{100} = 12$ Cost = $250 \times 12 = \text{£}30$	M1 A1 M1 M1 A1 M1 A1 B1ft M1 A1

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Subject **PURE MATHEMATICS 6671**

Paper No. **P1**

Question number	Scheme	Marks
6.	(a) P: $x = 0$ $y = -2$ Mid-point: $\left(\frac{(0+5)}{2}, \frac{(-2-3)}{2} \right) = \left(\frac{5}{2}, -\frac{5}{2} \right)$ (b) Gradient of l_1 is $\frac{3}{2}$, so gradient of l_2 is $-\frac{2}{3}$ l_2: $y - (-3) = -\frac{2}{3}(x - 5)$ $2x + 3y = 1$ (c) Solving: $3x - 2y = 4$ $2x + 3y = 1$ $x = \frac{14}{13}$ $y = \frac{-5}{13}$	B1 M1 A1ft B1 M1 A1ft A1 M1 A1 M1 A1ft

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Subject **PURE MATHEMATICS 6671**

Paper No. **P1**

Question number	Scheme	Marks
7.	(a) $BM = \sqrt{7^2 + 24^2} = 25$ (*) (b) $\tan \alpha = \frac{7}{24}$ or equiv. and $\angle BMC = 2\alpha$, or cosine rule $\angle BMC = 0.568$ radians (*) (c) ΔABM: $\frac{1}{2}(14 \times 24)$ (= 168 mm²) (or other appropriate Δ) Sector: $\frac{1}{2}(25^2 \times 0.568)$ Total: “168 + 168 + 177.5” = 513 mm² (or 514, or 510) (d) Volume = “513” $\times$ 85 mm³ (M requires unit conversion) M1 = 44 cm³ A1	B1 M1 A1 A1 B1 M1 A1 M1 A1 M1 A1

January 2003

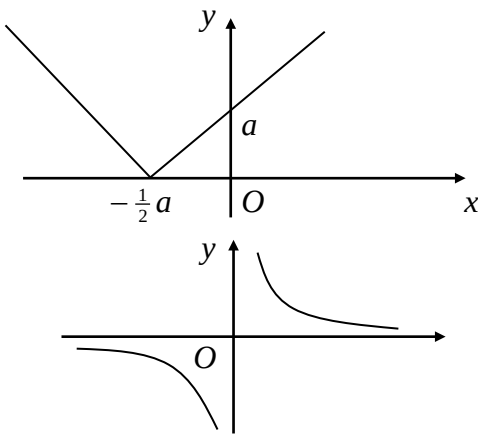
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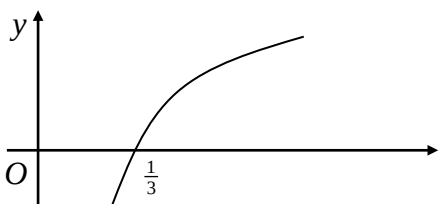
Subject **PURE MATHEMATICS 6671**

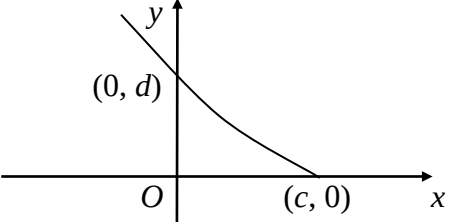
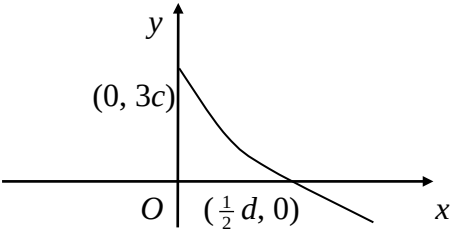
Paper No. **P1**

Question number	Scheme	Marks
8.	(a) $A: y = 1$ $B: y = 4$ (b) $\frac{dy}{dx} = \frac{2x}{25} = \frac{2}{5}$ where $x = 5$ Tangent: $y - 1 = \frac{2}{5}(x - 5)$ $(5y = 2x - 5)$ (c) $x = 5y^{\frac{1}{2}}$ (d) Integrate: $\frac{5y^{\frac{3}{2}}}{\frac{3}{2}} \left(= \frac{10y^{\frac{3}{2}}}{3} \right)$ M1 A1ft $[]^4 - []_1 = \left(\frac{10 \times 4^{\frac{3}{2}}}{3} \right) - \left(\frac{10 \times 1^{\frac{3}{2}}}{3} \right), = \frac{70}{3} \quad (23\frac{1}{3}, 23.3)$ M1 A1, A1	B1 M1 A1 M1 A1 B1 B1
	<u>Alternative for (d):</u> Integrate: $\frac{x^3}{75}$ Area = $(10 \times 4) - (5 \times 1) - \left(\frac{1000}{75} - \frac{125}{75} \right), = \frac{70}{3} \quad (23\frac{1}{3}, 23.3)$ In both (d) schemes, final M is scored using <u>candidate's</u> “4” and “1”.	M1 A1 M1 A1, A1

Question number	Scheme	Marks
1.	$x^2 - 9 = (x - 3)(x + 3)$ seen Attempt at forming single fraction $\frac{x(x - 3) + (x + 12)(x + 1)}{(x + 1)(x + 3)(x - 3)}; = \frac{2x^2 + 10x + 12}{(x + 1)(x + 3)(x - 3)}$ Factorising numerator = $\frac{2(x + 2)(x + 3)}{(x + 1)(x + 3)(x - 3)}$ or equivalent = $\frac{2(x + 2)}{(x + 1)(x - 3)}$	B1 M1; A1 M1 M1 A1 (6) (6 marks)
2.	$(1 + px)^n \equiv 1 + np x + \frac{n(n - 1)p^2 x^2}{2} + \dots$ Comparing coefficients: $np = -18$, $\frac{n(n - 1)}{2} = 36$ Solving $n(n - 1) = 72$ to give $n = 9$; $p = -2$	B1, B1 M1, A1 M1 A1; A1 ft (7) (7 marks)
3.	(a)  V graph with 'vertex' on x-axis $\{-\frac{1}{2}a, (0)\}$ and $\{(0), a\}$ seen Correct graph (could be separate) (c) Meet where $\frac{1}{x} = 2x + a \Rightarrow x 2x + a - 1 = 0$; only one meet (d) $2x^2 + x - 1$ Attempt to solve; $x = \frac{1}{2}$ (no other value)	M1 A1 (2) B1 (1) B1 (1) B1 M1; A1 (3) (7 marks)

Question number	Scheme	Marks														
4.	Volume = $\pi \int_1^4 \left(1 + \frac{1}{2\sqrt{x}}\right)^2 \mathrm{d}x$ $\int \left(1 + \frac{1}{2\sqrt{x}}\right)^2 \mathrm{d}x = \int \left(1 + \frac{1}{\sqrt{x}} + \frac{1}{4x}\right) \mathrm{d}x$ $= \left[x + 2\sqrt{x} + \frac{1}{4} \ln x \right]$ Using limits correctly Volume = $\pi \left[\left(8 + \frac{1}{4} \ln 4\right) + 3 \right]$ $= \pi \left[5 + \frac{1}{2} \ln 2 \right]$	M1 B1 M1 A1 A1ft M1 A1 A1 (8) (8 marks)														
5.	(a)<table><tr><td>Distance from one side (m)</td><td>0</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>Height (m)</td><td>0</td><td>6.13</td><td>7.80</td><td>7.80</td><td>6.13</td><td>0</td></tr></table> “y” = 7.80 when “x” = 4 or 6 Symmetry (b) Estimate area = $\frac{2}{2} [0 + 2(6.13 + 7.80 + 7.80 + 6.13)]$ $= 55.7 \text{ m}^2$ (c) $140 - (b) = 84.3 \text{ m}^2$ (d) Over-estimate; reason, e.g. area under curve is under-estimate (due to curvature)	Distance from one side (m)	0	2	4	6	8	10	Height (m)	0	6.13	7.80	7.80	6.13	0	B1 B1 ft (2) B1 M1 A1ft A1 (4) A1 ft (1) B1 B1 (2) (9 marks)
Distance from one side (m)	0	2	4	6	8	10										
Height (m)	0	6.13	7.80	7.80	6.13	0										

Question number	Scheme	Marks
6. (a)		Shape B1 $p = \frac{1}{3}$ or $\{\frac{1}{3}, 0\}$ seen B1 (2)
(b)	Gradient of tangent at $Q = \frac{1}{q}$ Gradient of normal $= -q$ Attempt at equation of OQ [$y = -qx$] and substituting $x = q, y = \ln 3q$ or attempt at equation of tangent [$y - 3 \ln q = -q(x - q)$] with $x = 0, y = 0$ or equating gradient of normal to $(\ln 3q)/q$ $q^2 + \ln 3q = 0$ (*)	B1 M1 M1 A1 (4)
(c)	$\ln 3x = -x^2 \Rightarrow 3x = e^{-x^2}; \Rightarrow x = \frac{1}{3}e^{-x^2}$	M1; A1 (2)
(d)	$x_1 = 0.298280; x_2 = 0.304957, x_3 = 0.303731, x_4 = 0.303958$ Root $= 0.304$ (3 decimal places)	M1; A1 A1 (3)
		(11 marks)
7. (a)	$\sin x + \sqrt{3} \cos x = R \sin(x + \alpha)$ $= R(\sin x \cos \alpha + \cos x \sin \alpha)$ $R \cos \alpha = 1, R \sin \alpha = \sqrt{3}$ Method for R or α , e.g. $R = \sqrt{1 + 3}$ or $\tan \alpha = \sqrt{3}$ Both $R = 2$ and $\alpha = 60$	M1 A1 M1 A1 (4)
(b)	$\sec x + \sqrt{3} \operatorname{cosec} x = 4 \Rightarrow \frac{1}{\cos x} + \frac{\sqrt{3}}{\sin x} = 4$ $\Rightarrow \sin x + \sqrt{3} \cos x = 4 \sin x \cos x$ $= 2 \sin 2x$ (*)	B1 M1 M1 (3)
(c)	Clearly producing $2 \sin 2x = 2 \sin(x + 60)$	A1 (1)
(d)	$\sin 2x - \sin(x + 60) = 0 \Rightarrow \cos \frac{3x + 60}{2} \sin \frac{x - 60}{2} = 0$ $\cos \frac{3x + 60}{2} = 0 \Rightarrow x = 40^\circ, 160^\circ$ $\sin \frac{x - 60}{2} = 0 \Rightarrow x = 60^\circ$	M1 M1 A1 A1 ft B1 (5)
		(13 marks)

Question number	Scheme	Marks
8. (a)	 shape intersections with axes $(c, 0), (0, d)$	B1 B1 (2)
(b)	 shape x intersection $(\frac{1}{2}d, 0)$ y intersection $(0, 3c)$	B1 B1 B1 (3)
(c)(i)	$c = 2$	B1
(ii)	$-1 < f(x) \leq$ (candidate's) c value	B1 B1 ft (3)
(d)	$3(2^{-x}) = 1 \Rightarrow 2^{-x} = \frac{1}{3}$ and take logs; $-x = \frac{\ln \frac{1}{3}}{\ln 2}$ d (or x) = 1.585 (3 decimal places)	M1; A1 A1 (3)
(e)	$fg(x) = f[\log_2 x] = [3(2^{-\log_2 x}) - 1]; = [3(2^{\log_2 \frac{1}{x}}) - 1]$ or $\frac{3}{2^{\log_2 x}} - 1$ $= \frac{3}{x} - 1$	M1; A1 A1 (3)
		(14 marks)

Question Number	Scheme	Marks
1(a)	$\frac{3(x+1)}{(x+2)(x-1)} \equiv \frac{A}{x+2} + \frac{B}{x-1}, \text{ and correct method for finding } A \text{ or } B$	M1
	$A = 1, \quad B = 2$	A1, A1 (3)
(b)	$f'(x) = -\frac{1}{(x+2)^2} - \frac{2}{(x-1)^2}$	M1 A1
	Argument for negative, including statement that square terms are positive for all values of x. (f.t. on wrong values of A and B)	A1ft✓ (3)
2		
(a)	$a = 4, \quad b = 5 \quad (\text{both are required})$	B1 (1)
(b)	$(x-4)^2 + (y-5)^2 = 25$	M1A1ft (2)
(c)	Finding the distance between centre and (8, 17), $\sqrt{[(8-a)^2 + (17-b)^2]}$	M1
	Complete method to find PT, i.e. use Pythagoras theorem and subtraction,	M1
	$PT = 11.6$	A1 (3)

Question Number	Scheme	Marks
3(a)	Using $f(\pm 2) = 3$	M1
	Showing that $p = 6$ ★, with no wrong working seen.	A1 (2)
	S.C. If $p = 6$ used and the remainder is shown to be 3 award B1	
(b)	Attempt to find quotient when dividing $(n + 2)$ into $f(n)$ or attempting to equate coefficients.	M1
	Quotient = $n^2 + 4n + 3$, or finding either $q = 1$ or $r = 3$	A1
	Finding both $q = 1$ and $r = 3$	A1 (3)
(c)	The product of three consecutive numbers must be divisible by 3	M1
	Complete argument	A1 (2)
4. (a)	$(1+3x)^{-2} = 1 + (-2)(3x) + \frac{(-2)(-3)}{2!}(3x)^2 + \frac{(-2)(-3)(-4)}{3!}(3x)^3 + \dots$ $= 1, -6x, +27x^2 \dots (-108x^3)$	M1 B1, A1, A1 (4)
(b)	Using (a) to expand $(x+4)(1+3x)^{-2}$ or complete method to find coefficients [e.g. Maclaurin or $\frac{1}{3}(1+3x)^{-1} + \frac{11}{3}(1+3x)^{-2}$]. $= 4 - 23x, +102x^2, -405x^3 = 4, -23x, +102x^2 \dots (-405x^3)$	M1 A1, A1ft, A1ft (4)

Question Number	Scheme	Marks
6(a)	$\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \pm \lambda(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$ or $\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} \pm \lambda(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$ (or any equivalent vector equation)	M1A1 (2)
(b)	Show that $\mu = -3$	B1 (1)
(c)	Using $\cos \theta = \frac{(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) \cdot (\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})}{\sqrt{(4^2 + 5^2 + 3^2)}\sqrt{(1^2 + 2^2 + 2^2)}}$ $= \frac{20}{15\sqrt{2}} = \frac{4}{3\sqrt{2}}$ (ft on $4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$) num, denom.	M1 A1ft A1ft
	$\theta = 19.5^\circ$ (allow 19 or 20 if no wrong working is seen)	A1 (4)
(d)	Shortest distance = $AC \sin \theta$	M1
	$AC = \sqrt{((a-1)^2 + 2^2 + (b+3)^2)}$ (= 3)	M1A1
	Shortest distance = 1 unit	A1 (4)
	<i>Alternatives</i> Since $X = (1+4\lambda, 2-5\lambda, -3+3\lambda)$ $\mathbf{CX} = (-1+4\lambda)\mathbf{i} + (2-5\lambda)\mathbf{j} + (-2+3\lambda)\mathbf{k}$ Use Scalar product $\mathbf{CX} \cdot (4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) = 0$, OR differentiate $ \mathbf{CX} $ or $ \mathbf{CX} ^2$ and equate to zero, to obtain $\lambda = 0.4$ and thus $ \mathbf{CX} = 1$	M1 M1 A1 A1 (4)

Question Number	Scheme	Marks
5. (a)	$\frac{dV}{dt} = 30 - \frac{2}{15}V$ $\Rightarrow -15 \frac{dV}{dt} = -450 + 2V, \quad \text{no wrong working seen}$	M1A1 A1* (3)
(b)	Separating the variables $\Rightarrow -\frac{15}{2V-450}dV = dt$ Integrating to obtain $-\frac{15}{2}\ln 2V-450 = t$ OR $-\frac{15}{2}\ln V-225 = t$ Using limits correctly or finding c ($-\frac{15}{2}\ln 1550$ OR $-\frac{15}{2}\ln 775$) $\ln \frac{2V-450}{1550} = -\frac{2}{15}t$, or equivalent Rearranging to give $V = 225 + 775e^{-\frac{2}{15}t}$.	M1 dM1 A1 M1 A1 dM1A1 (7)
(c)	$V = 225$	B1 (1)

Question Number	Scheme	Marks
7(a)	$\frac{dy}{dx} = -2e^{-2x}\sqrt{x} + \frac{e^{-2x}}{2\sqrt{x}}$ Putting $\frac{dy}{dx} = 0$ and attempting to solve $x = \frac{1}{4}$	M1 A1 A1 dM1 A1 (5)
(b)	$\text{Volume} = \pi \int_0^1 (\sqrt{x}e^{-2x})^2 dx = \pi \int_0^1 xe^{-4x} dx$ $\int xe^{-4x} dx = -\frac{1}{4}xe^{-4x} + \int \frac{1}{4}e^{-4x} dx$ $= -\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x}$ $\text{Volume} = \pi \left[-\frac{1}{4}e^{-4} - \frac{1}{16}e^{-4} \right] - \left[-\frac{1}{16} \right] = \frac{\pi}{16} [1 - 5e^{-4}]$	M1 A1 M1 A1 A1 ft M1 A1 (7)

Question Number	Scheme	Marks
8 (a)	$\cos(A + A) = \cos^2 A - \sin^2 A$ $= \cos^2 A - (1 - \cos^2 A) = 2 \cos^2 A - 1$	M1 A1 (2)
(b)	$[x = 2, \theta = \frac{\pi}{4}; x = \sqrt{6}, \theta = \frac{\pi}{3}]$ $x = 2\sqrt{2} \sin \theta, \frac{dx}{d\theta} = 2\sqrt{2} \cos \theta$ $\int \sqrt{8 - x^2} dx = \int 2\sqrt{2} \cos \theta 2\sqrt{2} \cos \theta d\theta = \int 8 \cos^2 \theta d\theta$ Using $\cos 2\theta = 2 \cos^2 \theta - 1$ to give $\int 4(1 + \cos 2\theta) d\theta$ $= 4\theta + 2 \sin 2\theta$ Substituting limits to give $\frac{1}{3}\pi + \sqrt{3} - 2$ or given result	B1 B1 M1A1 dM1 A1ft A1 (7)
(c)	$\frac{dy}{d\theta} = \frac{-2 \sin 2\theta}{1 + \cos 2\theta}$ Using the chain rule, with $\frac{dx}{d\theta} = \sec \theta \tan \theta$ to give $\frac{dy}{dx} (= -2 \cos \theta)$ Gradient at the point where $\theta = \frac{\pi}{3}$ is -1. Equation of tangent is $y + \ln 2 = -(x - 2)$ (o.a.e.)	B1 M1 A1ft M1A1 (5)

Question Number	Scheme	Marks
1. (a)	$y = 5x - x^{-1} + C$	M1 A2 (1,0)
(b)	$7 = 5 - 1 + C, \quad C = 3$	M1 A1 ft
	$x = 2: \quad y = 10 - \frac{1}{2} + 3 = 12\frac{1}{2}$	M1 A1
		(7 marks)
2. (a)	$6x - 2x < 3 + 7 \quad x < 2\frac{1}{2}$	M1 A1
(b)	$(2x - 1)(x - 5) \quad \text{Critical values } \frac{1}{2} \text{ and } 5$	M1 A1
	$\frac{1}{2} < x < 5$	M1 A1 ft
(c)	$\frac{1}{2} < x < 2\frac{1}{2}$	B1 ft
		(7 marks)
3. (a)(i)	$a + (n - 1)d = 280 + (35 \times 5) = 455$	M1 A1
(ii)	$\frac{1}{2}n [2a + (n - 1)d] = 18 [560 + (35 \times 5)] = 13\,230$	M1 A1 ft
(b)	$18 [560 + (35 \times d)] = 17\,000$	M1 A1
	$d = 10.98... \quad x = 11 \text{ (allow 11.0 or 10.98 or 10.99 or } 10\frac{62}{63} \text{)}$	M1 A1
		(8 marks)

(ft = follow-through mark)

Question Number	Scheme	Marks
4.	(a) $\frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times 1.5 = 15$ $r^2 = 20 = \sqrt{(4 \times 5)} \quad r = 2\sqrt{5} \quad (*)$ (b) $r\theta + 2r = 3\sqrt{5} + 4\sqrt{5} = 7\sqrt{5} \text{ cm} \quad (\text{or } 15.7, \text{ or a.w.r.t } 15.65\dots)$ (c) $\Delta OAB: \quad \frac{1}{2}r^2 \sin \theta = 10 \sin 1.5 (= 9.9749\dots)$ Segment area = $15 - \Delta OAB = 5.025 \text{ cm}^2$	M1 A1 A1 M1 A1 M1 M1 A1 (8 marks)
	$2 \cos^2 \theta - \cos \theta - 1 = 1 - \cos^2 \theta$ $3 \cos^2 \theta - \cos \theta - 2 = 0$ $(3 \cos \theta + 2)(\cos \theta - 1) = 0 \quad \cos \theta = -\frac{2}{3} \text{ or } 1$ $\theta = 0 \quad \theta = 131.8^\circ$ $\theta = (360 - "131.8")^\circ = 228.2^\circ$	M1 A1 M1 A1 B1 A1 M1 A1 ft (8 marks)
6.	(a) $m = \frac{2-6}{12-4} \left(= -\frac{1}{2} \right)$ $y - 6 = (\text{their } m)(x - 4) \quad x + 2y = 16$ (b) $y = -4x$ (c) $x + 2(-4x) = 16 \quad -7x = 16 \quad x = -\frac{16}{7}$ $y = \frac{64}{7}$ $A(4, 6), C\left(-\frac{16}{7}, \frac{64}{7}\right): \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \rightarrow \left(\frac{6}{7}, \frac{53}{7}\right)$	M1 A1 M1 A1 B1 M1 A1 A1 ft M1 A1 ft (10 marks)

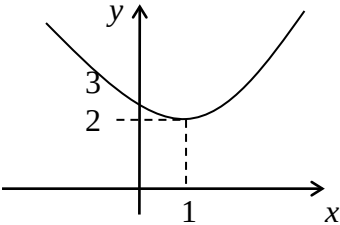
(ft = follow-through mark)

Question Number	Scheme	Marks
7. (a)	$x^2 - 2x + 3 = 9 - x$ $x^2 - x - 6 = 0 \quad (x + 2)(x - 3) = 0 \quad x = -2, 3$ $y = 11, 6$	M1 M1 A1 M1 A1 ft
(b)	$\int (x^2 - 2x + 3) dx = \frac{x^3}{3} - x^2 + 3x$ $\left[\frac{x^3}{3} - x^2 + 3x \right]_{-2}^3 = (9 - 9 + 9) - \left(\frac{-8}{3} - 4 - 6 \right) \quad \left(= 21\frac{2}{3} \right)$ Trapezium: $\frac{1}{2} (11 + 6) \times 5 \quad \left(= 42\frac{1}{2} \right)$ Area = $42\frac{1}{2} - 21\frac{2}{3} = 20\frac{5}{6}$ <u>Alternative:</u> $(9 - x) - (x^2 - 2x + 3) = 6 + x - x^2$ $\int (6 + x - x^2) dx = 6x + \frac{x^2}{2} - \frac{x^3}{3}$ $\left[6x + \frac{x^2}{2} - \frac{x^3}{3} \right]_{-2}^3 = \left(18 + \frac{9}{2} - 9 \right) - \left(-12 + 2 + \frac{8}{3} \right) = 20\frac{5}{6}$	M1 A1 M1 A1 B1 ft M1 A1 M1 A1 M1 A1 ft M1 A1, A1 (12 marks)

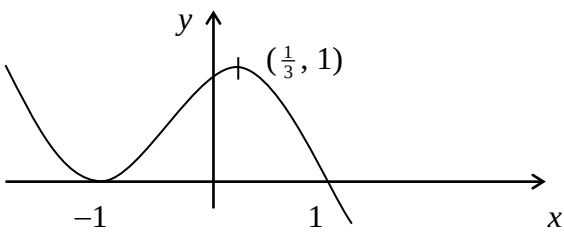
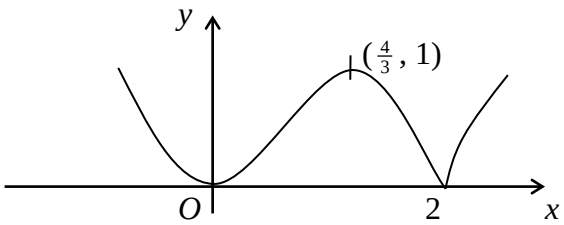
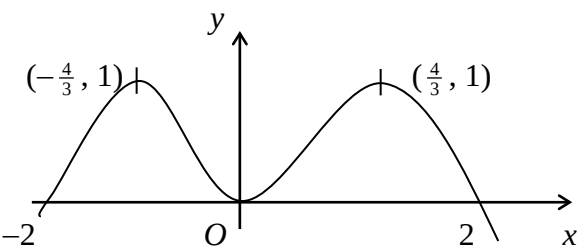
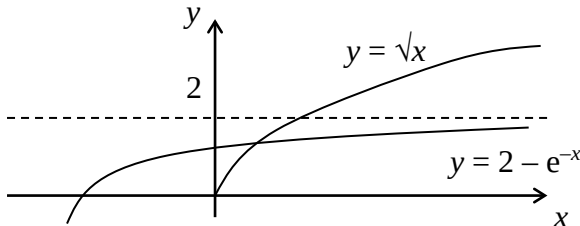
(ft = follow-through mark)

Question Number	Scheme	Marks
8.	(a) $\frac{dy}{dx} = 4x^3 - 16x$ (b) $4x^3 - 16x = 0$ $4x(x^2 - 4) = 0$ $x = 0, 2, -2$ $y = 3, -13, -13$ (c) $\frac{d^2y}{dx^2} = 12x^2 - 16$ $x = 0$ Max. } One of these, ft $x = 2$ Min. } $x = -2$ Min. } All three (d) $x = 1:$ $y = 1 - 8 + 3 = -4$ At $x = 1,$ $\frac{dy}{dx} = 4 - 16 = -12$ (m) Gradient of normal = $-\frac{1}{m}$ $\left(= \frac{1}{12} \right)$ $y - (-4) = \frac{1}{12}(x - 1)$ $x - 12y - 49 = 0$	M1 A1 M1 A2 (1, 0) M1 A1 M1 A1ft A1 B1 B1 ft M1 M1 A1 (15 marks)

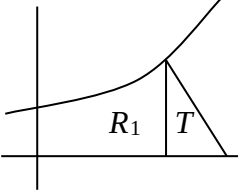
(ft = follow-through mark)

Question Number	Scheme	Marks
1.	(a) $\frac{x^2 + 4x + 3}{x^2 + x} = \frac{(x+3)(x+1)}{x(x+1)}$ Attempt to factorise numerator or denominator $= \frac{x+3}{x}$ or $1 + \frac{3}{x}$ (b) LHS = $\log_2 \left(\frac{x^2 + 4x + 3}{x^2 + x} \right)$ Use of $\log a - \log b$ RHS = 2^4 or 16 $x + 3 = 16x$ Linear or quadratic equation in x $x = \frac{3}{15}$ or $\frac{1}{5}$ or 0.2	M1 A1 (2) M1 B1 M1 A1 (4) (6 marks)
2.	(a)  $x^2 - 2x + 3 = (x-1)^2 + 2$ Full method to establish min. f $f(4) = 3^2 + 2 = 11$ $f \geq 2$ $f \leq 11$ (b) $f(2) = 3$; $\therefore 16 = gf(2) \Rightarrow 16 = 3\lambda + 1$ M for using their $f(2)$ for eqn $\therefore \lambda = 5$ ft their genuine $f(2)$	M1 A1 B1 (3) B1; M1 A1 ft (3) (6 marks)
3.	$(2 - px)^6 = 2^6 + \binom{6}{1} 2^5(-px) + \binom{6}{2} 2^4(-px)^2$ Coeff. of x or x^2 $= 64 + 6 \times 2^5(-px) + 15 \times 2^4(-px)^2$ No $\binom{n}{r}$ $15 \times 16p^2 = 135 \Rightarrow p^2 = \frac{9}{16}$ or $p = \frac{3}{4}$ (only) $-6.32p = A$ $\Rightarrow A = -144$	M1 $\binom{n}{r}$ okay A1; A1 M1, A1 M1 A1 ft (their $p > 0$) (7 marks)

(ft = follow-through mark)

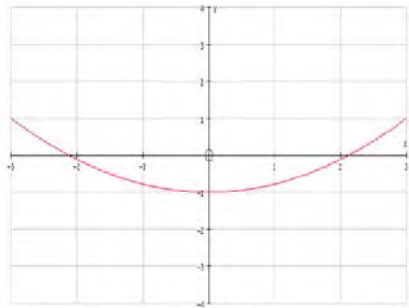
Question Number	Scheme	Marks
4. (a)	 Translation in $\leftarrow$ or $\rightarrow$ Points correct	B1 B2, 1, 0 (-1eeoo) (3)
(b)	 $x < 2$ including points $x > 2$ correct reflection cusp at (2, 0) (not $\cup$)	B1 B1 B1 (3)
(c)	 correct shape $x \geq 0$ symmetry in y-axis correct maxima correct x intercepts	B1 B1 B1 B1 (4) (10 marks)
5. (a)	 $y = \sqrt{x}$: starting (0,0) $y = 2 - e^{-x}$: shape & int. on + y-axis correct relative posns	B1 B1 B1 (3)
(b)	Where curves meet is solution to $f(x) = 0$; only one intersection	B1 (1)
(c)	$f(3) = -0.218...$ $f(4) = 0.018...$ change of sign $\therefore$ root in interval	one correct value to 1 sf M1 both correct (1 sf) + comment M1 (2)
(d)	$x_0 = 4$ $x_1 = (2 - e^{-4})^2 = 3.92707...$ dp $x_2 = 3.92158...$ dp $x_3 = 3.92115...$ x_4 $x_4 = 3.92111(9)...$ Approx. solution = 3.921 (3 dp)	expression or x_1 to 3 M1 x_1, x_2 to ≥ 4 A1 carry on to to ≥ 3 dp M1 A1 cao (4) (10 marks)

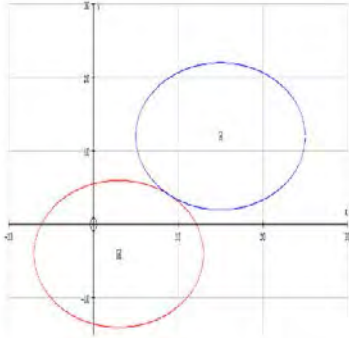
(-1eooo = minus 1 mark for each error or omission; cao = correct answer only)

Question Number	Scheme	Marks
6.	(a) $\frac{dy}{dx} = -\frac{c}{x^2}$ Attempt $\frac{dy}{dx}$ When $x = p \Rightarrow -4 = -\frac{c}{p^2} \therefore c = 4p^2$ (*) (b) $5 = 1 + \frac{c}{p}$ and solve with $c = 4p^2$ $5 = 1 + 4p \Rightarrow p = 1 \therefore c = 4$ (*) (c) $y^2 = 1 + \frac{8}{x} + \frac{16}{x^2}$ $y^2 = ; \geq 2$ terms correct $\int y^2 dx = \left[x + 8 \ln x - \frac{16}{x} \right]$ some correct $\int$; all correct $\int_1^2 y^2 dx = (2 + 8 \ln 2 - \frac{16}{2}) - (1 + 8 \ln 1 - 16)$ Use of correct limits $V = \pi \int_1^2 y^2 dx$ $\therefore V = \pi(9 + 8 \ln 2)$ $k = 9; q = 8$	M1 A1 cso (2) M1 A1 cso (2) M1 M1 ; A1 M1 B1 A1; A1 (7) (11 marks)
7.	(a) M is $(0, 7)$ $\frac{dy}{dx} = 2e^x$ Attempt $\frac{dy}{dx}$ $\therefore$ gradient of normal is $-\frac{1}{2}$ ft their $y'(0)$ $\therefore$ equation of normal is $y - 7 = -\frac{1}{2}(x - 0)$ or $x + 2y = 14$ (b) $y = 0, x = 14 \therefore N$ is $(14, 0)$ (*) (c)  $\int (2e^x + 5) dx = [2e^x + 5x]$ some correct $\int$ $R_1 = \int_0^{\ln 4} (2e^x + 5) dx = (2 \times 4 + 5 \ln 4) - (2 + 0)$ M1 limits used $= 6 + 5 \ln 4$ A1 $T = \frac{1}{2} \times 14 \times (7 - \ln 4)$ Area of T B1 $T = 13(7 - \ln 2) ; R_1 = 6 + 10 \ln 2$ Use of $\ln 4 = 2 \ln 2$ B1 $R = T + R_1, R = 97 - 3 \ln 2$ M1, A1 (7) (12marks)	B1 M1 M1 A1 (4) B1 cso (1) M1 M1 limits used A1 B1 B1 (7) (12marks)

Question Number	Scheme	Marks
8. (i)	$\cos x \cos 30 - \sin x \sin 30 = 3(\cos x \cos 30 + \sin x \sin 30)$ Use of $\cos(x \pm 30)$ $\Rightarrow \sqrt{3} \cos x - \sin x = 3\sqrt{3} \cos x + 3 \sin x$ Sub. for sin 30 etc decimals M1, surds A1 i.e. $-4 \sin x = 2\sqrt{3} \cos x \rightarrow \tan x = -\frac{\sqrt{3}}{2} (*)$ Use $\tan x = \frac{\sin x}{\cos x}$	M1 M1, A1 M1, A1cso (5)
(ii) (a)	$\text{LHS} = \frac{1 - (1 - 2 \sin^2 \theta)}{2 \sin \theta \cos \theta}$ $= \frac{\sin \theta}{\cos \theta} = \tan \theta (*)$	M1; A1 A1 cso (3)
(b)	Verifying: $0 = 2 - 2$ (since $\sin 360 = 0$, $\cos 360 = 1$)	B1 cso
(c)	Equation $\rightarrow 1 = \frac{2(1 - \cos 2\theta)}{\sin 2\theta}$ Rearrange to form $\frac{1 - \cos 2\theta}{\sin 2\theta}$ $\Rightarrow \tan \theta = \frac{1}{2}$ i.e. $\theta = 26.6^\circ$ or 206.6° (Accept 27° , 207°)	M1 A1 M1 (any acc.) A1 (both) (4) (13 marks)
Alt 1 (c)	$2 \sin \theta \cos \theta = 2 - 2(1 - 2 \sin^2 \theta)$ Use of $\cos 2A$ <u>and</u> $\sin 2A$ $0 = 2 \sin \theta (2 \sin \theta - \cos \theta)$ $\Rightarrow (\sin \theta = 0) \tan \theta = \frac{1}{2}$ etc, as in scheme	M1 A1
Alt 2 (c)	$2 \cos 2\theta + \sin 2\theta = 2 \Rightarrow \cos(2\theta - \alpha) = \frac{2}{\sqrt{5}}$ $\alpha = 22.6$ (or 27) $2\theta = 2\alpha, 360, 360 + 2\alpha$ $\theta = \alpha$ or $180 + \alpha$ $\theta = \alpha, 180 + \alpha$ i.e. $\theta = 27^\circ$ or 207° (or 1 dp)	M1 A1 M1 A1 both

((*) indicates final line is given on the paper; cso = correct solution only; ft = follow-through mark)

Question number	Scheme	Marks
1.	Attempt to use correctly stated double angle formula $\cos 2t = 2 \cos^2 t - 1$, or complete method using other double angle formula for $\cos 2t$ with $\cos^2 t + \sin^2 t = 1$ to eliminate t and obtain $y =$ $y = 2(\frac{x}{3})^2 - 1$ or any correct equivalent. (even $y = \cos 2(\cos^{-1}(\frac{x}{3}))$) shape position including restricted domain $-3 < x < 3$ 	M1 A1 (2) B1 B1 (2)
2.	(a) $p + 6 + 12 + q = -\frac{1}{8}p + \frac{6}{4} - 6 + q$ $\therefore \frac{9}{8}p = -22\frac{1}{2}$ $p = -20$ (b) Remainder = $p + q + 18 = p + 21 (=1)$	M1 , M1 M1 A1 (4) B1 $\sqrt{\text{ft on } p}$ (1)

Question number	Scheme	Marks
3.	(a) Centre is at (3,-4) $\text{radius} = \sqrt{(3^2 + (-4)^2 - -75)} = 10$ (b) 1st circle 2nd circle Circles touching At (9, 4) 	B1 M1 A1 (3) B1 B1 B1 B1 (4)
4.	(a) $14x + (48x \frac{dy}{dx} + 48y) - 14y \frac{dy}{dx} = 0$ Substitutes $\frac{dy}{dx} = \frac{2}{11}$ into derived expression to obtain $14x + \frac{96}{11}x + 48y - \frac{28}{11}y = 0$ $\therefore 250x + 500y = 0 \Rightarrow x + 2y = 0$ (b) Eliminates one variable to obtain, for example, $7(2y)^2 + 48(-2y)y - 7y^2 + 75 = 0$ and obtains y (or x) Substitutes y to obtain x (or y) Obtains coordinates (-2,1) and (2,-1)	M1 (B1) A1 M1 A1 (5) M1 M1 A1, A1 (4)

Question number	Scheme	Marks
5.	(a) $\frac{dy}{dx} = \sqrt{\sin x} + \frac{x}{2}(\sin x)^{-\frac{1}{2}} \cos x$ At A $\sqrt{\sin x} + \frac{x}{2}(\sin x)^{-\frac{1}{2}} \cos x = 0$ $\therefore \sin x + \frac{x}{2} \cos x = 0$ (essential to see intermediate line before given answer) $\therefore 2 \tan x + x = 0$ *	M1,A1 dM1 A1 (4)
	(b) $V = \pi \int y^2 dx = \pi \int x^2 \sin x dx$ $= \pi \left[-x^2 \cos x + \int 2x \cos x dx \right]_0^\pi$ $= \pi \left[-x^2 \cos x + 2x \sin x - \int 2 \sin x dx \right]_0^\pi$ $= \pi \left[-x^2 \cos x + 2x \sin x + 2 \cos x \right]_0^\pi$ $= \pi \left[\pi^2 - 2 - 2 \right]$ $= \pi \left[\pi^2 - 4 \right]$	M1 M1 A1 M1 A1 M1 A1 (7)

Question number	Scheme	Marks
6.	(a) $\overrightarrow{AB} = (2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, $\overrightarrow{CB} = (-2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, (or $\overrightarrow{BA}, \overrightarrow{BC}$, or $\overrightarrow{AB}, \overrightarrow{BC}$, stated in above form or column vector form. $\cos \angle ABC = \frac{\overrightarrow{CB} \cdot \overrightarrow{AB}}{ \overrightarrow{CB} \overrightarrow{AB} } = -\frac{4}{9}$	M1A1 M1 A1 (4)
	(b) Area of $\triangle ABC = \frac{1}{2} \times 3 \times 3 \times \sin B$ $\sin B = \sqrt{1 - \frac{16}{81}} = \frac{\sqrt{65}}{9}$ $\therefore \text{Area} = \frac{1}{2} \sqrt{65}$	M1 M1 A1 (3)
	(c) $\overrightarrow{AC} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$ $\overrightarrow{DC} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$ or given in alternative form with attempt at scalar product $\overrightarrow{AC} \cdot \overrightarrow{DC} = 0, \text{ therefore the lines are perpendicular.}$	M1 A1 (2)
	(d) $\overrightarrow{AD} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$ $\overrightarrow{DB} = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}$ and AD:DB = 2:-1 (allow 2:1)	M1, A1 (2)

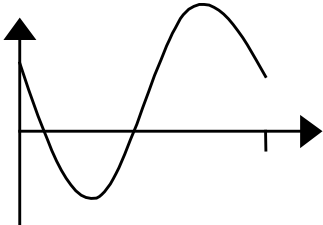
Question number	Scheme	Marks
7. (a)	$\frac{dV}{dt} = \pm c\sqrt{V} \text{ or } \frac{dV}{dt} \propto \sqrt{V}$ $\text{As } V = Ah, \frac{dV}{dh} = A \text{ or } V \propto h$ $\therefore \text{use chain rule to obtain } \frac{dh}{dt} = -\frac{c}{A}\sqrt{V} = \frac{-c}{\sqrt{A}}\sqrt{h} = -k\sqrt{h}$	M1 M1 A1 (3)
(b)	$\int \frac{dh}{h} = -\int k dt$ $2h^{\frac{1}{2}} = A - kt$ $h^{\frac{1}{2}} = \frac{A}{2} - \frac{kt}{2}$ $h = (A - Bt)^2 \quad *$	M1, M1 A1 A1 (4)
(c)	$t = 0, h = 1: \quad A = 1$ $t = 5, h = 0.5: \quad 0.5 = (1 - 5B)^2$ $B = \frac{(1 - \sqrt{0.5})}{5} \quad (B = 0.0586)$ $h = 0, t = \frac{A}{B} = \frac{5}{1 - \sqrt{0.5}} = 17.1 \text{ min}$	B1 B1 B1 (3)
(d)	$h = \frac{A^2}{4} = 0.25 \text{ m}$	M1 A1 (2)

Question number	Scheme	Marks
8. (a)	Method using either $\frac{A}{(1-x)} + \frac{B}{(2x+3)} + \frac{C}{(2x+3)^2} \text{ or } \frac{A}{1-x} + \frac{Dx+E}{(2x+3)^2}$ $A = 1$ $C = 10, B = 2 \text{ or } D = 4 \text{ and } E = 16$	M1 B1 A1, A1 (4)
(b)	$\int \left[\frac{1}{1-x} + \frac{2}{2x+3} + 10(2x+3)^{-2} \right] dx \text{ or } \int \frac{A}{1-x} + \frac{Dx+E}{(2x+3)^2} dx$ $-\ln 1-x + \ln 2x+3 - 5(2x+3)^{-1} (+c) \text{ or}$ $-\ln 1-x + \ln 2x+3 - (2x+8)(2x+3)^{-1} (+c)$	M1 M1A1√ A1√A1√ (5)
(c)	Either $(1-x)^{-1} + 2(3+2x)^{-1} + 10(3+2x)^{-2} =$ $1+x+x^2+\dots$ $+\frac{2}{3}\left(1-\frac{2x}{3}+\frac{4x^2}{9}\dots\right)$ $+\frac{10}{9}\left(1+(-2)\left(\frac{2x}{3}\right)+\frac{(-2)(-3)}{2}\left(\frac{2x}{3}\right)^2+\dots\right)$ $= \frac{25}{9} - \frac{25}{27}x + \frac{25}{9}x^2\dots$ Or $25\left[(9+12x+4x^2)(1-x)\right]^{-1} = 25\left[(9+3x-8x^2-4x^3)\right]^{-1}$ $\frac{25}{9}\left[1+\frac{3x}{9}-\frac{8x^2}{9}-\frac{4x^3}{9}\right]^{-1} = \frac{25}{9}\left[1-\left(\frac{3x}{9}-\frac{8x^2}{9}-\frac{4x^3}{9}\right)+\left(\frac{x^2}{9}\dots\right)\right]$ $= \frac{25}{9} - \frac{25}{27}x + \frac{25}{9}x^2\dots$	M1 A1 M1 A1 A1 M1A1 (7) M1 A1 M1 A1 A1 M1A1 (7)

EDEXCEL PURE MATHEMATICS P1 (6671)
PROVISIONAL MARK SCHEME NOVEMBER 2003

Question Number	Scheme	Marks
1.	(a) 77 74 (b) $d = 74 - 77 = -3$ (c) $S_{50} = \frac{1}{2}n[2a + (n-1)d] = 25[(2 \times 77) + (49 \times -3)]$ $= 175$	B1 B1 (2) B1 $\sqrt{}$ (1) M1 A1 $\sqrt{}$ (3) A1 6
2.	(a) $4x(x+3)$ or $x(4x+12)$ (or use of quadratic formula) $x = 0$ $x = -3$ (b) Using $b^2 - 4ac = 0$ $144 - 16c = 0$ $c = 9$ $(2x+3)(2x+3) = 0$ $x = \dots$ (or quadratic formula) $x = -\frac{3}{2}$	M1 A1 A1 (3) M1 A1 M1 A1 (4) 7
3.	$x = 3y - 1$ $(3y-1)^2 - 3y(3y-1) + y^2 = 11$ $y^2 - 3y - 10 = 0$ $(y-5)(y+2) = 0$ $y = 5$ $y = -2$ $x = 14$ $x = -7$	M1 M1 A1 M1 A1 M1 A1 $\sqrt{}$ (7) 7

EDEXCEL PURE MATHEMATICS P1 (6671)
PROVISIONAL MARK SCHEME NOVEMBER 2003

Question Number	Scheme	Marks
4.	(a) $4x+9, +12\sqrt{x}$ (b) $\int(4x+12x^{\frac{1}{2}}+9)dx = 2x^2 + 8x^{\frac{3}{2}} + 9x$ ($\sqrt{\quad}$ dep. on 3 terms) (c) $[\dots]_1^2 = (8 + (8 \times 2^{\frac{3}{2}}) + 18) - (2 + 8 + 9)$ $= 7 + 16\sqrt{2}$	B1, B1 (2) M1 A1 $\sqrt{\quad}$ M1 M1 A1 (5) 7
5.	(a)  Shape Position (b) $\left(0, \frac{1}{\sqrt{2}}\right), \left(\frac{\pi}{4}, 0\right), \left(\frac{5\pi}{4}, 0\right)$ (c) $\left(x + \frac{\pi}{4} = \right) \frac{\pi}{3}$ Other value $\left(2\pi - \frac{\pi}{3} = \right) \frac{5\pi}{3}$ Subtract $\frac{\pi}{4}$ $x = \frac{\pi}{12}, x = \frac{17\pi}{12}$	B1 B1 (2) B1 B1 B1 (3) B1 M1 M1 A1 (4) 9

EDEXCEL PURE MATHEMATICS P1 (6671)
PROVISIONAL MARK SCHEME NOVEMBER 2003

Question Number	Scheme	Marks
6.	(a) $V = \pi r^2 h = 500, \quad A = 2\pi r h + \pi r^2$ $A = 2\pi r \left(\frac{500}{\pi r^2} \right) + \pi r^2 = \pi r^2 + \frac{1000}{r} \quad *$	B1 M1 M1 A1 (4)
	(b) $\frac{dA}{dr} = 2\pi r - 1000r^{-2}$ $2\pi r - 1000r^{-2} = 0 \quad r = \sqrt[3]{\frac{500}{\pi}} \quad (\approx 5.42)$	M1 A1 M1 A1 (4)
	(c) $\frac{d^2A}{dr^2} = 2\pi + 2000r^{-3}, \quad > 0 \quad \text{therefore minimum}$	M1 A1 $\sqrt{\quad}$ (2)
	(d) $A = \pi r^2 + \frac{1000}{r} = 277 \quad (\text{nearest integer})$	M1 A1 (2)
		12

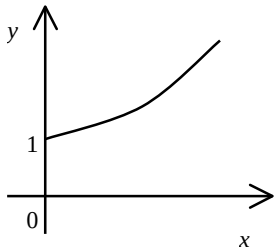
EDEXCEL PURE MATHEMATICS P1 (6671)
PROVISIONAL MARK SCHEME NOVEMBER 2003

Question Number	Scheme	Marks
7.	(a) $\frac{5 - (-3)}{8 - 2} = \frac{4}{3}$ (b) $M : \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad (5, 1)$ Gradient of CM is $-\frac{3}{4}$ Equation of CM: $y - 1 = -\frac{3}{4}(x - 5)$ $(4y = -3x + 19)$ (c) When $x=4$, $y = \frac{7}{4}$ (d) Radius = $\sqrt{(4 - 2)^2 + \left(\frac{7}{4} + 3\right)^2}$ $= \sqrt{4 + \frac{361}{16}} = \sqrt{\frac{425}{16}} = \sqrt{\frac{25}{16}}\sqrt{17} = \frac{5\sqrt{17}}{4} \quad *$	M1 A1 (2) M1 A1 B1 $\sqrt{\quad}$ M1 A1 (5) M1 A1 $\sqrt{\quad}$ (2) M1 A1 $\sqrt{\quad}$ M1 A1 (4) 13

EDEXCEL PURE MATHEMATICS P1 (6671)
PROVISIONAL MARK SCHEME NOVEMBER 2003

Question Number	Scheme	Marks
8.	(a) $x(x^2 - 6x + 5)$	M1
	$= x(x-1)(x-5)$	M1 A1 (3)
	(b) 1 and 5	B1 $\sqrt{\quad}$ (1)
	(c) $\frac{dy}{dx} = 3x^2 - 12x + 5$	M1 A1
	At $x = 1$. $\frac{dy}{dx} = 3 - 12 + 5 = -4$	A1 (3)
	(d) $\int (x^3 - 6x^2 + 5x) dx = \frac{x^4}{4} - \frac{6x^3}{3} + \frac{5x^2}{2}$	M1 A1
	$\left[\dots \right]_0^1 = \frac{1}{4} - 2 + \frac{5}{2} \quad \left(= \frac{3}{4} \right) \quad R$	M1 A1 $\sqrt{\quad}$
	Evaluating at 5: $\frac{625}{4} - 250 + \frac{125}{2} \quad \left(= -31\frac{1}{4} \right)$	A1
	To find S: $-31\frac{1}{4} - \frac{3}{4} = -32$	M1
	Total Area = $32 + \frac{3}{4} = 32\frac{3}{4}$	A1 (7)
		14

EDEXCEL PURE MATHEMATICS P2 (6672)
PROVISIONAL MARK SCHEME NOVEMBER 2003

Question Number	Scheme	Marks
1.	(a) $\frac{2}{x-3} + \frac{13}{(x-3)(x+7)}$ $= \frac{2(x+7)+13}{(x-3)(x+7)} = \frac{2x+27}{(x-3)(x+7)}$ (b) $2x+27 = x^2+4x-21$ $x^2+2x-48 = (x+8)(x-6) = 0$ $x = -8, 6$	M1 M1 A1 <u>3</u> M1 M1 A1 <u>3</u> <u>6</u>
2.	(a)  (b) $£800 \times 1.04^{10} \approx £1184$ (c) $1.04^x = 2$ $x = \frac{\ln 2}{\ln 1.04} \approx 18 \text{ (years)}$	Shape B1 domain, intercept B1 <u>2</u> cao M1 A1 <u>2</u> M1 M1 A1 <u>3</u> <u>7</u>

EDEXCEL PURE MATHEMATICS P2 (6672)
PROVISIONAL MARK SCHEME NOVEMBER 2003

Question Number	Scheme	Marks
3.	(a) $1 + nax, + \frac{n(n-1)}{2}(ax)^2 + \frac{n(n-1)(n-2)}{6}(ax)^3 + \dots$ accept 2!, 3! (b) $na = 8, \quad \frac{n(n-1)}{2}a^2 = 30$ both $\frac{n(n-1)}{2} \cdot \frac{64}{n^2} = 30, \quad \frac{\frac{8}{a}(\frac{8}{a}-1)a^2}{2} = 30$ either $n = 16, \quad a = \frac{1}{2}$ (c) $\frac{16 \cdot 15 \cdot 14}{6} \cdot \left(\frac{1}{2}\right)^3 = 70$	B1, B1 <u>2</u> M1 M1 A1, A1 <u>4</u> M1 A1 <u>2</u> 8
4.	(a) $\frac{8}{x} - x^2 = 0 \Rightarrow x^3 = 8 \Rightarrow x = 2$ (b) $\left(\frac{8}{x} - x^2\right) = x^4 - 16x + \frac{64}{x^2}$ M1 3(or 4) terms $\int (x^4 - 16x + 64x^{-2}) dx = \frac{x^5}{5} - 8x^2 - \frac{64}{x}$ $\left[\frac{x^5}{5} - 8x^2 - \frac{64}{x}\right]_1^2 = \left(\frac{32}{5} - 32 - 32\right) - \left(\frac{1}{5} - 8\right) - 64$ Volume is $\frac{71}{5}\pi$ (units³)	M1 A1 <u>2</u> M1 A1 M1 A1 M1 A1 ft A1 <u>7</u> 9

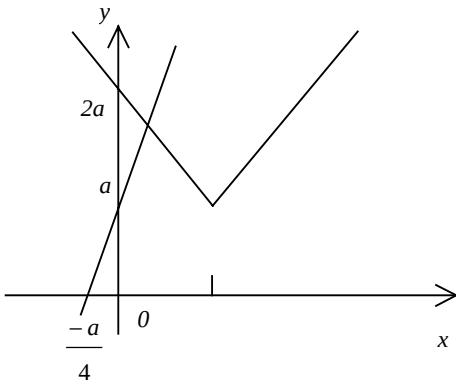
EDEXCEL PURE MATHEMATICS P2 (6672)
PROVISIONAL MARK SCHEME NOVEMBER 2003

Question Number	Scheme	Marks
5.	(a) $\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} = \frac{1 - \frac{\sin^2 \theta}{\cos^2 \theta}}{1 + \frac{\sin^2 \theta}{\cos^2 \theta}} \left(\text{or } \frac{1 - \frac{\sin^2 \theta}{\cos^2 \theta}}{\sec^2 \theta} \text{ or equivalent} \right)$ $\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta} = \frac{\cos 2\theta}{1} = \cos 2\theta \quad * \quad \text{cso}$ (b) $\theta = \frac{\pi}{8}, \quad \cos 2\theta = \frac{1}{\sqrt{2}}$ $\frac{1 - t^2}{1 + t^2} = \frac{1}{\sqrt{2}}$ $t^2 = \frac{\sqrt{2} - 1}{\sqrt{2} + 1}$ $= \frac{\sqrt{2} - 1}{\sqrt{2} + 1} \cdot \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = 3 - 2\sqrt{2} \quad *$ Alternative to 5(b) $\frac{2t}{1 - t^2} = \tan 2\theta = 1$ $t^2 + 2t - 1 = 0$ $t = \sqrt{2} - 1$ $t^2 = (\sqrt{2} - 1)^2 = 3 - 2\sqrt{2} \quad *$	M1 M1 M1 A1 <u>4</u> M1 M1 M1 M1 A1 <u>5</u> <u>9</u> M1 M1 M1 M1 A1 <u>5</u>

EDEXCEL PURE MATHEMATICS P2 (6672)
PROVISIONAL MARK SCHEME NOVEMBER 2003

Question Number	Scheme	Marks
6.	(a) $x \quad 0 \quad \frac{\pi}{6} \quad \frac{\pi}{3} \quad \frac{\pi}{2} \quad 1, 0$ B1 $y \quad 1 \quad 1.46 \quad 1.42 \quad 0 \quad 1.46, 1.42$ B1, B1 <u>3</u> <i>NB. Not giving 2 d.p. loses a maximum of one mark</i>	
	(b) $I \approx \frac{1}{2} \left(\frac{\pi}{6} \right) \dots$ B1 $\approx \dots (1 + 2(1.46 + 1.42) + 0)$ ft their ys M1 A1 ft ≈ 1.8 accept 1.77 A1 <u>4</u>	
	(c) underestimates diagram or explanation B1 B1 <u>2</u> <u>9</u>	
	<i>NB. Exact answer is $\frac{1}{2} \left(e^{\frac{\pi}{2}} - 1 \right) \approx 1.905 \dots$</i>	

EDEXCEL PURE MATHEMATICS P2 (6672)
PROVISIONAL MARK SCHEME NOVEMBER 2003

Question Number	Scheme	Marks
7.	(a) V shape right way up vertex in first quadrant g -1 eeo; $2a, a, -\frac{a}{4}$  (b) $4x + a = (a - x) + a$ $5x = a, \quad x = \frac{a}{5}$ $y = \frac{9a}{5}$ (c) $fg(x) = 4x + a - a + a = 4x + a$ (d) $ 4x + a = 3a \Rightarrow 4x = 2a$ $x = \frac{a}{2}, -\frac{a}{2}$	B1 B1 B1 B2 (1, 0) <u>5</u> M1 M1 both correct A1 <u>3</u> M1 A1 <u>2</u> M1 A1 A1, A1 <u>3</u> 13

EDEXCEL PURE MATHEMATICS P2 (6672)
PROVISIONAL MARK SCHEME NOVEMBER 2003

Question Number	Scheme	Marks
8.	(a) $f'(x) = \frac{3}{x} - \frac{1}{x^2}$	M1 A1
	$\frac{3}{x} - \frac{1}{x^2} = 0 \Rightarrow 3x^2 - x = 0 \Rightarrow x = \frac{1}{3}$	M1 A1 <u>4</u>
	(b) $y = 31\ln\left(\frac{1}{3}\right) + \frac{1}{\left(\frac{1}{3}\right)} = 3 - 31\ln 3 \quad (k = 3)$	M1 A1 <u>2</u>
	(c) $x = 1 \Rightarrow y = 1$	B1
	$f'(1) = 2 \Rightarrow m' = -\frac{1}{2}$	M1
	$y - 1 = -\frac{1}{2}(x - 1) \quad \left(y = -\frac{x}{2} + \frac{3}{2}\right)$	M1 A1 <u>4</u>
	(d) i $-\frac{x}{2} + \frac{3}{2} = 31\ln x + \frac{1}{x}$	M1
	leading to $61\ln x + x + \frac{2}{x} - 3 = 0$ *	cso A1
	ii $g(0.13) = 0.273\dots$ $g(0.14) = -0.370\dots$ Both, accept one d.p.	M1
	Sign change (and continuity) $\Rightarrow$ root $\in (0.13, 0.14)$	A1 <u>4</u> 14

EDEXCEL 6671 PURE MATHEMATICS P1 JANUARY 2004 MARK SCHEME

Question	Mark Scheme	Marks
1. (a)	$f(-2) = (-2)^3 - (19 \times -2) - 30$ M: Evaluate $f(-2)$ or $f(2)$ $f(-2) = 0$, so $(x+2)$ is a factor <u>Alternative:</u> $(x^3 - 19x - 30) \div (x+2) = (x^2 + ax + b)$, $a \neq 0, b \neq 0$ [M1] $= (x^2 - 2x - 15)$, so $(x+2)$ is a factor [A1]	M1 A1 (2)
(b)	$(x^3 - 19x - 30) = (x+2)(x^2 - 2x - 15)$ $= (x+2)(x+3)(x-5)$	M1 A1 M1 A1 (4) (6)
2. (a)	$\frac{1}{2}r^2\theta = \frac{1}{2} \times 6.5^2 \times 0.8 = 16.9$ (a.w.r.t. if changed to degrees)	M1 A1 (2)
(b)	$\sin 0.4 = \frac{x}{6.5}$, $x = 6.5 \sin 0.4$, (where x is half of AB) (n.b. $0.8 \text{ rad} = 45.8^\circ$) $AB = 2x = 5.06$ (a.w.r.t.) (*) <u>Alternative:</u> $AB^2 = 6.5^2 + 6.5^2 - 2 \times 6.5 \times 6.5 \cos 0.8$ [M1] $AB = \sqrt{6.5^2 + 6.5^2 - 2 \times 6.5 \times 6.5 \cos 0.8}$ [A1] $AB = 5.06$ [A1]	M1, A1 A1 (3)
(c)	$r\theta + 5.06 = (6.5 \times 0.8) + 5.06 = 10.26$ (a.w.r.t) (or 10.3)	M1 A1 (2) (7)
3.(a)	$(5p-8) - p = (3p+8) - (5p-8)$ Solve, showing steps, to get $p = 4$, or verify that $p = 4$. (*) <u>Alternative:</u> Using $p = 4$, finding terms (4, 12, 20), and indicating differences.[M1] Equal differences + conclusion (or “common difference = 8”). [A1]	M1 A1 c.s.o. (2)
(b)	$a = 4$ and $d = 8$ (stated or implied here or elsewhere). $T_{40} = a + (n-1)d = 4 + (39 \times 8) = 316$	B1 M1 A1 (3)
(c)	$S_n = \frac{1}{2}n[2a + (n-1)d] = \frac{1}{2}n[8 + 8(n-1)]$ $= 4n^2 = (2n)^2$	M1 A1ft A1 (3) (8)

EDEXCEL 6671 PURE MATHEMATICS P1 JANUARY 2004 MARK SCHEME

Question	Mark Scheme	Marks
4.(a)	$b^2 - 4ac = (-k)^2 - 36 = k^2 - 36$ Or, (completing the square), $\left(x - \frac{1}{2}k\right)^2 = \frac{1}{4}k^2 - 9$ Or, if b^2 and $4ac$ are compared directly, [M1] for finding both [A1] for k^2 and 36. No real solutions: $k^2 - 36 < 0$, $-6 < k < 6$ (ft their "36")	M1 A1 M1, A1ft (4)
(b)	$x^2 - 4x + 9 = (x - 2)^2 + 5$ ($p = 2$) Ignore statement $p = -2$ if otherwise correct. $x^2 - 4x + 9 = (x - 2)^2 - 4 + 9 = (x - 2)^2 + 5$ ($q = 5$) M: Attempting $(x \pm a)^2 \pm b \pm 9$, $a \neq 0$, $b \neq 0$.	B1 M1 A1 (3)
(c)	Min value 5 (or just q), occurs where $x = 2$ (or just p) <u>Alternative:</u> $f'(x) = 2x - 4$ (Min occurs where) $x = 2$ [B1] Where $x = 2$, $f(x) = 5$ [B1ft]	B1ft, B1ft (2) (9)
5.(a)	$\sqrt{8} = 2\sqrt{2}$ seen or used somewhere (possibly implied). $\frac{12}{\sqrt{8}} = \frac{12\sqrt{8}}{8}$ or $\frac{12}{2\sqrt{2}} = \frac{12\sqrt{2}}{4}$ Direct statement, e.g. $\frac{6}{\sqrt{2}} = 3\sqrt{2}$ (no indication of method) is M0.	B1 M1
(b)	At $x = 8$, $\frac{dy}{dx} = 3\sqrt{8} + \frac{12}{\sqrt{8}} = 6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$ (*) Integrating: $\frac{3x^{3/2}}{(3/2)} + \frac{12x^{1/2}}{(1/2)} + C$ (C not required) At (4, 30), $\frac{3 \times 4^{3/2}}{(3/2)} + \frac{12 \times 4^{1/2}}{(1/2)} + C = 30$ (C required) (f(x) =) $2x^{3/2} + 24x^{1/2}, -34$	A1 (3) M1 A1 A1 M1 A1, A1 (6) (9)

EDEXCEL 6671 PURE MATHEMATICS P1 JANUARY 2004 MARK SCHEME

Question	Mark Scheme	Marks
6.(a)	(2, 0) (or $x = 2, y = 0$)	B1 (1)
(b)	$y^2 = 4\left(\frac{3y+12}{2} - 2\right) \quad \text{or} \quad \left(\frac{2x-12}{3}\right)^2 = 4(x-2)$ $y^2 - 6y - 16 = 0 \quad \text{or} \quad x^2 - 21x + 54 = 0 \quad (\text{or equiv. 3 terms})$ $(y+2)(y-8) = 0, y = \dots \text{or} \quad (x-3)(x-18) = 0, x = \dots \quad (3 \text{ term quad.})$ $y = -2, y = 8 \quad \text{or} \quad x = 3, x = 18$ $x = 3, x = 18 \quad \text{or} \quad y = -2, y = 8 \quad (\text{attempt one for M mark})$ (A1ft requires both values)	M1 A1 M1 A1 M1 A1ft (6)
(c)	Grad. of $AQ = \frac{8-0}{18-2}$, Grad. of $AP = \frac{0-(-2)}{2-3}$ (attempt <u>one</u> for M mark) $m_1 \times m_2 = \frac{1}{2} \times -2 = -1$, so $\angle PAQ$ is a right angle (A1 is c.s.o.) <u>Alternative:</u> Pythagoras: Find 2 lengths [M1] $AQ = \sqrt{320}, AP = \sqrt{5}, PQ = \sqrt{325}$ (O.K. unsimplified) [A1ft] (if decimal values only are given, with no working shown, require at least 1 d.p. accuracy for M1(implied) A1) $AQ^2 + AP^2 = PQ^2$, so $\angle PAQ$ is a right angle [M1, A1] M1 requires attempt to use Pythag. for right angle at A, and A1 requires correct <u>exact</u> working + conclusion.	M1 A1ft M1 A1 (4) (11)

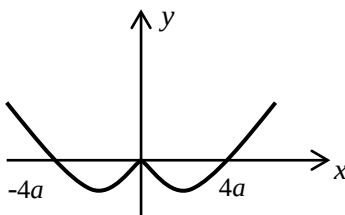
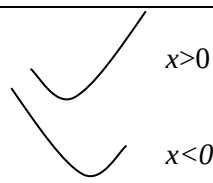
EDEXCEL 6671 PURE MATHEMATICS P1 JANUARY 2004 MARK SCHEME

Question	Mark Scheme	Marks
7.(a)	Solve $\frac{3}{2}x^2 - \frac{1}{4}x^3 = 0$ to find $p = 6$, or verify: $\frac{3}{2} \times 6^2 - \frac{1}{4} \times 6^3 = 0$ (*)	B1 (1)
(b)	$\frac{dy}{dx} = 3x - \frac{3x^2}{4}$ $m = -9, \quad y - 0 = -9(x - 6)$ (Any correct form)	M1 A1 M1 A1 (4)
(c)	$3x - \frac{3x^2}{4} = 0, \quad x = 4$	M1, A1ft (2)
(d)	$\int \left(\frac{3x^2}{2} - \frac{x^3}{4} \right) dx = \frac{x^3}{2} - \frac{x^4}{16}$ (Allow unsimplified versions) $\left[\dots \right]_0^6 = \frac{6^3}{2} - \frac{6^4}{16} = 27$ M: Need 6 and 0 as limits.	M1 A1 M1 A1 (4)
		(11)
8.(a)	$\theta - 10 = 15 \quad \theta = 25$ ($\cos(\theta - 10) = \cos \theta - \cos 10$, etc, is B0)	B1
	$\theta - 10 = 345 \quad \theta = 355$ M: Using 360 – “15” (can be implied) Stating $\theta = 345$ scores M1 A0 (Other methods: M1 for <u>complete</u> method, A1 for 25 and A1 for 355)	M1 A1 (3)
(b)	$2\theta = 21.8\dots$ (α) (At least 1 d.p.) (Could be implied by a correct θ). $2\theta = \alpha + 180$ or $2\theta = \alpha + 360$ or $2\theta = \alpha + 540$ (One more solution) $\theta = 10.9, 100.9, 190.9, 280.9$ (M1: divide by 2) (A1ft: 2 correct, ft their α) (A1: all 4 correct cao, at least 1 d.p.)	B1 M1 M1 A1ft A1 (5)
(c)	$2\sin \theta \left(\frac{\sin \theta}{\cos \theta} \right) = 3, \quad 2\sin^2 \theta = 3\cos \theta$ $2(1 - \cos^2 \theta) = 3\cos \theta$ $2\cos^2 \theta + 3\cos \theta - 2 = 0$ $(2\cos \theta - 1)(\cos \theta + 2) = 0 \quad \cos \theta = \frac{1}{2}$ (M: solve 3 term quadratic up to $\cos \theta = \dots$ or $x = \dots$) $\theta = 60, \quad \theta = 300$	M1, A1 M1 M1 A1 A1 (6)
		(14)

Question number	Mark Scheme		Marks
1(a)	$2 + \frac{3}{x+2} \left(= \frac{2(x+2)+3}{x+2} \right) \quad \ddot{=} \quad \underline{\underline{\frac{2x+7}{x+2}}} \text{ or } \frac{2(x+2)+3}{x+2}$		B1 (1)
(b)	$y = 2 + \frac{3}{x+2}$ $y - 2 = \frac{3}{x+2}$ $x + 2 = \frac{3}{y-2}$ $x = \frac{3}{y-2} - 2$ $\therefore \underline{\underline{f^{-1}(x) = \frac{3}{x-2} - 2}}$ <u>OR</u> 		

Question number	Mark scheme		Marks
2(a)	$u_2 = \sqrt{\left(\frac{3}{2} + \frac{20}{3}\right)} = 2.85773.... = \underline{\underline{2.86}}$		M1
	$u_3 = 2.90300... = \underline{\underline{2.90}}$		A1 c.a.o
	$u_4 = 2.88806.... = \underline{\underline{2.89}}$		A1 c.a.o
S.C.	[If $u_3 = \text{AWRT } 2.90$ and $u_4 = \text{AWRT } 2.89$ penalise once only]		(3)
(b)	$(i) \quad 3 = \sqrt{\left(\frac{3}{2} + \frac{a}{3}\right)} \quad \text{or} \quad 9 = \frac{3}{2} + \frac{a}{3}$		M1
	$\frac{a}{3} = 9 - \frac{3}{2} \quad \text{or} \quad a = 3\left(9 - \frac{3}{2}\right)$		M1
		$\underline{\underline{a = 22.5}}$	A1
	(ii) (If $u_1 = u_2$, then $u_2 = u_3.....$)	$u_5 = \underline{\underline{3}}$	B1
			(1)
			(7)
	Notes		
2(a)	M1	Correct expression or AWRT 2.86	
(b)(i)	M1	A correct equation for a, with or without $\sqrt{}$.	
	M1	Attempt correct manipulation to $ka = , (k > 0)$.	

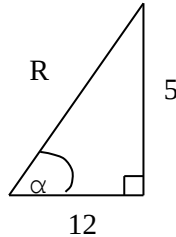
Question number	Mark Scheme		Marks
3(a)	$\log_2 (16x) = \log_2 16 + \log_2 x$ $= \underline{\underline{4 + a}}$		M1 A1 c.a.o (2)
(b)	$\log_2 \left(\frac{x^4}{2} \right) = \log_2 x^4 - \log_2 2$ $= 4\log_2 x - \log_2 2$ $= \underline{\underline{4a - 1}}$ (accept <u>$4\log_2 x - 1$</u>)		M1 M1 A1 (3)
(c)	$\frac{1}{2} = 4 + a - (4a - 1)$ $a = \frac{3}{2}$ $\log_2 x = \frac{3}{2} \Rightarrow x = 2^{\frac{3}{2}}$ <u>$x = \sqrt{8} \text{ or } 2\sqrt{2}$</u> or <u>$\sqrt{2^3} \text{ or } (\sqrt{2})^3$</u>		M1 A1 M1 A1 $\sqrt{\quad}$ (4) (9)
	Notes		
3(a)	M1	Correct use of $\log(ab) = \log a + \log b$	
(b)	M1	Correct use of $\log\left(\frac{a}{b}\right) = \dots$	
	M1	Use of $\log x^n = n \log x$	
(c)	M1	Use their (a)&(b) to form equ in a	
	M1	Out of logs: $x = 2^a$	
	A1 $\sqrt{\quad}$	Must write x in surd form, follow through their rational a .	

Question number	Mark Scheme		Marks
4(a)	  $(4a,0)$ & $(-4a,0)$ and shape at $(0,0)$		B1 B1 ✓ B1 (3)
(b)	$f(2a) = (2a)^2 - 4a(2a) = 4a^2 - 8a^2 = \underline{\underline{-4a^2}}$ $f(-2a) [= f(2a) (\because \text{even function})] = \underline{\underline{-4a^2}}$		B1 B1 ✓ (2)
(c)	$a=3 \text{ and } f(x) = 45 \Rightarrow 45 = x^2 - 12x \quad (x > 0)$ $0 = x^2 - 12x - 45$ $0 = (x-15)(x+3)$ $x=15 \text{ (or } -3)$ $\therefore \text{Solutions are } \underline{\underline{x = \pm 15}} \quad \underline{\underline{\text{only}}}$		M1 M1 A1 A1 (4) (9)
Notes			
4(b)	B1 ✓	✓ their $f(2a)$	
(c)	M1	Attempt 3TQ in x	
	M1	Attempt to solve	
	A1	At least $x=15$ can ignore $x=-3$	
	A1	To get final A1 must make clear <u>only</u> answers are ± 15 .	

Question number	Mark Scheme		Marks
5(a)(i)	<u><u>$x = a^y$</u></u>		B1 (1)
(ii)	In both sides of (i) i.e $\ln x = \ln a^y$ or $(y =) \log_a x = \frac{\ln x}{\ln a}$ $= \underline{y \ln a}$ * $\Rightarrow y \ln a = \ln x$		B1 _{CSO} (1)
(b) ALT.	$y = \frac{1}{\ln a} \cdot \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{\ln a} \times \frac{1}{x}$ * $\left[\text{or } \frac{1}{x} = \frac{dy}{dx} \cdot \ln a \Rightarrow \frac{dy}{dx} = \frac{1}{x \ln a} \right]$ *		M1, A1 _{CSO} (2)
(c)	$\log_{10} 10 = 1 \Rightarrow A \text{ is } (10, \underline{1})$ $y_A = 1$ from(b) $m = \frac{1}{10 \ln a}$ or $\frac{1}{10 \ln 10}$ or 0.043 (or better) equ of target $y - 1 = m(x - 10)$ i.e $y - 1 = \frac{1}{10 \ln 10} (x - 10)$ or $y = \frac{1}{10 \ln 10} x + 1 - \frac{1}{\ln 10}$ (o.e)		B1 B1 M1 A1 (4)
(d)	$y = 0 \text{ in (c)} \Rightarrow 0 = \frac{x}{10 \ln 10} + 1 - \frac{1}{\ln 10} \Rightarrow x = 10 \ln 10 \left(\frac{1}{\ln 10} - 1 \right)$ $\underline{x = 10 - 10 \ln 10}$ or $10(1 - \ln 10)$ or $\underline{10 \ln 10 \left(\frac{1}{\ln 10} - 1 \right)}$		M1 A1 (2) (10)
	Notes		
5(a)	B1	$x = e^{y \ln a}$ is BO	
	B1	Must see $\ln a^y$ or use of change of base formula.	
(b)	M1, A1 _{CSO}	M1 needs some correct attempt at differentiating.	
(c)	B1 M1	Allow either $\sqrt{\quad}$ their y_A and m	
(d)	M1	Attempt to solve correct equation. Allow if a not = 10.	

Question number	Mark Scheme		Marks
6(a)	$f'(x)=0$ for maximum (or stationary point or turning point) $f'(1.48) = e^{1.48} - 2 \times 1.48^2 = 0.0121\dots$ $f'(1.49) = \dots = -0.0031\dots$ change of sign $\therefore$ root / maximum in range		B1 M1 A1 (3)
(b)	$y = e^x - \frac{2}{3}x^3 + c$ at(0,5) $5 = e^0 - 0 + c$ <u>$c = 4$</u> $\left(y = e^x - \frac{2}{3}x^3 + 4 \right)$ ($c=4$)		M1 A1 M1 A1 (4)
(c)	Area $= \int_0^2 \left(e^x - \frac{2}{3}x^3 + 4 \right) dx$ $= \left[e^x - \frac{2}{12}x^4 + 4x \right]_0^2$ $= \left(e^2 - \frac{16}{6} + 8 \right) - (e^0 - 0 + 0)$ <u>$= e^2 + 4\frac{1}{3}$ or $e^2 + \frac{13}{3}$</u>		M1 A1 $\checkmark$ M1 A1 _{cao} (4) (11)
	Notes		
6(a)	M1 M1 A1	May be $\Rightarrow$ if maximum mentioned at A1 One value correct to 1 S.F. Both correct and comment	
(b)	M1 A1 M1	Some correct $\int$ $e^x - \frac{2}{3}x^3$ Attempt to use (0,5) No + c is M0	
(c)	M1 A1 $\checkmark$ M1	Some correct $\int$ <u>other</u> than $e^x \rightarrow e^x$. [] $\checkmark$ their $c(\neq 0)$. Attempt both limits	

Question number	Mark Scheme		Marks
7(a)	<u>4, 4.84, 7.06</u>		B2/1/0 (2)
(b)	$I \approx \frac{1}{2} \times 0.25 [6.06 + 7.06 + 2(4.32 + 4 + 4.84)]$ $= \frac{1}{2} \times 0.25 [39.44]$ $= \underline{\underline{4.93}} \text{ or } \underline{\underline{4.9}} \qquad \qquad \qquad \text{(AWRT 4.93 or just 4.9)}$		B1 [M1 A1] ✓ A1 (4)
(c)	$\int_{0.5}^{1.5} \left(\frac{3}{x} + x^4 \right) dx = \left[3\ln x + \frac{1}{5} x^5 \right]_{0.5}^{1.5}$ $= \left(3\ln 1.5 + \frac{1}{5} 1.5^5 \right) - \left(3\ln 0.5 + \frac{1}{5} 0.5^5 \right)$ $= \underline{\underline{3\ln 3 + 1.5125}} \text{ or } \underline{\underline{3\ln 3 + \frac{121}{80}}}$		M1 A1 M1 A1 (4)
(d)	$\frac{[4.93 - (c)]}{(c)} \times 100, = 2.53\% \text{ (i.e } < 3\%)$ AWRT <u>2.5%</u>		M1, A1 (2) (12)
	Notes		
7(b)	B1 M1 A1	$\frac{1}{2} \times 0.25$ ✓ []	
(c)	M1 A1 M1	Some correct $\int$ $3\ln x + \frac{1}{5} x^5$ Use of limits	

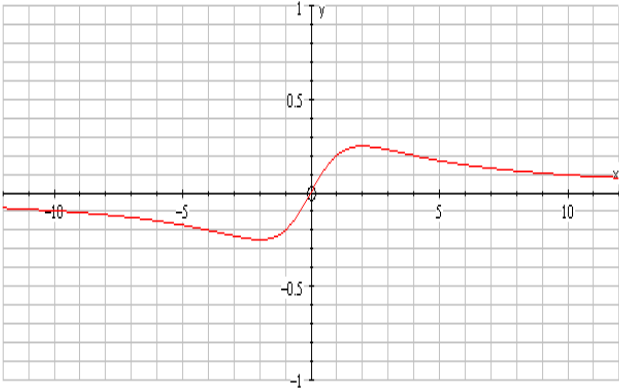
Question number	Mark Scheme		Marks
8(a)(i)	$12 \cos \theta - 5 \sin \theta = R \cos \theta \cos \alpha - R \sin \theta \sin \alpha.$  $R^2 = 5^2 + 12^2 \Rightarrow \underline{R=13}$ $\tan \alpha = \frac{5}{12} \Rightarrow \alpha = 22.6^\circ \text{ (AWRT 22.6)}$ or $0.39^\circ \text{ (AWRT 0.39}^\circ\text{)}$		M1, A1 M1, A1 (4)
(b)	$\cos (\theta + 22.6) = \frac{4}{13}$ $\theta + 22.6 = 72.1,$ <u>$\theta = 49.5$</u> (only)		M1 M1 A1 (3)
(ii)	$\frac{8}{\tan \theta} - 3 \tan \theta = 2$ i.e. $0 = 3 \tan^2 \theta + 2 \tan \theta - 8$ $0 = (3 \tan \theta - 4)(\tan \theta + 2)$ $\tan \theta = \frac{4}{3} \text{ or } -2$ $\tan \theta = \frac{4}{3} \Rightarrow \underline{\underline{\theta = 53.1}}$ [ignore θ not in range e.g. $\theta = 116.6$]		M1 M1 M1 A1 A1 (5) (12)
	Notes		
8(a)(i)	M1, A1 M1, A1	M1 for correct expression for R or R^2 M1 for correct trig expression for α	
(b)	M1 M1	$\cos (\theta + \alpha) = \frac{4}{R}$ $\theta + \alpha = \dots$ their R	
(ii)	M1 M1 M1 A1	Use of $\cot \theta = \frac{1}{\tan \theta}$ 3TQ in $\tan \theta = 0$ Attempt to solve 3TQ = 0 For Final A mark must deal with $\tan \theta = -2$	

Question Number	Scheme	Marks
1.	Either 1 f.t. on $\frac{1}{a}$ Obtains centre (0, 6.5) Finds radius or diameter by Pythagoras Theorem, to obtain $r = 2.5$ or $r^2 = 6.25$ $x^2 + (y - 6.5)^2 = 2.5^2$ or $x^2 + y^2 - 13y + 36 = 0$ Or $\frac{y-8}{x+2} \times \frac{y-5}{x-2} = -1$ Gradients multiplied and put = to -1 $x^2 + y^2 - 13y + 36 = 0$ Or Obtains centre (0, 6.5) $x^2 + (y - 6.5)^2 = r^2$ or $x^2 + y^2 - 13y + c = 0$ substitutes either (2 , 5) or (-2 , 8) $x^2 + (y - 6.5)^2 = 2.5^2$ or $x^2 + y^2 - 13y + 36 = 0$	B1 M1, A1 B1 (4) B1 M1A1 B1 (4) B1 B1 M1 A1 (4)
2.	(a) $na = -6,$ $\frac{n(n-1)}{2}a^2 = 27$ Attempts solution by eliminating variable e.g. $\frac{n(n-1)36}{n^2} = 54$ or $-\frac{6}{a}(-\frac{6}{a}-1)a^2 = 54$ $n = -2,$ $a = 3$ (b) $\frac{(-2)(-3)(-4)3^3}{6} = -108$ for M1 allow a instead of a^3 (c) $x < \frac{1}{3}$ or $-\frac{1}{3} < x < \frac{1}{3}$	B1, B1 M1 A1, A1 (5) M1 A1 (2) B1 f.t. (1)

Question Number	Scheme	Marks
3.	(a) $10x, +(2y + 2x \frac{dy}{dx}), -6y \frac{dy}{dx} = 0$ At (1, 2) $10 + (4 + 2 \frac{dy}{dx}) - 12 \frac{dy}{dx} = 0$ $\therefore \frac{dy}{dx} = \frac{14}{10} = 1.4 \text{ or } \frac{7}{5} \text{ or } 1\frac{2}{5}$ (b) The gradient of the normal is $-\frac{5}{7}$ Its equation is $y - 2 = -\frac{5}{7}(x - 1)$ (allow tangent) $y = -\frac{5}{7}x + 2\frac{5}{7} \text{ or } y = -\frac{5}{7}x + \frac{19}{7}$	M1,(B1),A1 M1 A1 (5) M1 M1 A1cao (3)
4.	(a) Uses the remainder theorem with $x = \frac{1}{2}$, or long division, and puts remainder = 0 To obtain $p + 2q = -35$ or any correct equivalent (allow more than 3 terms) Uses the remainder theorem with $x = 1$, or long division, and puts remainder = ± 7 To obtain $p + q = -21$ or any correct equivalent (allow more than 3 terms) Solves simultaneous equations to give $p = -7$, and $q = -14$ (b) Then $6x^3 - 7x^2 - 14x + 8 = (2x - 1)(3x^2 - 2x - 8)$ So $f(x) = (2x - 1)(3x + 4)(x - 2)$	M1 A1 M1 A1 M1 A1 (6) M1 A1 ft B1 (3)

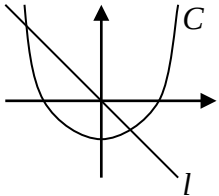
Question Number	Scheme	Marks
5. (a)	Area of triangle = $\frac{1}{2} \times 30 \times 3\pi^2$ (= 444.132) Accept 440 or 450	B1 (1)
(b)	Either Area shaded = $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 30 \sin 2t \cdot 32t dt$ $= \left[-480t \cos 2t + \int 480 \cos 2t \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= \left[-480t \cos 2t + 240 \sin 2t \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= 240(\pi - 1)$ or $\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} 60 \cos 2t \cdot (16t^2 - \pi^2) dt$ $= \left[(30 \sin 2t(\pi^2 - 16t^2) - 480t \cos 2t + \int 480 \cos 2t \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= \left[-480t \cos 2t + 240 \sin 2t \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= 240(\pi - 1)$	M1 A1 M1 A1 A1 ft M1A1 (7) M1 A1 M1 A1 A1 ft M1 A1 (7)
(c)	Percentage error = $\frac{240(\pi - 1) - \text{estimate}}{240(\pi - 1)} \times 100 = 13.6\%$ (Accept answers in the range 12.4% to 14.4%)	M1 A1 (2)

Question Number	Scheme	Marks
6. (a)	Uses $\frac{A}{(2x-3)} + \frac{B}{(x+1)}$ Considers $-2x + 13 = A(x+1) + B(2x-3)$ and substitutes $x = -1$ or $x = 1.5$, or compares coefficients and solves simultaneous equations To obtain $A = 4$ and $B = -3$.	M1 M1 A1, A1 (4)
(b)	Separates variables $\int \frac{1}{y} dy = \int \frac{4}{2x-3} - \frac{3}{x+1} dx$ $\ln y = 2\ln(2x-3) - 3\ln(x+1) + C$ Substitutes to give $\ln 4 = 2\ln 1 - 3\ln 3 + C$ and finds C ($\ln 108$) $\ln y = \ln(2x-3)^2 - \ln(x+1)^3 (+\ln 108)$ $= \ln \frac{C(2x-3)^2}{(x+1)^3}$ $\therefore y = \frac{108(2x-3)^2}{(x+1)^3}$ Or $y = e^{2\ln(2x-3) - 3\ln(x+1) + \ln 108}$ special case M1 A2	M1 A1, B1 ft M1 M1 A1 A1 cso (7)

Question Number	Scheme	Marks
7.		
(a)	$\frac{dy}{dx} = \frac{(4+x^2) - x(2x)}{(4+x^2)^2}$ Need numerical answers for M1 or from $(x^2)^{-2}$ Solve $\frac{dy}{dx} = 0$ to obtain $(2, \frac{1}{4})$, and $(-2, -\frac{1}{4})$ or $(2$ and -2 A1, full solution A1)	M1 A1 M1 A1, A1 (5)
(b)	When $x = 2$, $\frac{d^2y}{dx^2} = -0.0625 < 0$ thus maximum When $x = -2$, $\frac{d^2y}{dx^2} = 0.0625 > 0$ thus minimum.	B1 M1 B1 (3)
(c)	 Shape for $-2 \leq x \leq 2$ Shape for $x > 2$ Shape for $x < -2$	B1 B1 B1 (3)

Question Number	Scheme	Marks
8. (a)	$1 + \lambda = -2 + 4\mu$ $3 + 2\lambda = 3 + 4\mu$ $5 - \lambda = -4 + 4\mu$ Need two of these for M1 Solve simultaneous equations to obtain $\mu = 2$, or $\lambda = 1$ $\therefore$ intersect at (2, 5, 4) Check in the third equation or on second line	M1 M1 A1 M1 A1 B1 (6)
(b)	$1 \times 2 + 2 \times 1 + (-1) \times 4 = 0 \quad \therefore \text{perpendicular}$	M1 A1 (2)
(c)	P is the point (3, 7, 3) [i.e. $\lambda = 2$] and R is the point (4, 6, 8) [i.e. $\mu = 3$] $PQ = \sqrt{1^2 + 2^2 + (-1)^2} = \sqrt{6}$ $RQ = \sqrt{2^2 + 1^2 + 4^2} = \sqrt{21}$ $PR = \sqrt{27}$ The area of the triangle = $\frac{1}{2} \times \sqrt{6} \times \sqrt{21} = \frac{3\sqrt{14}}{2}$ Or area = $\frac{1}{2} \times \sqrt{6} \times \sqrt{27} \sin P$ where $\sin P = \frac{\sqrt{7}}{3} = \frac{3\sqrt{14}}{2}$ Or area = $\frac{1}{2} \times \sqrt{21} \times \sqrt{27} \sin R$ where $\sin R = \frac{\sqrt{2}}{3} = \frac{3\sqrt{14}}{2}$ (must be simplified)	M1 A1 M1 A1 ft M1 A1 (6)

Question number	Scheme	Marks
1.	(a) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1+5}{2}, \frac{2+8}{2}\right) = (3, 5)$ (b) Gradient = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{8-2}{5-1}$ $y - 2 = m(x - 1) \quad y = \frac{3}{2}x + \frac{1}{2}$ Allow $y = \frac{3x+1}{2}$ or $y = \frac{1}{2}(3x+1)$	M1 A1 (2) M1 A1 M1 A1 (4) 6
2.	(a) $\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$ (seen or implied) $(3 - \sqrt{8})(3 - \sqrt{8}) = 9 - 6\sqrt{8} + 8 = 17 - 12\sqrt{2}$ (b) $\frac{1}{4 - \sqrt{8}} \times \frac{4 + \sqrt{8}}{4 + \sqrt{8}}, = \frac{4 + \sqrt{8}}{16 - 8} = \frac{1}{2} + \frac{1}{4}\sqrt{2}$ Allow $\frac{1}{4}(2 + \sqrt{2})$ or equiv. (in terms of $\sqrt{2}$)	B1 M1 A1 (3) M1, M1 A1 (3) 6
3.	(a) $2x + 2(x + 20) < 300$ (Using $x - 20$ is A0) (b) $x(x + 20) > 4800$ (Using $x - 20$ is A0) (c) 65 (i.e. Allow wrong inequality sign or $x = 65$). Solving 3 term quadratic, $(x + 80)(x - 60) = 0 \quad x = \dots$ $x > 60$ ($x < -80$ may be included here, but there must be no other <u>wrong</u> solution to the quadratic inequality such as $x > -80$) $60 < x < 65$	M1 A1 (2) M1 A1 (2) B1ft M1 A1 A1 (4) 8

Question number	Scheme	Marks
4.	(a)  C : “U” shape C : Position l : Straight line through origin with negative gradient (b) (2, 0), (−2, 0), (0, −4) 2 of these correct: All 3 correct: (c) $x^2 - 4 = -3x$ $x^2 + 3x - 4 = 0$ $(x + 4)(x - 1) = 0$ $x = \dots$ $x = -4$ $x = 1$ $y = 12$ $y = -3$ M: Attempt one y value	B1 B1 B1 (3) B1 B1 (2) M1 A1 M1 A1 (4) 9
5.	(a) $\tan x = \frac{8}{3}$ (or exact equivalent, or 3 s.f. or better) (b) $\tan x = \frac{8}{3}$ $x = 69.4^\circ (\alpha)$, $x = 249.4^\circ (180 + \alpha)$ (c) $3(1 - \cos^2 y) - 8\cos y = 0$ $3\cos^2 y + 8\cos y - 3 = 0$ $(3\cos y - 1)(\cos y + 3) = 0$ $\cos y = \dots$, $\frac{1}{3}$ (or -3) $y = 70.5^\circ (\beta)$, $x = 289.5^\circ (360 - \beta)$	B1 (1) M1 A1, A1ft (3) M1 A1 M1 A1 A1 A1ft (6) 10

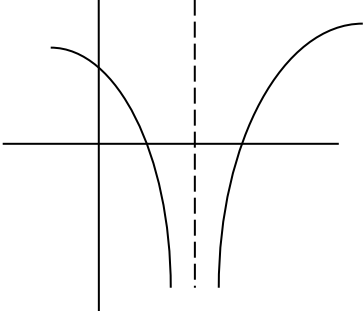
Question number	Scheme	Marks
6.	(a) $(x^4 - 6x^2 + 9)$ $(x^4 - 6x^2 + 9) \div x^3 = x - 6x^{-1} + 9x^{-3}$ (*) (b) $f'(x) = 1 + 6x^{-2} - 27x^{-4}$ First A1: 2 terms correct (unsimplified) Second A1: all 3 correct (simplified) (c) When $x = \pm\sqrt{3}$, $f'(x) = 1 + \frac{6}{(\sqrt{3})^2} - \frac{27}{(\sqrt{3})^4}$ $\left(= 1 + \frac{6}{3} - \frac{27}{9}\right) = 0, \therefore \text{Stationary}$ (d) $f''(x) = -12x^{-3} + 108x^{-5}$ M: Attempt to diff. $f'(x)$, <u>not</u> $g(x)f'(x)$. $f''(\sqrt{3}) = -\frac{12}{(\sqrt{3})^3} + \frac{108}{(\sqrt{3})^5} \quad (\approx -2.309 + 6.928 = 4.619) \left(= \frac{8}{\sqrt{3}}\right)$ $> 0, \therefore \text{Minimum (not dependent on a numerical version of } f''(x))$	M1 A1 (2) M1 A1 A1 (3) M1 A1 (2) M1 A1 A1ft (3) 10
7.	(a) $(S =) a + ar + \dots + ar^{n-1}$ "S =" not required. Addition required. $(rS =) ar + ar^2 + \dots + ar^n$ "rS =" not required (M: Multiply by r) $S(1-r) = a(1-r^n)$ $S = \frac{a(1-r^n)}{1-r}$ (M: Subtract and factorise each side) (*) (b) $r = 0.9$ $S_{20} = \frac{10(1-0.9^{20})}{1-0.9} = 87.8$ (c) Sum to infinity $= \frac{a}{1-r} = \frac{10}{1-0.9} = 100$ (ft only for $r < 1$) (d) $\frac{a}{1-r} = \frac{r}{1-r} = 10$ (Put $a = r$ in the formula from (c), and equate to 10) $r = 10(1-r)$ $r = \dots, \frac{10}{11}$ (or exact equivalent)	B1 M1 M1 A1 (4) B1 M1 A1 (3) M1 A1ft (2) M1 M1, A1 (3) 12

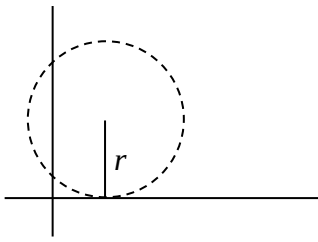
Question number	Scheme	Marks
8.	(a) $\frac{dy}{dx} = 3x^2 - 14x + 15$ (b) $3x^2 - 14x + 15 = 0$ $(3x - 5)(x - 3) = 0$ $x = \dots, 3$ (A1 requires <u>correct</u> quadratic factors). $y = 12$ (Following from $x = 3$) (c) $P: x = 1 \quad y = 12$ Same y-coord. as Q (or “zero gradient”), so PQ is parallel to the x-axis (d) $\int (x^3 - 7x^2 + 15x + 3)dx = \frac{x^4}{4} - \frac{7x^3}{3} + \frac{15x^2}{2} + 3x$ (First A1: 3 terms correct, Second A1: all correct) $\left[\frac{x^4}{4} - \frac{7x^3}{3} + \frac{15x^2}{2} + 3x \right]_1^3 = \left(\frac{81}{4} - 63 + \frac{135}{2} + 9 \right) - \left(\frac{1}{4} - \frac{7}{3} + \frac{15}{2} + 3 \right)$ $\left(33\frac{3}{4} - 8\frac{5}{12} \right) - 24 = 25\frac{1}{3} - (2 \times 12) = 1\frac{1}{3}$ (or equiv. or 3 s.f or better)	M1 A1 (2) M1 M1, A1 A1 (4) B1 B1 (2) M1 A1 A1 M1 M1 A1 (6) 14

Question Number	Scheme	Marks
1.	$\frac{(x-3)(x-5)}{(x-3)(x+3)} \times \frac{2x(x+3)}{(x-5)^2} \quad (3 \times \text{factorising})$ $= \frac{2x}{x-5}$	B1 B1 B1 B1 (4 marks)
2.	(i) A correct form of $\cos 2x$ used $1 - 2\left(\frac{3}{5}\right)^2 \text{ or } \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 \text{ or } 2\left(\frac{4}{5}\right)^2 - 1 \quad \left\{\frac{7}{25}\right\}$ $\sec 2x = \frac{1}{\cos 2x} \quad ; \quad = \frac{25}{7} \text{ or } 3\frac{4}{7}$ (ii) (a) $\frac{\cos 2x}{\sin 2x} + \frac{1}{\sin 2x}$ or (b) $\frac{1}{\tan 2x} + \frac{1}{\sin 2x}$ Forming single fraction (or multiplying both sides by $\sin 2x$) Use of correct trig. formulae throughout and producing expression in terms of $\sin x$ and $\cos x$ Completion (cso) e.g. $\frac{2\cos^2 x}{2\sin x \cos x} = \frac{\cos x}{\sin x} = \cot x \quad (*)$	M1 A1 M1A1 (4) M1 M1 A1 (4) (8 marks)
3.	(a) $(x^3)^{12}; \dots + \binom{12}{1}(x^3)^{11}\left(-\frac{1}{2x}\right) + \binom{12}{2}(x^3)^{10}\left(-\frac{1}{2x}\right)^2 + \dots$ [For M1, needs binomial coefficients, nC_r form OK, at least as far as shown] Correct values for nC_r s: 12, 66, 220 used (may be implied) $(x^3)^{12} + 12(x^3)^{11}\left(-\frac{1}{2x}\right) + 66(x^3)^{10}\left(-\frac{1}{2x}\right)^2 + 220(x^3)^9\left(-\frac{1}{2x}\right)^3 \dots$ $x^{36} - 6x^{32} + \frac{33}{2}x^{28} - \frac{55}{2}x^{24}$ (b) Term involving $(x^3)^3\left(-\frac{1}{2x}\right)^9$; $\text{coeff} = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} \left(-\frac{1}{2}\right)^9$ $= -\frac{55}{128} \quad (\text{or } -0.4296875)$	B1; M1 B1 A2(1,0) (5) M1 A1 A1 (3) (8 marks)

Question Number	Scheme	Marks
4.	(a) $y^2 = \left(\frac{x+2}{\sqrt{x}}\right)^2 = \frac{x^2 + 4x + 4}{x} = x + 4 + \frac{4}{x}$ $\pi \int y^2 dx$ [dependent on attempt at squaring y] $\int y^2 dx = \int \left(\frac{x^2 + 4x + 4}{x}\right) dx; = \frac{x^2}{2} + 4x + 4 \ln x$ [A1√ must have $\ln x$ term] Correct use of limits: $\left[\right]_1^4 = \left[\right]^4 - \left[\right]_1$ [M dependent on prev. M1] Volume = $\left(\frac{39}{2} + 4 \ln 4\right)\pi$ or equivalent exact (b) Showing that $y = 3$ at $x = 1$ and $x = 4$ (c) Volume = $2^3 \times$ answer to (a) ; = $629.5 \text{ cm}^3 \approx 630 \text{ cm}^3$ (*) [allow 629 – 630]	M1A1 B1 M1;A1 ft M1 A1 (7) B1 (1) M1;A1 (2) (10 marks)
5.	(a) Attempting to reach at least the stage $x^2(x+1) = 4x+1$ Conclusion (no errors seen) $x = \sqrt{\frac{4x+1}{x+1}}$ (*) [Reverse process: need to square and clear fractions for M1] (b) $x_2 = \sqrt{\frac{4+1}{1+1}} = 1.58\dots$ $x_3 = 1.68, \quad x_4 = 1.70$ [Max. deduction of 1 for more than 2 d.p.] (c) Suitable interval; e.g. [1.695, 1.705] (or “tighter”) $f(1.695) = -0.037\dots, \quad f(1.705) = +0.0435\dots$ Change of sign, no errors seen, so root = 1.70 (correct to 2 d.p.) (d) $x = -1$, “division by zero not possible”, or equivalent or any number in interval $-1 < x < -\frac{1}{4}$, “square root of neg. no.”	M1 A1 (2) M1 A1A1 (3) M1 Dep. M1 A1 (3) B1,B1 (2) (10 marks)

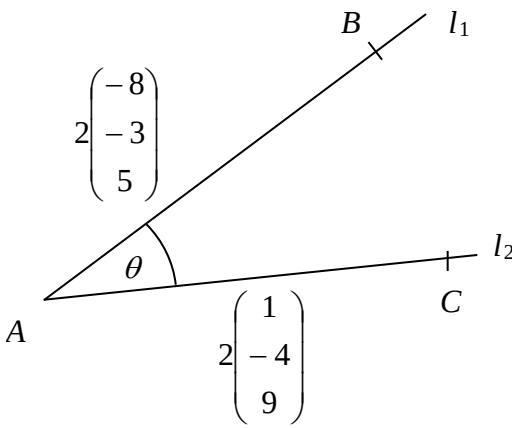
Question Number	Scheme	Marks
6.	(a) $\log_5 x^2 - \log_5 y$; $= 2\log_5 x - \log_5 y = 2a - b$ (b) $\log_5 25 = 2$ or $\log_5 y$ $\log_5 25 + \log_5 x + \log_5 y^{\frac{1}{2}}; = 2 + a + \frac{1}{2}b$ (c) $2a - b = 1, 2 + a + \frac{1}{2}b = 1$ (must be in a and b) (d) Using both correct equations to show that $a = -0.25$ (*) $b = -1.5$ [Mark for (c) can be gained in (d)] (e) Using correct method to find a value for x or a value of y: $x = 5^{-0.25} = 0.669, y = 5^{-1.5} = 0.089$ [max. penalty -1 for more than 3 d.p.]	M1A1 (2) B1 M1;A1 (3) B1 ft (1) M1 B1 (2) M1 A1 A1 ft (3) (11 marks)
7.	(a) Differentiating; $f'(x) = 1 + \frac{e^x}{5}$ (b) A: $\left(0, \frac{1}{5}\right)$ Attempt at $y - f(0) = f'(0)x$; $y - \frac{1}{5} = \frac{6}{5}x$ or equivalent “one line” 3 termed equation (c) 1.24, 1.55, 1.86 (d) Estimate = $\frac{0.5}{2}$; ($\times$) $[(0.45 + 1.86) + 2(0.91 + 1.24 + 1.55)]$ $= 2.4275 \quad \left(\begin{smallmatrix} 2.428 \\ 2.429 \end{smallmatrix}, 2.43\right)$	M1;A1 (2) B1 M1 A1 ft (3) B2(1,0) (2) B1 M1 A1 ft A1 (4) (11 marks)

Question Number	Scheme	Marks
8.	(a) $y = \ln(3x - 6) \Rightarrow 3x - 6 = e^y$ $\Rightarrow x = \frac{e^y + 6}{3}; \quad \{f^{-1}(x)\} = \frac{e^x + 6}{3}$	M1 M1;A1 (3)
	(b) Domain: $x \in \mathbb{R}$ Range: $f^{-1}(x) > 2$	B1 B1 (2)
	(c) Attempting to find $f^{-1}(3) [= \frac{e^3 + 6}{3}]; = 8.70$	M1;A1 (2)
	(d) 	ln curve passing through $y = 0$ Symmetry in $x = k, k > 0$ All correct and asymptote at $x = 2$ labelled B1 M1 A1
	(e) Meets y-axis: $(x = 0), y = \ln 6$ Meets x-axis: $x = \frac{5}{3}, (0); \quad x = \frac{7}{3}, (0)$ [May be seen on graph]	B1 B1B1 (3) (13 marks)

Question Number	Scheme	Marks
1.	$\frac{dy}{dx} = \frac{1}{\csc x + \cot x} (-\csc x \cot x + -\csc^2 x)$ Full attempt at chain rule $= -\csc x \frac{(\cot x + \csc x)}{\csc x + \cot x}$ Factorise cosec x $= -\csc x \quad (*)$	M1 M1 A1 cso (3) (3 marks)
2.	(a) 3 (b) $f(2) = 24 \Rightarrow 24 = (4 + p) \times 7 + 3$ $\Rightarrow p = -1 \quad (*)$ (c) $f(x) = (x^2 - 1)(2x + 3) + 3$ $= 2x^3 + 3x^2 - 2x - 3 + 3$ $= x(2x^2 + 3x - 2)$ $= x(2x - 1)(x + 2)$ Attempt to multiply out Factor of x Attempt to factorise 3 term quadratic	B1 (1) M1 A1 cso (2) M1 M1 M1, A1 (4) (7 marks)
3.	(a)  Eqn: $(x - 5)^2 + (y - 13)^2 = r^2$ $r = 13 \quad (x - 5)^2 + (y - 13)^2 = 13^2$ (b) Differentiate: $2(x - 5) + 2(y - 13) \frac{dy}{dx} = 0$ At (10, 1) $(2 \times 5) + 2 \times -12 \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{10}{24} \text{ or } \frac{5}{12}$ Eqn. of tangent $y - 1 = \frac{5}{12}(x - 10)$ $5x - 12y - 38 = 0$ Attempt to diff. Use of (10, 1) f.t. on their m	M1 A1 (2) M1 M1 A1 M1 A1 (5) (7 marks)

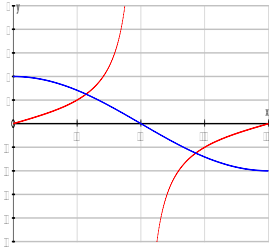
Question Number	Scheme	Marks
4.	$u = 1 + \sin x \Rightarrow \frac{du}{dx} = \cos x \quad \text{or} \quad du = \cos x dx \quad \text{or} \quad dx = \frac{du}{\cos x}$ $I = \int (u-1)u^5 du \quad \text{Full sub. to } I \text{ in terms of } u, \text{ correct}$ $= \int (u^6 - u^5) du \quad \text{Correct split}$ $= \frac{u^7}{7} - \frac{u^6}{6} (+c) \quad \text{M1 for } u^n \rightarrow u^{n+1}$ $= \frac{u^6}{42} (6u-7) (+c) \quad \text{Attempt to factorise}$ $= \frac{(1+\sin x)^6}{42} (6\sin x + 6-7) (+c) = \frac{(1+\sin x)^6}{42} (6\sin x - 1) (+c) (*)$	M1 M1, A1 M1 M1, A1 M1 A1 cso (8 marks)
Alt	Integration by parts $I = (u-1)\frac{u^6}{6} - \frac{1}{6} \int u^6 du \quad \text{Attempt first stage}$ $= (u-1)\frac{u^6}{6} - \frac{u^7}{42} \quad \text{Full integration}$ $= \frac{u^7}{6} - \frac{u^6}{6} - \frac{u^7}{42} \quad \text{or} \quad \frac{6u^7 - 7u^6}{42}$	M1 M1 A1 rest as scheme
5.	(a) $3 + 5x \equiv A(1-x) + B(1+3x)$ Method for A or B $(x=1) \Rightarrow 8 = 4B \quad B = 2$ $(x=-\frac{1}{3}) \Rightarrow \frac{4}{3} = \frac{4}{3}A \quad A = 1$ (b) $2(1-x)^{-1} = 2[1+x+x^2+\dots]$ Use of binomial with $n=-1$ scores M1(×2) $(1+3x)^{-1} = [1-3x + \frac{(-1)(-2)}{2!}(3x)^2 + \dots]$ $\therefore \frac{3+5x}{(1-x)(1+3x)} = 2 + 2x + 2x^2 + 1 - 3x + 9x^2 = 3 - x + 11x^2$ (c) $(1+3x)^{-1}$ requires $x < \frac{1}{3}$, so expansion is <i>not</i> valid.	M1 A1 A1 (3) M1 [A1] M1 [A1] A1 (5) M1, A1 (2) (10 marks)

Question Number	Scheme	Marks
6.	(a) $4 = 2 \sec t \Rightarrow \cos t = \frac{1}{2}, \Rightarrow t = \frac{\pi}{3}$ $\therefore a = 3 \times \frac{\pi}{3} \times \sin \frac{\pi}{3} = \frac{\pi\sqrt{3}}{2}$ (b) $A = \int_0^a y \, dx = \int y \frac{dx}{dt} dt$ Change of variable $= \int 2 \sec t \times [3 \sin t + 3t \cos t] dt$ Attempt $\frac{dx}{dt}$ $= \int_0^{\frac{\pi}{3}} (6 \tan t + 6t) dt \quad (*)$ Final A1 requires limit stated (c) $A = [6 \ln \sec t + 3t^2]_0^{\frac{\pi}{3}}$ Some integration (M1) both correct (A1) ignore lim. $= (6 \ln 2 + 3 \times \frac{\pi^2}{9}) - (0)$ Use of $\frac{\pi}{3}$ $= 6 \ln 2 + \frac{\pi^2}{3}$	M1, A1 B1 (3) M1 M1 A1, A1cso (4) M1, A1 M1 A1 (4) (11 marks)
7.	(a) $A = \pi r^2, \frac{dr}{dt} = 4\lambda e^{-\lambda t}$ $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}, \Rightarrow \frac{dA}{dt} = 2\pi \times 4(1 - e^{-\lambda t}) \times 4\lambda e^{-\lambda t}$ $\frac{dA}{dt} = 32\pi\lambda(e^{-\lambda t} - e^{-2\lambda t})$ (b) $\int A^{-\frac{3}{2}} dA = \int t^{-2} dt$ Separation $\frac{A^{-\frac{1}{2}}}{-\frac{1}{2}} = \frac{t^{-1}}{-1} (+c)$ $-2 = -1 + c$ Use of (1, 1) $c = -1$ So $2A^{-\frac{1}{2}} = \frac{1}{t} + 1 \Rightarrow \sqrt{A} = \frac{2t}{1+t}$ Attempt $\sqrt{A} =$ or $A =$ i.e. $A = \frac{4t^2}{(1+t)^2}$ (c) Because $\frac{t^2}{(1+t)^2} < 1$ or $t^2 < (1+t)^2 \quad (\Rightarrow A < 4)$	B1, B1 M1, M1 A1cso (5) M1 M1, A1 M1 A1 M1 A1 (7) B1 (1) (13 marks)

Question Number	Scheme	Marks
8.	(a) $9 - 8t = -16 + s$ Attempt a correct equation M1 $4 + 5t = 10 + 9s$ Both correct A1 Sub. $s = 25 - 8t \Rightarrow 5t = 6 + 225 - 72t$ Solving either M1 $77t = 231$ or $t = 3, s = 1$ A1 Sub. into 'j' $2 - 3t = \alpha - 4s$ Use of 3rd equation M1 $\Rightarrow \alpha = -3$ A1 (6) (b) $\overrightarrow{OA} = \begin{pmatrix} -15 \\ -7 \\ 19 \end{pmatrix}$ B1 (1) (c) $\begin{pmatrix} -8 \\ -3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -4 \\ 9 \end{pmatrix} = -8 + 12 + 45 (= 49)$ Attempt scalar product M1 $\cos \theta = \frac{49}{\sqrt{8^2 + 3^2 + 5^2} \sqrt{1^2 + 4^2 + 9^2}} = \frac{49}{\sqrt{98} \sqrt{98}} = \frac{1}{2}$ Use of $\frac{\mathbf{a} \cdot \mathbf{b}}{ \mathbf{a} \mathbf{b} }$, $\mathbf{a}$ or $\mathbf{b}$ M1, M1 $\cos \theta = \frac{1}{2}$ A1 $\theta = 60^\circ$ (*) A1 cso (5)  $14m = 2 \times 7 \sqrt{2} = 2 \sqrt{98}$ B1 $\overrightarrow{OB} = \overrightarrow{OA} \pm 2 \begin{pmatrix} -8 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} -31 \\ -13 \\ 29 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ -1 \\ 9 \end{pmatrix}$ M1: $\mathbf{a} \pm 2()$, A1: any one M1, A1 $\overrightarrow{OC} = \overrightarrow{OA} \pm 2 \begin{pmatrix} 1 \\ -4 \\ 9 \end{pmatrix} = \begin{pmatrix} -13 \\ -15 \\ 37 \end{pmatrix}$ or $\begin{pmatrix} -17 \\ 1 \\ 1 \end{pmatrix}$ any correct pair A1 (4)	(16 marks)

Question number	Scheme	Marks
1.	$4 = 2^2$ (or $\log 4 = 2\log 2$) Linear equation in x : $1 - x = 2x$ $x = \frac{1}{3}$	B1 M1 A1 3
2.	(a) $ar^3 = 12$ $ar^4 = -8$ $r = \dots, -\frac{2}{3}$ (or exact equivalent) (b) Using r with $ar^3 = 12$ or $ar^4 = -8$ to find $a = \dots$ $a = -40\frac{1}{2}$ (*) (c) $\frac{a}{1-r} = \frac{-40\frac{1}{2}}{1 - \left(-\frac{2}{3}\right)}, = -24.3 \left(-24\frac{3}{10} \text{ or } -\frac{243}{10}\right)$ A1ft requires $ r < 1$	M1, A1 (2) M1 A1 (2) M1 A1ft, A1 (3) 7
3.	(a) $y = 4x - x^2$ $\frac{dy}{dx} = 4 - 2x$ " $4 - 2x$ " = -2 , $x = \dots$ $x = 3, y = 3$ (b) x -coordinate of A is 4 $\int (4x - x^2) dx = \left[\frac{4x^2}{2} - \frac{x^3}{3} \right]$ $\left[\frac{4x^2}{2} - \frac{x^3}{3} \right]_0^4 = 32 - \frac{64}{3} = \frac{32}{3} \left(= 10\frac{2}{3} \right)$ (or exact equivalent)	M1 A1 M1 A1 (4) B1 M1 A1ft M1 A1 (5) 9

Question number	Scheme	Marks
4.	(a) $\cos A = \frac{6^2 + 2^2 - (2\sqrt{7})^2}{2 \times 6 \times 2}$ $\cos A = \frac{1}{2} \quad A = \frac{\pi}{3} \text{ radians} \quad (*)$ (b) $r\theta = \frac{2\pi}{3} \quad (= 2.09) \quad (\text{Exact or at least 3 s.f.})$ (c) Sector ABD: $\frac{1}{2}r^2\theta = \frac{1}{2} \times 2^2 \times \frac{\pi}{3} \quad \left(= \frac{2\pi}{3} \approx 2.094... \right)$ Triangle ACB: $\frac{1}{2} \times 2 \times 6 \times \sin \frac{\pi}{3} \quad \left(= 3\sqrt{3} \approx 5.196... \right)$ Triangle – Sector = $3\sqrt{3} - \frac{2\pi}{3} \quad (= 3.10175...) \quad \text{Allow 3.1 or a.w.r.t. 3.10}$	M1 A1 A1 (3) M1 A1 (2) M1 M1 M1 A1 (4) 9
5.	(a) Gradient of AB = $\frac{12 - 4}{11 - (-1)} = \frac{2}{3}$ (or equiv.) (b) Using $m_1 m_2 = -1$, gradient of BC = $-\frac{3}{2}$ Equation of BC: $y - 12 = -\frac{3}{2}(x - 11)$ $3x + 2y - 57 = 0$ (Allow rearranged versions, e.g. $2y = 57 - 3x$) (c) D: $y = 0$ in equation of BC: $x = 19$ Coordinates of C: (3, 24) (d) $AC = \sqrt{("3" - (-1))^2 + ("24" - 4)^2} = \sqrt{416} = 4\sqrt{26} \quad (*)$	M1 A1 (2) B1ft M1 A1ft A1 (4) B1ft B1, B1 (3) M1 A1 (2) 11

Question number	Scheme	Marks
6.	(a)  Tangent graph shape 180 indicated Cosine graph shape 2 and 90 indicated Allow separate sketches. (b) Using $\tan x = \frac{\sin x}{\cos x}$ and multiplying both sides by $\cos x$. ($\sin x = 2 \cos^2 x$) Using $\sin^2 x + \cos^2 x = 1$ $2 \sin^2 x + \sin x - 2 = 0$ (*) (c) Solving quadratic: $\sin x = \frac{-1 \pm \sqrt{17}}{4}$ (or equiv.) $x = 51.3$ (3 s.f. or better, 51.33...) α $x = 128.7$ (accept 129) (3 s.f. or better) $180 - \alpha$ ($\alpha \neq 90n$)	M1 A1 M1 A1 (4) M1 M1 A1 (3) M1 A1 A1 B1ft (4) 11

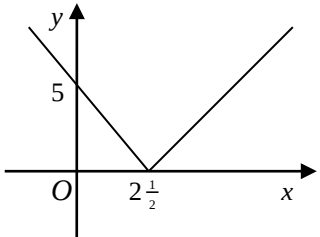
Question number	Scheme	Marks
7.	(a) $\pi r^2 h = 780, \quad h = \frac{780}{\pi r^2}$ (b) $A = 2\pi r^2 + 2\pi rh$ and substitute for h. $A = 2\pi r^2 + \frac{1560}{r} \quad (*)$ (c) $\frac{dA}{dr} = 4\pi r - 1560r^{-2}$ Equate to zero and proceed to $r^3 = \dots$ or $r = \dots$, coping with indices. $r = \sqrt[3]{\frac{1560}{4\pi}} \left(= \sqrt[3]{\frac{390}{\pi}} \approx 4.99 \approx 5.0 \right)$ (d) Attempt second derivative and consider its sign/value. $\frac{d^2 A}{dr^2} = 4\pi + 3120r^{-3} \quad \text{Correct second derivative, } > 0, \therefore \text{minimum.}$ (e) Substitute value of r (or values of r and h) into their A formula. 469 (or a.w.r.t.) or 470 (2 s.f.)	M1, A1 (2) M1 A1 (2) M1 A1 M1 A1 (4) M1 A1 (2) M1 A1cso (2) 12

Question number	Scheme	Marks
8.	(a) $f(4) = 0 \Rightarrow 64 + 16(p + 1) - 72 + q = 0$ M1: $f(4)$ or $f(-4)$ $16p + q + 8 = 0 \quad (*)$ (b) $f(-p) = 0 \Rightarrow -p^3 + p^2(p + 1) + 18p + q = 0$ M1: $f(-p)$ or $f(p)$ $p^2 + 18p + q = 0 \quad (*)$ (c) Combine to form a quadratic equation in one unknown. $p^2 + 18p - (16p + 8) = 0 \quad p^2 + 2p - 8 = 0$ $(p + 4)(p - 2) = 0 \quad p = \dots, \quad 2$ $q = -40 \quad (\text{ft only for a positive } p)$ (d) Complete method to find third factor. $x^3 + 3x - 18x - 40 = (x - 4)(x + 2)(x + 5)$	M1 A1 A1 (3) M1 A1 A1 (3) M1 A1 M1, A1cso B1ft (5) M1 A1 (2) 13

Question Number	Scheme	Marks
1.	$\int \left(1 + \frac{5}{x}\right) dx = x + 5 \ln x$ $[x + 5 \ln x]_1^e = (e + 5) - 1 = e + 4 \quad \text{M1 Correct use of limits}$	M1 A1 M1 A1 (4) [4]
2.	(a) $\left(k + \frac{x}{2}\right)^5 = k^5 + 5k^4 \frac{x}{2} + 10k^3 \left(\frac{x}{2}\right)^2 + 10k^2 \left(\frac{x}{2}\right)^3 + \dots$ Need not be simplified. Accept ${}^5C_1, {}^5C_2, {}^5C_3$ (b) $10k^3 \left(\frac{x}{2}\right)^2 = 540x^2$ with or without x^2 Leading to $k = 6$ * cso Or substituting $k = 6$ into ${}^5C_2 k^3 \left(\frac{x}{2}\right)^2$ and simplifying to $540x^2$. (c) Coefficient of x^3 is $10 \times 6^2 \times \frac{1}{2^3} = 45$	M1 A1 (2) M1 A1 (2) M1 A1 (2) [6]

Question Number	Scheme	Marks
3.	Use of $V = \pi \int y^2 dx$ $y^2 = 16x^2 + \frac{36}{x^2}; -48$ Integrating to obtain $(\pi) \left[\frac{16x^3}{3} - \frac{36}{x}; -48x \right]$ ft constants only $(\pi) \left[\frac{16x^3}{3} - \frac{36}{x} - 48x \right]_2^4 = (\pi) \left[140\frac{1}{3} - (-71\frac{1}{3}) \right] \text{ correct use of limits}$ $V = 211\frac{2}{3}\pi \quad (\text{units}^3)$	M1 B1; B1 M1 A1ft; A1ft M1 A1 (8) [8]
4.	(a) $f(1) = -2, f(2) = 5\frac{1}{2}$ Change of sign (and continuity) $\Rightarrow$ root $\in (1, 2)$ (b) $x_1 = 1.38672..., x_2 = 1.39609...$ awrt 4dp $x_3 = 1.39527..., x_4 = 1.39534...$ same to 3dp Root is 1.395 (to 3dp) cao (c) Choosing a suitable interval, (1.3945, 1.3955) or tighter. $f(1.3945) \approx -0.005, f(1.3955) \approx +0.001$ Change of sign (and continuity) $\Rightarrow$ root $\in (1.3945, 1.3955)$ $\Rightarrow$ root is 1.395 correct to 3dp	M1 A1 (2) B1, B1 M1 A1 (4) M1 A1 (2) [8]

Question Number	Scheme	Marks
5.	(a) $\frac{2x+5}{x+3} - \frac{1}{(x+3)(x+2)} = \frac{(2x+5)(x+2)-1}{(x+3)(x+2)}$ $= \frac{2x^2+9x+9}{(x+3)(x+2)}$ $= \frac{(2x+3)(x+3)}{(x+3)(x+2)}$ $= \frac{2x+3}{x+2}$ (b) $2 - \frac{1}{x+2} = \frac{2(x+2)-1}{x+2} = \frac{2x+3}{x+2}$ or the reverse (c) T_1: Translation of -2 in x direction T_2: Reflection in the x-axis T_3: Translation of $(+)$2 in y direction All three fully correct One alternative is T_1: Translation of -2 in x direction T_2: Rotation of 90° clockwise about O T_3: Translation of -2 in x direction	M1 A1 M1 A1 A1 (5) M1 A1 (2) B1 B1 B1 B1 (4) [11]

Question Number	Scheme	Marks
6.	(a)  Correct shape, vertex on x-axis $(0, 5)$ or 5 on y-axis $(2\frac{1}{2}, 0)$ or $2\frac{1}{2}$ on x-axis (b) $2x - 5 = x \Rightarrow x = 5$ $2x - 5 = -x$ $x = 1\frac{2}{3}$ accept stated or equivalent accept exact equivalents (c) Method for finding either coordinate of the lowest point (differentiating and equating to zero, completing the square, using symmetry). $x = 3 \text{ or } g(x) = -9$ $g(x) \square -9$ (d) $fg(1) = f(-5)$ $= 15$	B1 B1 B1 (3) M1 A1 M1 A1 (4) M1 A1 A1 (3) M1 A1 (2) [12]

Question Number	Scheme	Marks
7.	(a) $0 = k + \ln 2 \left(\frac{1}{2e} \right) \Rightarrow 0 = k - 1 \Rightarrow k = 1 *$ (Allow also substituting $k = 1$ and $x = \frac{1}{2e}$ into equation and showing $y = 0$ and substituting $k = 1$ and $y = 0$ and showing $x = \frac{1}{2e}$.) (b) $\frac{dy}{dx} = \frac{1}{x}$ At A gradient of tangent is $\frac{1}{\frac{1}{2e}} = 2e$ Equations of tangent: $y = 2e \left(x - \frac{1}{2e} \right)$ Simplifying to $y = 2ex - 1 *$ (c) $y_1 = 1.69, y_2 = 2.39$ (d) $\int_1^3 (1 + \ln 2x) dx \approx \frac{1}{2} \times \frac{1}{2} \times (\dots)$ $\approx \dots \times (1.69 + 2.79 + 2(2.10 + 2.39 + 2.61))$ ft their (c) ≈ 4.7 accept 4.67	M1 A1 (2) B1 M1 M1 CSO A1 (4) B1, B1 (2) B1 M1 A1ft A1 (4) [12]

Question Number	Scheme	Marks
8.	(i) Using $\tan \theta = \frac{\sin \theta}{\cos \theta}$ and $\cot \theta = \frac{\cos \theta}{\sin \theta}$ or $\cot \theta = \frac{1}{\tan \theta}$	M1
	Forming a single fraction	
	$\text{LHS} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \quad \text{or} \quad \text{LHS} = \frac{1 + \tan^2 \theta}{\tan \theta}$	M1
	Reaching the expression	
	$\frac{1}{\sin \theta \cos \theta}$	A1
	Using $\sin 2\theta = 2 \sin \theta \cos \theta$	M1
	$\text{LHS} = \frac{2}{2 \sin \theta \cos \theta} = \frac{2}{\sin 2\theta} = 2 \operatorname{cosec} 2\theta = \text{RHS} \quad *$	A1 (5)
	(ii)(a) $\cos \alpha = \frac{12}{13}$ M1 Use of $\sin^2 \alpha + \cos^2 \alpha = 1$ or right angled triangle but accept stated	M1 A1
	(b) Use of $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ (or $1 - 2 \sin^2 \alpha$ or $2 \cos^2 \alpha - 1$)	M1
	$\cos 2\alpha = \frac{119}{169}$	A1 (4)
	(c) Use of $\cos(x + \alpha) = \cos x \cos \alpha - \sin x \sin \alpha$	M1
	Substituting for $\sin \alpha$ and $\cos \alpha$	M1
	$12 \cos x - 5 \sin x + 5 \sin x = 6 \quad (12 \cos x = 6)$	A1
	$x = \frac{\pi}{3}$	M1 A1 (5)
	awrt 1.05	
		[14]

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Paper: P1

Question Number	Scheme	Marks
1.	Forming equation in x or y by attempt to eliminate one variable $(3-y)^2 + y = 15$ or $x^2 + (3-x) = 15$ <u>Attempt at solution</u> i.e. solving 3 term quadratic: $(y-6)(y+1) = 0$, $y = \dots$ or $(x-4)(x+3) = 0$, $x = \dots$ or correct use of formula or correct use of completing the square $x = 4$ and $x = -3$ or $y = -1$ and $y = 6$ Finding the values of the other coordinates	M1 A1 M1 A1 M1 A1 ft (6)
2.	Using $\sin^2 \theta + \cos^2 \theta = 1$ to give a quadratic in $\cos \theta$. Attempt to solve $\cos^2 \theta + \cos \theta = 0$ $(\cos \theta = 0) \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $(\cos \theta = -1) \Rightarrow \theta = \pi$ (Candidate who writes down 3 answers only with no working scores a maximum of 3)	M1 M1 B1, B1 B1 (5)
3.	(a) Attempt $f(2) = 16 - 4 + 4 - 16 = 0 \Rightarrow (x-2)$ is a factor (b) $c = 8$ <u>A complete method to find b</u> Either compare coefficients of x or x^2: $-2b + 8 = 2$, or $-4 + b = -1$ Or substitute value of x (may be implied): e.g. $(x=1) \Rightarrow -13 = (-1)(10+b)$ $b = 3$ (c) Checking $b^2 - 8c$; $< 0 \Rightarrow$ no real roots to the quadratic $\Rightarrow x = 2$ is the only solution	M1; A1 (2) B1 M1 A1 (3) M1; A1 A1 (3)

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Paper: **P1**

Question Number	Scheme	Marks
4 (a)	Correct strategy for differentiation e.g. $y = 4x^2 + 5 - \frac{1}{x}$, or product or quotient rules applied correctly to $\frac{5x-1}{x}$. $\frac{dy}{dx} = 8x, + \frac{1}{x^2}$ B1 for $8x$ seen anywhere.	M1 B1, A1 (3)
(b)	Using $\frac{dy}{dx} = 0$ So $8x^3 + 1 = 0 \Rightarrow x = -\frac{1}{2}$. M1 requires multiplication by denominator and use of a root in the solution	M1 M1 A1 (3)
(c)	Complete method: Either $\frac{d^2y}{dx^2} = 8 - \frac{2}{x^3}$, with x value substituted, or gradient either side checked Completely correct argument, either > 0 with no error seen, or $-ve$ to $+ve$ gradient, then minimum stated	M1 A1 (2)
5(a)	$p = 15, q = -3$ Special case if B0 B0, allow M1 for method, e.g. $8 = \frac{1+p}{2}$	B1, B1 (2)
(b)	Gradient of line $ADC = -\frac{5}{7}$, gradient of perpendicular line = $-\frac{1}{\text{gradient } ADC} \left(\frac{7}{5} \right)$ Equation of l: $y - 2 = \left(\frac{7}{5} \right)(x - 8)$ $\Rightarrow 7x - 5y - 46 = 0$	B1, M1 M1 A1ft A1cao (5)
(c)	Substituting $y = 7$ and finding value for x, $x = \frac{81}{7}$ or $11\frac{4}{7}$	M1 A1 (2)

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Question Number	Scheme	Marks
6 (a)	$P = r\theta + 2r, \quad A = \frac{1}{2}r^2\theta$	B1, B1 (2)
(b)	Substituting value for r and equating P to A . $[2\sqrt{2}(2+\theta) = \frac{1}{2}(2\sqrt{2})^2\theta]$ Correct process to find θ $[\theta(\sqrt{2}-1) = 2]$ $\theta = \frac{2}{\sqrt{2}-1} \quad *$	M1 M1 A1 c.s.o. (3)
(c)	Multiply numerator and denominator by $(\sqrt{2}+1)$ $2, +2\sqrt{2}$	M1 A1, A1 (3)
7 (a)	Applying correct formula $[325 = 120 + 5(n-1)]$ Solving to give $n=42 \quad * \quad$ (or verifying in correct equation)	M1 A1 (2)
(b)	Using formula for sum of AP: $S = \frac{42}{2}\{240 + 5(42-1)\}$ $= 9345$	M1 A1 A1 (3)
(c)	Recognising GP with $r = 0.98$ Value (in £) $= 7200 (0.98)^{24}$ $= 4434 \text{ (only this value)}$	M1 M1 A1 (3)

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Paper: P1

Question Number	Scheme	Marks
8 (a)	Substitute $x=0, y=\sqrt{3}$ to give $\sqrt{3} = k\frac{\sqrt{3}}{2} \Rightarrow k=2$ (or verify result)	B1 (1)
(b)	$p = 120, \quad q = 300$ (f.t. on $p+180$)	B1, B1ft (2)
(c)	$\arcsin(-0.8) = -53.1$ or $\arcsin(0.8) = 53.1$ $(x+60) = 180 - \arcsin(-0.8)$ OR $360 + \arcsin(-0.8)$ (only need one) or equivalent [e.g. $180 + \arcsin 0.8$ OR $360 - \arcsin 0.8$] First value of $x = 233.1 - 60$, i.e. $x = 173.1$ Second value of $x = 306.9 - 60$, i.e. $x = 246.9$ (special case ft on $p+q-1^{\text{st}}$ value)	B1 M1 A1 M1, A1 (5)
9 (a)	$(x-3)^2, +9$ Value $a=3$ and $b=9$ may just be written down with no method shown.	B1, M1 A1 (3)
(b)	P is $(3, 9)$	B1 ft, B1ft (2)
(c)	$A = (0, 18)$ $\frac{dy}{dx} = 2x-6$, at A $m = -6$ Equation of tangent is $y - 18 = -6x$ (in any form)	B1 M1 A1 A1ft (4)
(d)	Showing that line meets x axis directly below P , i.e. at $x = 3$.	A1cso (1)
(e)	$A = \int x^2 - 6x + 18 dx = [\frac{1}{3}x^3 - 3x^2 + 18x]$ Substituting $x=3$ to find area A under curve $A = 36$ Area of $R = A - \text{area of triangle} = A - \frac{1}{2} \times 18 \times 3 = 9$ Alternative: $\int x^2 - 6x + 18 - (18 - 6x) dx$ M1 $= \frac{1}{3}x^3$ M1 A1 ft Use $x=3$ to give answer 9 M1 A1	M1 A1 M1 M1 A1 (5)

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GENERAL CERTIFICATE OF EDUCATION

Advanced Subsidiary/Advanced Level

Pure Mathematics P2

MARKING SCHEME

January 2005

Principal Examiner:

**Miss Jane Dyer
10 The Crofts
St Bees
Cumbria
CA27 0BH**

Tel.: 01946 822508

Marking should be completed by 16th February 2005.

General Instructions

1. The total number of marks for the paper is 75.
2. Method (M) marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
3. Accuracy (A) marks can only be awarded if the relevant method (M) marks have been earned.
4. (B) marks are independent of method marks.
5. Method marks should not be subdivided.
6. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. Indicate this action by 'MR' in the body of the script (but see also note 10).
7. If a candidate makes more than one attempt at any question:
 - (a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - (b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
8. Marks for each question, or part of a question, must appear in the right-hand margin and, in addition, total marks for each question, even where zero, must be ringed and appear in the right-hand margin and on the grid on the front of the answer book. It is important that a check is made to ensure that the totals in the right-hand margin of the ringed marks and of the unringed marks are equal. The total mark for the paper must be put on the top right-hand corner of the front cover of the answer book.
9. For methods of solution not in the mark scheme, allocate the available M and A marks in as closely equivalent a way as possible, and indicate this by the letters 'OS' (outside scheme) put alongside in the body of the script.
10. All A marks are 'correct answer only' (c.a.o.) unless shown, for example, as A1 f.t. to indicate that previous wrong working is to be followed through. In the body of the script the symbol $\checkmark$ should be used for correct f.t. and $\times$ for incorrect f.t. After a misread, however, the subsequent A marks affected are treated as A f.t., but manifestly absurd answers should never be awarded A marks.
11. Ignore wrong working or incorrect statements following a correct answer.

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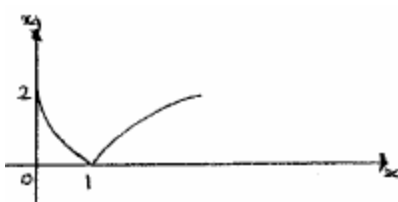
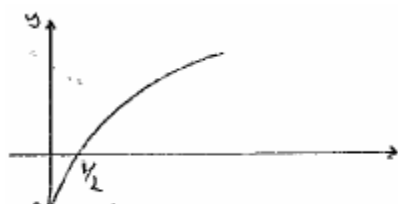
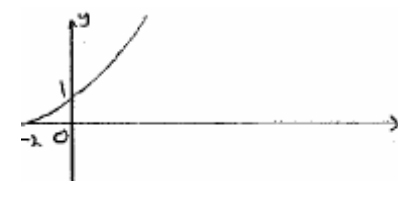
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Subject: **Pure Mathematics**

Paper: **P2**

Question Number	Scheme	Marks
1a)	$\frac{(x-3)(x+2)}{x(x-3)}; = \frac{(x+2)}{x} \text{ or } 1 + \frac{2}{x}$ B1 numerator, B1 denominator ; B1 either form of answer	B1,B1,B1 (3)
1b)	$\frac{(x+2)}{x} = x+1 \Rightarrow x^2 = 2$ M1 for equating f(x) to x + 1 and forming quadratic. A1 candidate's correct quadratic $x = \pm\sqrt{2}$	M1 A1√ A1 (3)
2a)	 Shape reflected $0 < x < 1$ Cusp + coords  General shaped -2 (1/2, 0)  Rough reflection $y = x$ (0,1) or 1 on $y - ax$ is (-2, 0) or -2 on $x - ax$ is	M1 A1 (2) B1 B1 (2) B1 B1 B1 (3)

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Paper: **P2**

Question Number	Scheme	Marks
3a)	$u_2 = (-1)(2) + d = -2 + d$ $u_3 = (-1)^2(-2 + d) + d = -2 + 2d$ Attempting to find u_3 and u_4 $u_4 = (-1)^3(-2 + 2d) + d = 2 - d$ either u_3 or u_4 correct $u_5 = (-1)^4(2 - d) + d = 2$ * cao	B1 M1 A1 A1* (4)
b)	$u_{10} = u_1 = d - 2$	B1✓ (1)
c)	$-2 + 2d = 3(-2 + d) \Rightarrow d = 4$ M1 equating their u_3 to their u_2	M1 A1 (2)
4a)	(0,4), or 4 on y-axis	B1 (1)
b)	$V = \pi \int x^2 dy$ attempt use of $x^2 = y - 4$ $V = (\pi) \int (y - 4) dy$ attempt to integrate $= (\pi) \left[\frac{y^2}{2} - 4y \right]$ using limits to give $\pi[(32 - 32) - (8 - 16)] = 8\pi$. (c.a.o)	M1 B1 M1 A1 M1 A1 (6)

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Paper: **P2**

Question Number	Scheme	Marks
5a)	$3^x = 5 \Rightarrow \ln 3^x = \ln 5 \quad [x \ln 3 = \ln 5]$ taking logs $x = \frac{\ln 5}{\ln 3}$ $= 1.46 \text{ cao}$	M1 A1 A1 (3)
b)	$\log_2 (2x + 1) - \log_2 x = \log_2 \frac{2x + 1}{x}$ $2 = \log_2 4$ Forming non-log equation $\frac{2x + 1}{x} = 4 \text{ or equivalent; } x = \frac{1}{2}$	M1 B1 M1 A1 (4)
c)	$-\ln \sec x = \ln \cos x$ $\sin x = \cos x \Rightarrow \tan x = 1 \quad x = 45$ use of $\tan x$	B1 M1, A1 (3)

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Paper: **P2**

Question Number	Scheme	Marks
6a)	$I = 3x + 2e^x$ Using limits correctly to give $1 + 2e$. (c.a.o.)	B1 M1 A1 (3)
b)	$A = (0, 5);$ $\frac{dy}{dx} = 2e^x$ Equation of tangent: $y = 2x + 5; c = -2.5$	B1 B1 M1; A1 attempting to find eq. of tangent and subst in $y = 0$ (4)
c)	$y = \frac{5x+2}{x+4} \Rightarrow yx + 4y = 5x + 2 \Rightarrow 4y - 2 = 5x - xy$ $g^{-1}(x) = \frac{4x-2}{5-x}$ or equivalent	M1; A1 A1 putting $y =$ and att. to rearrange to find x (3)
d)	$gf(0) = g(5); =3$	att to put 0 into f and then their answer into g M1; A1 (2)

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Paper: **P2**

Question Number	Scheme	Marks
7a)	Complete method for DE [e.g. split triangle ADE and sin, or sine or cos rule] $DE = 4 \sin \theta * (\text{c.s.o.})$	M1 A1* (2)
b)	$P = 2 DE + 2EF$ or equivalent. With attempt at EF $= 8 \sin \theta + 4 \cos \theta * (\text{c.s.o.})$	M1 A1* (2)
c)	$8 \sin \theta + 4 \cos \theta = R \sin (\theta + \alpha)$ $= R \sin \theta \cos \alpha + R \cos \theta \sin \alpha$ Method for R , method for α [$R \cos \alpha = 8$, $R \sin \alpha = 4$ $\tan \alpha = 0.5$, $R = \sqrt{8^2 + 4^2}$] $\alpha = 0.464$ (allow 26.6), $R = 4\sqrt{5}$ or 8.94	M1 M1 A1 A1 (4)
d)	Using candidate's $R \sin (\theta + \alpha) = 8.5$ to give $(\theta + \alpha) = \sin^{-1} \frac{8.5}{R}$ Solving to give $\theta = \sin^{-1} \frac{8.5}{R} - \alpha$, $\theta = 0.791$ (allow 45.3) Considering second angle: $\theta + \alpha = \pi$ (or 180) $-\sin^{-1} \frac{8.5}{R}$; $\theta = 1.42$ (allow 81.6)	M1 M1 A1 M1 A1 (5)

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Paper: **P2**

Question Number	Scheme	Marks
8a)	$f'(x) = -\frac{1}{2x^2}; +\frac{1}{x}$ M1 for evidence of differentiation $f'(0) = 0 \Rightarrow \frac{-1+x}{2x^2} = 0; \Rightarrow x = 0.5$ (or subst x = 0.5)	M1A1;A1 M1A1 * (5)
b)	$y = 1 + \ln\left(\frac{1}{4}\right) - 1; = -2 \ln 2$ Subst their value for x in	M1;A1 (2)
c)	$f(4.905) = < 0$ (−0.000955), $f(4.915) = > 0$ (+ 0.000874) Change of sign indicates root between (accept correct values above)	M1 A1 (2)
d)	$\frac{1}{2x} + \ln\left(\frac{x}{2}\right) - 1 = 0; \Rightarrow 1 - \frac{1}{2x} = \ln\left(\frac{x}{2}\right)$ $\Rightarrow \frac{x}{2} = e^{\left(1 - \frac{1}{2x}\right)} \Rightarrow x = 2e^{\left(1 - \frac{1}{2x}\right)} \quad * \text{ (c.s.o.)}$ M1 for use of e	M1A1 (2)
e)	$x_1 = 4.9192$ $x_2 = 4.9111, x_3 = 4.9103,$ both	B1 B1 (2)

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Paper: P3

Question Number	Scheme	Marks
1. (a)	$f(-2) = -16a - 4a + 6 + 7$ $f(-2) = -3 \Rightarrow -20a + 13 = -3$ $\text{i.e. } 20a = 16$ $a = \frac{4}{5}$	MI MI AI (3)
(b)	$f(\frac{1}{2}) = \frac{a}{4} - \frac{a}{4} - \frac{3}{2} + 7 = \frac{11}{2}$ (o.e.) $a = \frac{4}{5}$	MI AI (2) ⑤
2. (a)	$\sin(3x+x) = \sin 3x \cos x + \cos 3x \sin x$ $\sin(3x-x) = \sin 3x \cos x - \cos 3x \sin x$ (Subtract) $\Rightarrow \sin 4x - \sin 2x = 2 \sin x \cos 3x$	MI AI AI c.s.o. (3)
(b)	$\int 2 \sin x \cos 3x \, dx = \int (\sin 4x - \sin 2x) \, dx$ $= -\frac{\cos 4x}{4} + \frac{\cos 2x}{2} + c$	MI AI (2)
(c)	$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} 2 \sin x \cos 3x \, dx = \left(-\frac{1}{4} \cos \frac{10\pi}{3} + \frac{1}{2} \cos \frac{5\pi}{3}\right) - \left(-\frac{1}{4} \cos 2\pi + \frac{1}{2} \cos \pi\right)$ $= \frac{9}{8}$	MI AI (2) ⑦
3. (a)	$x^2 + y^2 - 12x + 4y + 20 = 0$ $(x-6)^2 + (y+2)^2 + k = 0$	MI AI (2)
(b)	$(x-6)^2 + (y+2)^2 = 20$	MI AI (2)
Use of Formulae	$2g = -12, 2f = 4, c = 20$ Centre $(-g, -f)$ $(\pm 6, \pm 2)$ (b) Radius $= \sqrt{36 + 4 - 20}$ Centre $(6, -2)$ $36, 4, 20$ and $\sqrt{}$ Radius $\sqrt{20}$	MI AI (2) MI AI (2)
(c)		P.T.O.

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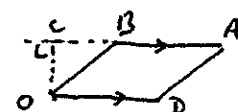
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Subject: Pure Mathematics

Paper: P3

Question Number	Scheme	Marks
3. (c)	$x^2 - 12x + 20 = 0$ $\Rightarrow x = 2, 10$ Centre of C_2 is <u>(6, 0)</u> Radius of C_2 is $6 - 2$ or $10 - 6 = 4$ Equation of C_2 is <u>$(x - 6)^2 + y^2 = 4^2$</u> o.e.	Sub $y = 0$ in $C_1 \rightarrow 3x^2 = 0$ M1 A1 B1 B1 M1 $\checkmark$ Centre and radius A1 (6) (10)
4. (a)	$(1 - x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(-x)^3$ $(= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 \dots)$ $(1 + x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}x^3$ $(= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 \dots)$ $\therefore f(x) = \underline{\underline{\frac{1}{2}x^2 + \frac{1}{4}x^3}}$	M1 need attempt at binomial $\rightarrow$ at least x^2 term. M1 A1 M1 A1 subtract c.a.o. M1 A1 (6)
(b)	$f'(x) = x + \frac{3}{4}x^2 \dots$, $f''(x) = 1 + \frac{3}{2}x \dots$ $f(0) = 0$ and $f'(0) = 0$ $f''(0) > 0$, $\therefore$ Minimum at origin (*)	Attempt f' & f'' M1 B1 M1, A1 c.a.o. (4) (10)
5. (a)	$\vec{AB} = \vec{b} - \vec{a} = \begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix}$; $\therefore$ Equation of L <u>$\vec{r} = \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix} + t \begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix}$</u> (o.e.)	M1; A1 (2)
(b)	$\begin{pmatrix} 3t \\ 5-3t \\ 5-6t \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix} = 0$ $\Rightarrow 9t - 15 + 9t - 30 + 36t = 0$ $\therefore t = \frac{5}{6}$ $\therefore \vec{OC} = \underline{\underline{\begin{pmatrix} 2.5 \\ 2.5 \\ 0 \end{pmatrix}}}$	Attempt $\vec{OC}$ in terms of t $\vec{OC} \cdot \vec{AB}$ attempt Evaluation of $\cdot$ product $t =$ $\vec{OC} =$ M1 M1 M1 A1 A1 (5)
(c)	 $\vec{OB} = \vec{BA} = \vec{a} - \vec{b}$ or $-\vec{AB}$, $= \underline{\underline{\begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix}}}$	M1, A1 (2)
(d)	$ \vec{OC} = 2.5\sqrt{2}$; $ \vec{OB} = 3\sqrt{1^2 + 1^2 + 2^2} = 3\sqrt{6}$ Area = $ \vec{OC} \times \vec{OB} $ (o.e.) , $= 7.5\sqrt{12}$ or <u>$15\sqrt{3}$</u> or AWR 26.0	Attempt at least one relevant length both correct M1 A1 M1, A1 (4) (13)

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January 2005

Advanced Subsidiary/Advanced Level

General Certificate of Education

Subject: Pure Mathematics

Paper: P3

Question Number	Scheme	Marks
6. (a)	$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{3}{2t}$ Gradient of normal is $-\frac{2t}{3}$ At P $t = 2$ $\therefore$ Gradient of normal @ P is $-\frac{4}{3}$ Equation of normal @ P is $y - 9 = -\frac{4}{3}(x - 5)$ Q is where $y = 0 \therefore x = \frac{27}{4} + 5 = \frac{47}{4}$ (o.e)	M1 M1 B1 A1 M1 A1 (6)
(b)	Curved Area = $\int y dx = \int y \frac{dx}{dt} dt$ $= \int 3(1+t) \cdot 2t dt$ $= [3t^2 + 2t^3]$ Curve cuts x-axis when $t = -1$ Curved Area = $[3t^2 + 2t^3]_{-1}^2 = (12 + 16) - (3 - 2) (= 27)$ Area of $\triangle P Q$ triangle = $\frac{1}{2}(12 - 5) \times 9 (= 30.375)$ Total area of R = Curved Area + Δ $= 57.375$ or AWAAT <u>57.4</u>	M1 A1 M1 A1 B1 M1 M1 A1 (9)
		(15)

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Advanced Subsidiary/Advanced Level

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Subject: Pure Mathematics

Paper: P3

Question Number	Scheme	Marks
7. (a)	$I = \int x \operatorname{cosec}^2(x + \frac{\pi}{6}) dx = \int x d(-\cot(x + \frac{\pi}{6}))$ Attempt Parts $= -x \cot(x + \frac{\pi}{6}) + \int \cot(x + \frac{\pi}{6}) dx$ + $\int \cot(x + \frac{\pi}{6}) dx$ $= -x \cot(x + \frac{\pi}{6}) + \ln \sin(x + \frac{\pi}{6}) + c$ (*)	M1 A1 A1 c.s.o. (3)
(b)	$\int \frac{1}{y(1+y)} dy = \int 2x \operatorname{cosec}^2(x + \frac{\pi}{6}) dx$ Attempt to separate $\text{LHS} = \int (\frac{1}{y} - \frac{1}{1+y}) dy$ Attempt PIF on LHS Correct PIF $\therefore \ln y - \ln 1+y \text{ or } \ln \frac{y}{1+y} = 2(a)$ LHS same correct $\ln$ Use of (a) in RHS $\therefore \frac{1}{2} \ln \frac{y}{1+y} = -x \cot(x + \frac{\pi}{6}) + \ln \sin(x + \frac{\pi}{6}) + c$ (*)	M1 M1 A1 M1 M1 A1 c.s.o. (6)
(c)	$y=1, x=0 \Rightarrow \frac{1}{2} \ln \frac{1}{2} = \ln(\sin \frac{\pi}{6}) + c$ Use of $y=1, x=0$ $\therefore c = -\frac{1}{2} \ln \frac{1}{2}$ Sub $x = \frac{\pi}{12}$ $x = \frac{\pi}{12} \Rightarrow \frac{1}{2} \ln \frac{y}{1+y} = -\frac{\pi}{12} \cdot 1 + \ln \frac{1}{\sqrt{2}} - \frac{1}{2} \ln \frac{1}{2}$ "e" (i.e. $\ln \frac{y}{1+y} = -\frac{\pi}{6}$) Out of $\ln$ $\frac{y+1}{y} = e^{\frac{\pi}{6}} \quad (\text{o.e.})$ $1 = y(e^{\frac{\pi}{6}} - 1)$ $\therefore y = \frac{1}{e^{\frac{\pi}{6}} - 1} \quad (\text{o.e.})$ A correct $y =$ expression	M1 A1 M1 A1 M1 A1 (6)

(15)

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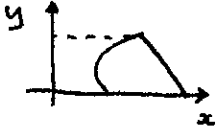
January 2005

Advanced Subsidiary/Advanced Level

General Certificate of Education

Subject: Pure Mathematics

Paper: P3

Question Number	Scheme	Marks
4 (a)	<u>ALTERNATIVE SOLUTIONS</u> $f(x) = \frac{1 - \sqrt{1-x^2}}{\sqrt{1-x}}$ $(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \dots$ $1 - (1-x^2)^{\frac{1}{2}} = 1 - \left(1 - \frac{1}{2}x^2 + \dots\right) = \frac{x^2}{2} + \dots$ $f(x) = \frac{x^2}{2} \left\{ 1 + \frac{x}{2} + \dots \right\}$ $= \frac{x^2}{2} + \frac{x^3}{4}$ Use of binomial to attempt both Multiply	MI AI MI AI MI AI (6)
6. (a).	Cartesian Equation: $(y-3)^2 = 9(x-1)$ or $y = 3 + 3\sqrt{x-1}$ $\frac{dy}{dx} = \frac{3}{2\sqrt{x-1}}$ [Rest as in scheme]	BI MI
(b)	$x = 2$ ($y = 0$), $x = 1$ ($y = 3$) (Two functions) $\pm$ Curved Area = $\int (3+3\sqrt{x-1}) dx - \int (3-3\sqrt{x-1}) dx$ [Rest as in scheme]	BI MI AI
<u>ALT</u>	Trapezium - $\int x dy$  $\int x dy = \frac{1}{9} \int [(y-3)^2 + 1] dy$ [Rest as in scheme + MI for $\int + \Delta$ is for Trap - $\int$] $\text{Area} = \frac{1}{2} \cdot 9 \cdot (5 + 11.75)$ Limits $\frac{9}{0}$ Attempt $x = x(y)$ correct integral	MI BI MI AI

June 2005
6671 Pure P1
Mark Scheme

Question Number	Scheme	Marks
1	(a) $\frac{dy}{dx} = 6 + 8x^{-3}$ or equiv. [M1 is for correct power of x in at least one term , 6 or x^{-3} is sufficient.] (b) $\int y dx = \frac{6x^2}{2} + 4x^{-1} + C$ or equiv. [A1: $\frac{6x^2}{2} + C$; A1: $+4x^{-1}$]	M1 A1 (2) M1 A1A1 (3) [5]
2	(a) $a = -4$ or $(x - 4)^2$ $x^2 - 8x - 29 \equiv (x \pm 4)^2 - 16$ (-29), $b = -45$ [Comparing coefficients: M1 is for $a^2 + b = -29$, and comparing x coefficient] (b) Method to find x: [$x + "a" = \sqrt{\dots\dots\dots}$ or x using the quadratic formula $x = 4 \pm 3\sqrt{5}$ or $c = 4$, $d = 3$	B1 M1A1 (3) M1 A1 A1 (3) [6]

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Question Number	Scheme	Marks
3	(a) $r\theta = 45\theta = 63, \quad \theta = 1.4$ (*) (b) Area of sector $OAB = \frac{1}{2}r^2\theta = \frac{1}{2}45^2 \times 1.4$ (= 1417.5) Complete method for area of triangle OCD Correct numerical expression for area : e.g. $\frac{1}{2}30^2 \times \sin 1.4$ (= 443.45...) Shaded area = $1417.5 - 443.45... = 974 \text{ m}^2$ cao	M1A1 (2) M1A1 M1 A1 A1 (5) [7]
4	(a) Complete method for equation of line e.g. $y - (-4) = \frac{1}{3}(x - 9)$ $x - 3y - 21 = 0$ or $3y - x + 21 = 0$ (b) Equation of l_2: $y = -2x$ Solve l_1 and l_2 simultaneously to find P: $x = 3, \quad y = -6$ [Follow through on first co-ord substituted in $y = -2x$] (c) C: $(0, -7)$ Complete method for area of triangle OCP Area = $10\frac{1}{2}$ (must be exact)	M1A1 A1 (3) B1 M1 A1A1√ (4) B1√ M1 A1 (3) [10]

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5	(a) $\arctan \frac{3}{2} = 56.3^\circ (= \alpha)$ seen anywhere $\alpha - 20^\circ,$ $(\alpha - 20^\circ) \div 3$ (that order) $\alpha + 180^\circ (= 236.3^\circ),$ $\alpha - 180^\circ (= -123.7^\circ)$ (Third quadrant) $x = -47.9^\circ, 12.1^\circ, 72.1^\circ$ [First A1 for two correct solutions, second A1 for third] (b) Equation in one trig. function, using correct identities [e.g. $2\sin^2 x + (1 - \sin^2 x) = \frac{10}{9}$ or $2(1 - \cos^2 x) + \cos^2 x = \frac{10}{9}$] $\sin^2 x = \frac{1}{9}$ or $\cos^2 x = \frac{8}{9}$ or $\tan^2 x = \frac{1}{8}$ or $\cos 2x = \frac{7}{9}$ $x = 19.5^\circ, -19.5^\circ$ Notes : Max. deduction of 1 overall for not correcting to 1 dec. place. Answers outside given interval, ignore Extra answers in range, max. deduction of 1 in each part (i.e. 4 or more answers within interval in (a), -1 from any gained A marks; 3 or more answers within interval in (b), -1 from any gained A marks	B1 M1M1 M1 A1A1 (6) M1 A1 A1A1√ (4) [10]

6	(a) $S = a + (a + d) + (a + 2d) + \dots + [a + (n - 1)d]$ or equiv. form $S = [a + (n - 1)d] + [a + (n - 2)d] + \dots + a$ or equiv. Add: $2S = n[2a + (n - 1)d] \Rightarrow S = \frac{1}{2}n[2a + (n - 1)d]$ cso (*) [If using “l”, second M not gained until “$l = a + (n - 1)d$” substituted.] (b) 3, 8, 13 (c) $a = 3$ $d = 5$ [$a = 3, l = 5n - 2$] Sum = $\frac{1}{2}n[(2 \times 3) + 5(n - 1)]$ or $\frac{1}{2}n[3 + 5n - 2] = \frac{1}{2}n(5n + 1)$ (*) Alt: $5 \sum r - \sum 2$ B1, $= \frac{5n(n+1)}{2} - 2n$ M1, $= \frac{n(5n+1)}{2}$ A1 (d) Finding $\sum_{r=1}^{200}$ e.g. $\sum_{r=1}^{200} (5r - 2) = \frac{1}{2} \times 200 \times 1001$ (= 100100) Sum of first 4 terms: $\sum_{r=1}^4 (5r - 2) = \frac{1}{2} \times 4 \times 21$ or 42 stated $\sum_{r=5}^{200} (5r - 2) = S(200) - S(4) = 100100 - 42 = 100058$ [Allow $S(200) - S(5)$ for second M1] ALT: Working with 23, 28, 33, $a = 23$ B1; Finding “n” and d, or equiv. M1 Applying $S = \frac{1}{2}n[2a + (n - 1)d]$, or equivalent, with 23, $n = 196$, $d = 5$ M1 Answer: 100058 A1	B1 M1 M1 A1 (4) B1 (1) B1✓ M1 A1 (3) M1 B1 M1 A1 (4) [12]
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7	(a) $\frac{dy}{dx} = 3x^{1/2} - 6$ Setting = 0 and solving , $x = 4$ (*) (b) $\int (2x^{3/2} - 6x + 10) dx = \left[\frac{4x^{5/2}}{5} - 3x^2 + 10x \right]$ $\left[\frac{4x^{5/2}}{5} - 3x^2 + 10x \right]_1^4 = \left(\frac{4 \times 4^{5/2}}{5} - (3 \times 16) + 40 \right) - \left(\frac{4}{5} - 3 + 10 \right)$ $(= 17.6 - 7.8 = 9.8)$ [A1√ requires 1 and 4 substituted in candidate's 3-termed integrand (unsimplified)] Correct method for finding area under line Correct unsimplified form e.g $= \frac{1}{2}(6 + 2) \times 3$ (= 12) Area of R (=12 - 9.8) = 2.2 Alt: Working with "line - curve" Area = $\int \left(-\frac{8}{3} + \frac{14}{3}x - 2x^2 \right) dx$ M1A1 $= \left[\frac{4x^{5/2}}{5}, \frac{7}{3}x^2 - \frac{8}{3}x \right]$ A1 A1 ft. Use of correct limits, as in main scheme M1A1 ft. 2.2 A1	M1 A1 M1 A1 (4) M1 A1 A1 M1 A1√ M1 A1 A1 (8) [12]
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8	(a) Substitution of $x = 3$ in $f(x)$ $f(3) = 27 - 117 + 165 - 75$ $= 0$, so $(x - 3)$ is a factor of $f(x)$ (b) Finding quadratic factor: $(x - 3)(x^2 - 10x + 25)$ $(x - 3)(x - 5)(x - 5)$ [S.C.: Allow M1 if just a second linear factor found] (c) 3 and 5 (d) $f'(x) = 3x^2 - 26x + 55$ $f'(3) = 27 - 78 + 55 = 4$ (e) “$3x^2 - 26x + 55$” = “4” $3x^2 - 26x + 51 = 0 \Rightarrow (3x - 17)(x - 3) = 0$ or $x = \dots$ if using “formula” x-coordinate of S is $\frac{17}{3} \left(\frac{34}{6} \text{ or } 5\frac{2}{3} \text{ or } 5.\dot{6} \text{ or } 5.67 \right)$	M1 A1 (2) M1 A1 A1 (3) B1 (1) M1 A1 A1 (3) M1 M1 A1√ A1 (4) [13]
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June 2005
6672 Pure P2
Mark Scheme (Final)

Question Number	Scheme	Marks
1	(a) $\log 5^x = \log 8$ or $x = \log_5 8$ Complete method for finding x: $x = \frac{\log 8}{\log 5}$ or $\frac{\ln 8}{\ln 5}$ $= 1.29$ only (b) Combining two logs: $\log_2 \frac{(x+1)}{x}$ or $\log_2 7x$ Forming equation in x (eliminating logs) legitimately $x = \frac{1}{6}$ or $0.1\dot{6}$	M1 M1 A1 (3) M1 M1 A1 (3) [6]
2	(a) $1 + 12px, + 66p^2x^2$ (accept any correct equivalent) (b) $12p = -q, 66p^2 = 11q$ Forming 2 equations by comparing coefficients Solving for p or q $p = -2, q = 24$	B1,B1 (2) M1 M1 A1A1 (4) [6]

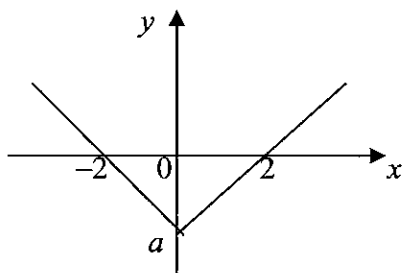
3	(a) <table border="1" data-bbox="327 253 1016 333"> <tr> <td>x</td><td>0</td><td>4</td><td>8</td><td>12</td><td>16</td><td>20</td></tr> <tr> <td>y</td><td>0</td><td>1.6(00)</td><td>2.771</td><td>3.394</td><td>3.2(00)</td><td>0</td></tr> </table> 1.6(00), 3.2(00) 3.394 (b) $A \approx \frac{1}{2} \times 4$, $x [(0 + 0) + 2\{1.60 + 2.771 + 3.394 + 3.20\}]$ follow through on candidate's y values $\approx 43.8(6), 43.9$ or 44 m^2 (c) $\text{Vol/min} \approx [\text{answer to (b)} \times 2] \times 60 = 5260, 5270$ or $5280 \text{ (m}^3 \text{ per min)}$	x	0	4	8	12	16	20	y	0	1.6(00)	2.771	3.394	3.2(00)	0	B1 B1 (2) B1, [M1A1√] A1 (4) M1 A1 (2) [8]
x	0	4	8	12	16	20										
y	0	1.6(00)	2.771	3.394	3.2(00)	0										
4	(a) $f(x) = \frac{5x+1}{(x+2)(x-1)} - \frac{3}{x+2}$ factors of quadratic denominator $= \frac{5x+1-3(x-1)}{(x+2)(x-1)}$ common denominator simplify to linear numerator $= \frac{2x+4}{(x+2)(x-1)} = \frac{2(x+2)}{(x+2)(x-1)} = \frac{2}{x-1}$ AG (b) $y = \frac{2}{x-1} \Rightarrow xy - y = 2 \Rightarrow$ $xy = 2 + y$ or $x - 1 = \frac{2}{y}$ $f^{-1}(x) = \frac{2+x}{x}$ or equiv. (c) $fg(x) = \frac{2}{x^2+4}$ (attempt) $[\frac{2}{"g"-1}]$ Setting $\frac{2}{x^2+4} = \frac{1}{4}$ and finding $x^2 = \dots$; $x = \pm 2$	B1 M1 M1 A1 (4) (cso) M1 A1 A1 (3) M1 DM1; A1 (3) [10]														

Question Number	Scheme	Marks
5	(a) $\left(\frac{x+1}{x}\right)^2 = 1 + \frac{2}{x} + \frac{1}{x^2}$ anywhere $V = \pi \int \left(\frac{x+1}{x}\right)^2 dx$ $\int \left(\frac{x+1}{x}\right)^2 dx = x - \frac{1}{x} + 2\ln x$ [M1 attempt to $\int$] Using limits correctly in their integral: $(\pi) \left\{ \left[x + 2\ln x - \frac{1}{x} \right]^3 - \left[x + 2\ln x - \frac{1}{x} \right]_1 \right\}$ $V = \pi [2\frac{2}{3} + 2\ln 3]$ (must be exact) (b) Volume of cone (or vol. generated by line) = $\frac{1}{3} \pi \times 2^2 \times 2$ $V_R = V_S - \text{volume of cone} = V_S - \frac{1}{3} \pi \times 2^2 \times 2$ $= 2\pi \ln 3$ or $\pi \ln 9$	B1 M1 M1A1,A1 M1 A1 (7) B1 M1 A1 (3) [10]

6	(a) $f'(x) = 3e^x - \frac{1}{2x}$ [M1: any evidence to suggest that tried to differentiate] (b) $3e^\alpha - \frac{1}{2\alpha} = 0$ [Equating $f'(x)$ to zero] $\Rightarrow 6\alpha e^\alpha = 1 \Rightarrow \alpha = \frac{1}{6} e^{-\alpha}$ AG (c) $x_1 = 0.0613\dots, x_2 = 0.1568\dots, x_3 = 0.1425\dots, x_4 = 0.1445\dots$ [M1 at least x_1 correct, A1 all correct to 4 d.p.] (d) Using $f'(x) \{= 3e^x - \frac{1}{2x}\}$ with suitable interval [e.g. $f(0.14425) = -0.0007, f(0.14435) = +0.002(1)$] Both correct with concluding statement.	M1A1A1 (3) M1 A1 (cso) (2) M1A1 (2) M1 A1 (2) [9]
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7

(a)

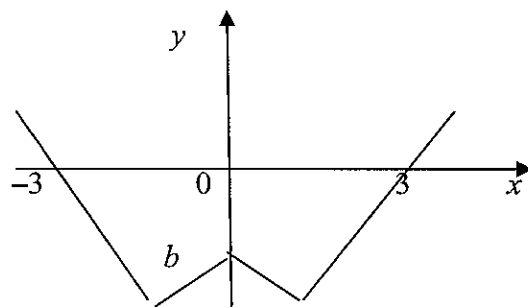
Translation $\leftarrow$ by 1

Intercepts correct

M1

A1 (2)

(b)

 $x \geq 0$, correct "shape"

[provided not just original]

Reflection in y -axis

Intercepts correct

B1

B1✓

B1 (3)

(c) $a = -2$, $b = -1$

B1B1(2)

(d) Intersection of $y = 5x$ with $y = -x - 1$

M1A1

Solving to give $x = -\frac{1}{6}$

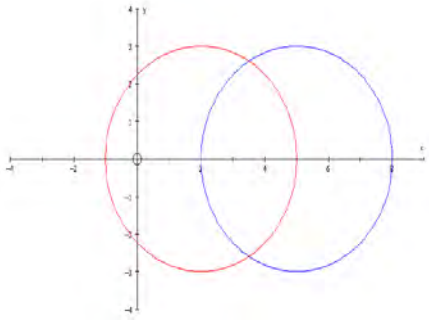
M1A1 (4)

[11]

8	(a) $2\sin(\theta + 30)^\circ = \cos(\theta + 60)^\circ$ $2\sin\theta^\circ\cos 30^\circ + 2\cos\theta^\circ\sin 30^\circ = \cos\theta^\circ\cos 60^\circ - \sin\theta^\circ\sin 60^\circ$ $\frac{2\sqrt{3}}{2}\sin\theta^\circ + \frac{2}{2}\cos\theta^\circ = \frac{1}{2}\cos\theta^\circ - \frac{\sqrt{3}}{2}\sin\theta^\circ$ Finding $\tan\theta^\circ$, $\tan\theta^\circ = -\frac{1}{3\sqrt{3}}$ or equiv. exact (b) (i) Setting $A = B$ to give $\cos 2A = \cos^2 A - \sin^2 A$ Correct completion: $= (1 - \sin^2 A) - \sin^2 A = 1 - 2\sin^2 A$ [Need to see intermediate step above for A1] (ii) Forming quadratic in $\sin x$ $[2\sin^2 x + \sin x - 1 = 0]$ Solving $[(2\sin x - 1)(\sin x + 1) = 0]$ or formula $[\sin\theta = \frac{1}{2} \text{ or } \sin\theta = -1]$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}; \quad [A1\checkmark \text{ for } \pi - " \alpha "]$ $\theta = \frac{3\pi}{2}$ (iii) LHS $= 2\sin y \cos y \frac{\sin y}{\cos y} + (1 - 2\sin^2 y)$ [B1 use of $\tan y = \frac{\sin y}{\cos y}$, M1 forming expression in $\sin y, \cos y$ only] Completion: $= 2\sin^2 y + (1 - 2\sin^2 y) = 1$ AG [Alternative: LHS $= \frac{\sin 2y \sin y + \cos 2y \cos y}{\cos y}$ B1M1 $= \frac{\cos(2y - y)}{\cos y} = 1$ A1]	B1B1 M1 M1,A1 (5) M1 A1 (2) M1 M1 A1,A1✓ A1 (5) B1M1 A1 (3) [15]
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June 2005
6673 Pure P3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) Finding $f(\pm 2)$, and obtaining $16 - 32 + 10 + 6 = 0$ Or uses division and obtains $2x^2 - kx \dots$, obtaining $2x^2 - 4x - 3$ and concluding remainder $= 0$ (b) Finding $f(\pm \frac{1}{2})$, and obtaining $-\frac{1}{4} - 2 - \frac{5}{2} + 6 = 1\frac{1}{4}$ Or uses division and obtains $x^2 - kx \dots$, obtaining $x^2 - \frac{9}{2}x + \frac{19}{4}$ and concluding remainder $= \frac{5}{4}$ (c) $x = 2$ (also allow $\frac{2 \pm \sqrt{10}}{2}$ or $\frac{4 \pm \sqrt{40}}{4}$)	M1, A1 M1 A1 M1, A1 B1 (5)
2.	(a) Writes down binomial expansion up to and including term in x^3, allow nC_r notation $1 + nax + n(n-1)\frac{a^2x^2}{2} + \frac{n(n-1)(n-2)}{6}a^3x^3$ (condone errors in powers of a) States $na = 15$ Puts $\frac{n(n-1)a^2}{2} = \frac{n(n-1)(n-2)a^3}{6}$ (condone errors in powers of a) $3 = (n-2)a$ Solves simultaneous equations in n and a to obtain $a = 6$, and $n = 2.5$ [n.b. Just writes $a = 6$, and $n = 2.5$ following no working or following errors allow the last M1 A1 A1] (b) Coefficient of $x^3 = 2.5 \times 1.5 \times 0.5 \times 6^3 \div 6 = 67.5$ (or equals coefficient of $x^2 = 2.5 \times 1.5 \times 6^2 \div 2 = 67.5$)	M1 B1 dM1 M1 A1 A1 (6) B1 (1) [7]

Question Number	Scheme	Marks
3. (a)	Attempt at integration by parts, i.e. $kx \sin 2x \pm \int k \sin 2x dx$, with $k = 2$ or $\frac{1}{2}$ $= \frac{1}{2} x \sin 2x - \int \frac{1}{2} \sin 2x dx$ Integrates $\sin 2x$ correctly, to obtain $\frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + c$ (penalise lack of constant of integration first time only)	M1 A1 M1, A1 (4)
4. (b)	Hence method : Uses $\cos 2x = 2 \cos^2 x - 1$ to connect integrals Obtains $\int x \cos^2 x dx = \frac{1}{2} \left\{ \frac{x^2}{2} + \text{answer to part(a)} \right\} = \frac{x^2}{4} + \frac{x}{4} \sin 2x + \frac{1}{8} \cos 2x + k$ Otherwise method $\int x \cos^2 x dx = x \left(\frac{1}{4} \sin 2x + \frac{x}{2} \right) - \int \frac{1}{4} \sin 2x + \frac{x}{2} dx$ B1 for $(\frac{1}{4} \sin 2x + \frac{x}{2})$ $= \frac{x^2}{4} + \frac{x}{4} \sin 2x + \frac{1}{8} \cos 2x + k$	M1 A1 (3) B1, M1 A1 (3)
4. (a)	$r = 3$ (both circles) Centres are at (2, 0) and (5, 0)	B1 B1, B1 (3)
4. (b)	 1st circle correct quadrants centre on x axis 2nd circle correct quadrants centre on x axis circles same size and passing through centres of other circle	B1 B1 B1 (3)
4. (c)	Finds circles meet at $x = 3.5$, by mid point of centres or by solving algebraically Establishes $y = \pm \frac{3\sqrt{3}}{2}$, and thus distance is $3\sqrt{3}$. Or uses trig or Pythagoras with lengths 3, angles 60 degrees, or 120 degrees. Complete and accurate method to find required distance Establishes distance is $3\sqrt{3}$.	M1 M1, A1 (3) M1 M1 A1 (3)

Question Number	Scheme	Marks
5.	(a) Substitutes $t = 4$ to give $V, = 1975.31$ or 1975.30 or 1975 or 1980 (3 s.f) (b) $\frac{dV}{dt} = -\ln 1.5 \times V ;= -800.92$ or -800.9 or -801 M1 needs $\ln 1.5$ term (c) rate of decrease in value on 1st January 2005	M1 , A1 (2) M1 A1; A1 (3) B1 (1)
6,	(a) $\overrightarrow{AB} = \begin{pmatrix} c \\ d-5 \\ 10 \end{pmatrix} = k \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$ or $11+5\lambda = 21, \Rightarrow \lambda = 2$, $\therefore c = 4$ $d = 7$ (b) $\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} 2\lambda \\ 5+\lambda \\ 11+5\lambda \end{pmatrix} = 0$ $\therefore 4\lambda + 5 + \lambda + 55 + 25\lambda = 0$ $\therefore \lambda = -2$ Substitutes to give the point P, $-4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ (Accept $(-4, 3, 1)$) (c) Finds the length of OA, or OB or OP or AB as $\sqrt{146}$ or $\sqrt{506}$ or $\sqrt{26}$ or $\sqrt{120}$ resp. Uses area formula- either Area = $\frac{1}{2} \mathbf{AB} \times \mathbf{OP}$ or $= \frac{1}{2} \mathbf{OA} \times \mathbf{OB} \sin \angle AOB$ or $= \frac{1}{2} \mathbf{OA} \times \mathbf{AB} \sin \angle OAB$ or $= \frac{1}{2} \mathbf{AB} \times \mathbf{OB} \sin \angle ABO$ $= \frac{1}{2}\sqrt{120}\sqrt{26}$ or $\frac{1}{2}\sqrt{146}\sqrt{506} \sin 11.86$ or $\frac{1}{2}\sqrt{146}\sqrt{120} \sin 155.04$ or $\frac{1}{2}\sqrt{120}\sqrt{506} \sin 13.10$ $= 27.9$	M1 , A1 A1 (3) M1 A1 M1 A1 M1, A1 (6) M1 M1 M1 A1 (4)

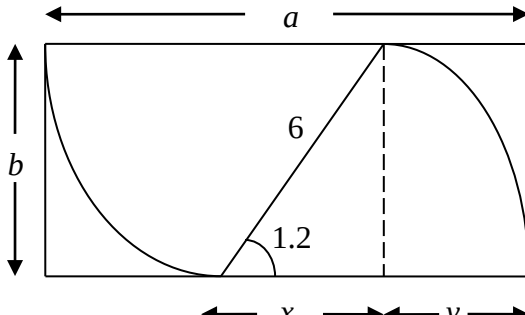
Question Number	Scheme	Marks
7 (a)	As $V = \frac{4}{3}\pi r^3$, then $\frac{dV}{dr} = 4\pi r^2$ Using chain rule $\frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr} = \frac{k}{\frac{4}{3}\pi r^3} \times \frac{1}{4\pi r^2}$ $= \frac{B}{r^5} \quad *$	M1 M1 A1 A1 (4)
(b)	$\int r^5 dr = \int B dt$ $\therefore \frac{r^6}{6} = Bt + c$ (allow mark at this stage, does not need $r =$)	B1 M1 A1 (3)
(c)	Use $r = 5$ at $t = 0$ to give $c = \frac{5^6}{6}$ or 2604 or 2600 Use $r = 6$ at $t = 2$ to give $B = \frac{6^5}{2} - \frac{5^6}{12}$ or 2586 or 2588 or 2590 Put $t = 4$ to obtain r^6 (approx 78000) Then take sixth root to obtain $r = 6.53$ (cm)	M1 M1 M1 A1 A1 (5)

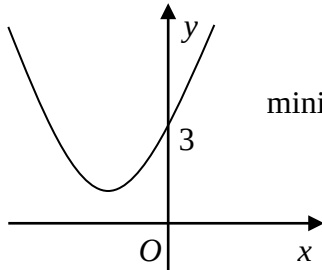
Question Number	Scheme	Marks
8. (a)	$\frac{dx}{dt} = -\frac{1}{(1+t)^2} \quad \text{and} \quad \frac{dy}{dt} = \frac{1}{(1-t)^2}$ $\therefore \frac{dy}{dx} = \frac{-(1+t)^2}{(1-t)^2} \quad \text{and at } t = \frac{1}{2}, \text{ gradient is } -9$ M1 requires their dy/dt / their dx/dt and substitution of t. At the point of contact $x = \frac{2}{3}$ and $y = 2$ Equation is $y - 2 = -9 \left(x - \frac{2}{3} \right)$	B1, B1 M1 A1cao B1 M1 A1
(b)	Either obtain t in terms of x and y i.e, $t = \frac{1}{x} - 1$ or $t = 1 - \frac{1}{y}$ (or both) Then substitute into other expression $y = f(x)$ or $x = g(y)$ and rearrange (or put $\frac{1}{x} - 1 = 1 - \frac{1}{y}$ and rearrange) To obtain $y = \frac{x}{2x-1}$ *	M1 M1 A1 (7)
	Or Substitute into $\frac{x}{2x-1} = \frac{\frac{1}{(1+t)^2}}{\frac{2}{1+t} - 1}$ $= \frac{1}{2 - (1+t)} = \frac{1}{1-t}$ $= y$ *	M1 A1 M1 (3)
(c)	Area = $\int_{\frac{2}{3}}^1 \frac{x}{2x-1} dx$ $= \int \frac{u+1}{2u} \frac{du}{2} = \frac{1}{4} \int 1 + \frac{1}{u} du$ putting into a form to integrate $= \left[\frac{1}{4}u + \frac{1}{4} \ln u \right]_{\frac{1}{3}}^1$ $= \frac{1}{4} - \left(\frac{1}{12} + \frac{1}{4} \ln \frac{1}{3} \right)$ $= \frac{1}{6} + \frac{1}{4} \ln 3$ or any correct equivalent.	B1 M1 M1 A1 M1 A1 (6)

Question Number	Scheme	Marks
8.	(c)	
	Or Area = $\int_{\frac{2}{3}}^1 \frac{x}{2x-1} dx$ $= \int_{\frac{2}{3}}^1 \frac{\frac{1}{2} + \frac{\frac{1}{2}}{2x-1}}{1} dx$ putting into a form to integrate $= \left[\frac{1}{2}x + \frac{1}{4}\ln(2x-1) \right]_{\frac{2}{3}}^1$ $= \frac{1}{2} - \frac{1}{3} - \frac{1}{4}\ln \frac{1}{3} = \frac{1}{6} - \frac{1}{4}\ln \frac{1}{3}$ Or Area = $\int \frac{1}{1-t} \frac{-1}{(1+t)^2} dt$ $= \int \frac{A}{(1-t)} + \frac{B}{(1+t)} + \frac{C}{(1+t)^2} dt$ putting into a form to integrate $= \left[\frac{1}{4}\ln(1-t) - \frac{1}{4}\ln(1+t) + \frac{1}{2}(1+t)^{-1} \right]$ = Using limits 0 and $\frac{1}{2}$ and subtracting (either way round) $= \frac{1}{6} + \frac{1}{4}\ln 3$ or any correct equivalent. Or Area = $\int_{\frac{2}{3}}^1 \frac{x}{2x-1} dx$ then use parts $= \frac{1}{2}x \ln(2x-1) - \int_{\frac{2}{3}}^1 \frac{1}{2} \ln(2x-1) dx$ $= \frac{1}{2}x \ln(2x-1) - \left[\frac{1}{4}(2x-1) \ln(2x-1) - \frac{1}{2}x \right]$ $= \frac{1}{2} - \left(\frac{1}{3}\ln \frac{1}{3} - \frac{1}{12}\ln \frac{1}{3} + \frac{1}{3} \right)$ $= \frac{1}{6} - \frac{1}{4}\ln \frac{1}{3}$	B1 M1 M1A1 dM1 A1 (6) B1 M1 M1 A1ft dM1 A1 (6) B1 M1 M1A1 DM1 A1

Question Number	Scheme	Marks
1.	(a) $(y=)5-2\times 3=-1$ * cso (b) Gradient of perpendicular line is $\frac{1}{2}$ $y-(-1)=\frac{1}{2}(x-3)$ ft their $m \neq -2$ (or substituting $(3, -1)$ into $y=(\text{their } m)x+c$) $x-2y-5=0$	B1 (1) B1 M1 A1ft A1 (4) Total 5 marks
2.	(a) $\frac{dy}{dx}=4x+18x^{-4}$ $x^n \mapsto x^{n-1}$ (b) $\int(2x^2-6x^{-3})dx=\frac{2}{3}x^3+3x^{-2}$ $x^n \mapsto x^{n+1}$ $\left[\dots \right]_1^3 = \frac{2}{3} \times 3^3 + \frac{3}{9} - \left(\frac{2}{3} + 3 \right)$ $= 14\frac{2}{3}$ $\frac{44}{3}, \frac{132}{9}$ or equivalent	M1 A1 (2) M1 A1 M1 A1 (4) Total 6 marks
3.	(a) $\tan \theta = \frac{3}{2}$ Use of $\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\theta = 56.3^\circ$ cao $= 236.3^\circ$ ft 180° + their principle value Maximum of one mark is lost if answers not to 1 decimal place (b) $2 - \cos \theta = 2(1 - \cos^2 \theta)$ Use of $\sin^2 \theta + \cos^2 \theta = 1$ $2 \cos^2 \theta - \cos \theta = 0$ Allow this A1 if both $\cos \theta = 0$ and $\cos \theta = \frac{1}{2}$ are given $\cos \theta = 0 \Rightarrow \theta = 90^\circ, 270^\circ$ M1 one solution $\cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ, 300^\circ$ M1 one solution	M1 A1 A1 ft (3) M1 A1 M1 A1 M1 A1 (6) Total 9 marks

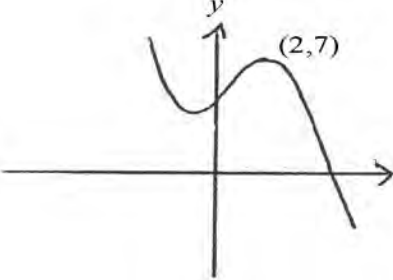
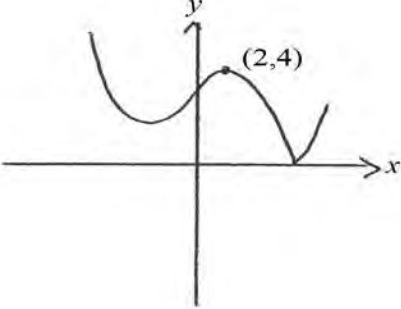
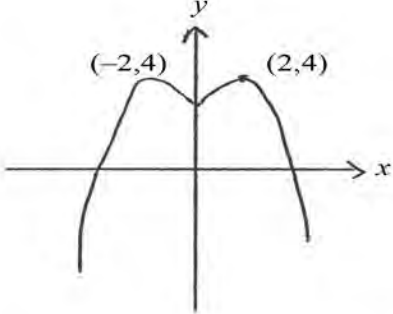
Question Number	Scheme	Marks
4.	(a) $y = 0 \Rightarrow x^{\frac{1}{2}}(3-x) = 0 \Rightarrow x = 3 *$ or $3\sqrt{3} - 3^{\frac{3}{2}} = 3\sqrt{3} - 3\sqrt{3} = 0$	B1 (1)
	(b) $\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$ $x^n \mapsto x^{n-1}$ $\frac{dy}{dx} = 0 \Rightarrow x^{\frac{1}{2}} = x^{-\frac{1}{2}}$ Use of $\frac{dy}{dx} = 0$ $\Rightarrow x = 1$ A: (1, 2)	M1 A1 M1 A1 A1 (5)
	(c) $\int (3x^{\frac{1}{2}} - x^{\frac{3}{2}}) dx = 2x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}$ M1 $x^n \mapsto x^{n+1}$ Accept unsimplified expressions for As $\text{Area} = [\dots]_0^3 = 2 \times 3\sqrt{3} - \frac{2}{5} \times 9\sqrt{3}$ Use of correct limits $\text{Area is } \frac{12}{5}\sqrt{3} \text{ (units}^2\text{)}$	M1 A1+A1 M1 A1 (5)
	For final A1, terms must be collected together but accept exact equivalents, e.g. $\frac{4}{5}\sqrt{27}$	
		Total 11 marks

Question Number	Scheme	Marks
5.	(a) Arc is 6×1.2 Use of $r\theta$ Perimeter is $6 \times 1.2 + 6 + 6 = 19.2$ (cm) A1 (2) (b) Area is $\frac{1}{2} \times 6^2 \times 1.2 = 21.6$ (cm²) Use of $\frac{1}{2}r^2\theta$ M1 A1 (2) (c)  $b = 6 \sin 1.2 \approx 5.59$ (cm) B1 $x = 6 \cos 1.2$ ($\approx 2.174...$) M1 $y = 6 - x$ ($\approx 3.825...$) $a = 6 + y \approx 9.83$ (cm) M1 A1 (4) Total 8 marks	

Question Number	Scheme	Marks
6.	(a) $x^2 + 2x + 3 = (x+1)^2 + 2$ $a = 1, b = 2$	B1, B1 (2)
	(b)  U shape anywhere minimum ft their a and positive b $(0, 3)$ marked	M1 A1ft B1 (3)
	(c) $\Delta = b^2 - 4ac = 2^2 - 4 \times 3 = -8$ The negative sign implies there are no real roots and, hence, the curve in (b) does not intersect (meet, cut, ...) the x-axis. Accept equivalent statements and the statement that the whole curve is above the x-axis.	B1 B1 (2)
	(d) $\Delta = k^2 - 12$ $\Delta < 0 \Rightarrow k^2 - 12 < 0$ (or $k^2 < 12$) $-2\sqrt{3} < k < 2\sqrt{3}$ Allow $\sqrt{12}$ If just $k < 2\sqrt{3}$ allow M1 A0	M1 A1 M1 A1 (4)
	Alternative to (d) $\frac{dy}{dx} = 0 \Rightarrow 2x + k = 0 \Rightarrow x = -\frac{k}{2}$ Minimum greater than 0 implies $\frac{k^2}{4} - \frac{k^2}{2} + 3 > 0$ $k^2 < 12$ Then as before.	M1 A1
		Total 11 marks

Question Number	Scheme	Marks
7.	(a) x-coordinate of P is -2, x-coordinate of Q is 2. (b) $y = x^3 - x^2 - 4x + 4$ Multiplying out $\frac{dy}{dx} = 3x^2 - 2x - 4$ * cso Alternatively Using product rule $\frac{dy}{dx} = 1(x^2 - 4) + (x - 1)2x$ $= 3x^2 - 2x - 4$ * (c) $x = -1 \Rightarrow m = 3 + 2 - 4 = 1$ Substituting $x = -1$ into (b) $y - 6 = 1(x - (-1)) \Rightarrow y = x + 7$ * cso (d) $x^3 - x^2 - 4x + 4 = x + 7$ line = curve $x^3 - x^2 - 5x - 3 = 0$ $(x + 1)(x^2 - 2x - 3) = 0$ Obtaining linear $\times$ quadratic $(x + 1)(x + 1)(x - 3) = 0$ Obtaining 3 linear factors $R: (3, 10)$ In (d) if the correct cubic is obtained the factors can just be written down by inspection. Parts (c) and (d) can be done together. On obtaining $(x + 1)^2(x - 3)$, the repeated root shows that $y = x + 7$ is a tangent to the curve at $(-1, 6)$ and, if this is stated, the M1 A1 for (c) should be given at this point.	B1, B1 (2) M1 M1 A1 (3) M1 M1 A1 M1 A1 (2) M1 M1 A1, A1 (5) Total 12 marks

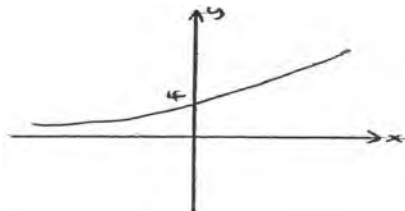
Question Number	Scheme	Marks
8.	(a) $a + (a + d) = \text{£} (500 + 500 + 200) = \text{£}1200$ *	cs0 B1 (1)
	(b) $a = 500, d = 200; \quad u_8 = a + (8 - 1)d$ $= \text{£} (500 + 7 \times 200) = \text{£}1900$	M1 A1 (2)
	(c) $S_8 = \frac{8}{2} (2 \times 500 + (8 - 1) \times 200)$ $= \text{£} 9600$	M1 A1 A1 (3)
	(d) $\frac{n}{2} (1000 + (n - 1) 200) = 32\,000$	M1 A1
	$n^2 + 4n - 320 = 0$ M1 reducing to a 3 term quadratic	M1 A1
	A1 any multiple of the above	
	$(n + 20)(n - 16) = 0$	M1
	$n = 16$	A1
	Age is 26	A1 (7)
	In (b) if the sum is found by repeated addition, i.e. $u_1 = \text{£}500, u_2 = \text{£}700, u_3 = \text{£}900, u_4 = \text{£}1100, u_5 = \text{£}1300,$ $u_6 = \text{£}1500, u_7 = \text{£}1700, u_8 = \text{£}1900,$ allow M1 A1 at completion.	Total 13 marks

Question Number	Scheme	Marks
1.	(a) $(1+6x)^4 = 1 + 4(6x) + 6(6x)^2 + 4(6x)^3 + (6x)^4$ $= 1 + 24x + 216x^2 + 864x^3 + 1296x^4$ (b) substitute $x=100$ to obtain $601^4 = 1 + 2400 + 2160000 + 864000000 + 1296000000000$ $= 130,466,162,401$	M1 A1 A1 (3) M1o.e. A1 (2)
2.	(a)  (b)  (c) 	B1 Shape B1 Point (2) B1 Shape B1 Point (2) B1 Shape >0 B1 Shape $x<0$ B1 Point $(-2,4)$ (3)

3.	$\frac{x(2x+3)}{(2x+3)(x-2)} - \frac{6}{(x-2)(x+1)}$ $= \frac{x(x+1)-6}{(x-2)(x+1)}$ $= \frac{(x+3)(x-2)}{(x-2)(x+1)}$ $= \frac{x+3}{x+1}$ Alternative 1 $\frac{2x^2+3x}{(2x+3)(x-2)} - \frac{6}{(x-2)(x+1)}$ $= \frac{(2x^2+3x)(x+1)-6(2x+3)}{(2x+3)(x-2)(x+1)}$ $= \frac{(2x^3+5x^2-9x-18)}{(2x+3)(x-2)(x+1)}$ $= \frac{(x-2)(2x^2+9x+9)}{(2x+3)(x-2)(x+1)}$ $= \frac{(x-2)(2x+3)(x+3)}{(2x+3)(x-2)(x+1)}, = \frac{x+3}{x+1}$ Alternative 2: $\frac{2x^2+3x}{(2x+3)(x-2)} - \frac{6}{x^2-x-2}$ $= \frac{x(2x+3)}{(2x+3)(x-2)} - \frac{6}{x^2-x-2}$ $= \frac{x(x^2-x-2)-6(x-2)}{(x-2)(x^2-x-2)}, = \frac{x^3-x^2-2x-6x+12}{(x-2)(x^2-x-2)}$ $= \frac{x^3-x^2-8x+12}{(x-2)(x^2-x-2)}$ $= \frac{(x-2)(x^2+x-6)}{(x-2)(x^2-x-2)}$ $= \frac{(x+3)(x-2)}{(x-2)(x+1)}, = \frac{x+3}{x+1}$	B1, B1 M1 A1ft M1 A1 A1 (7) B1 M1 A1ft A1 M1 A1, A1 B1 M1A1ft A1 M1 A1,A1
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Question Number	Scheme	Marks
4.	$\frac{dy}{dx} = \frac{1}{x}$ At $x=3$, gradient of normal $= \frac{-1}{\frac{1}{3}} = -3$ $y - \ln 1 = -3(x - 3)$ $y = -3x + 9$	M1 A1 M1 M1 A1 (5)
5. (a)	$x(2x^2 - 1) = 4$ $2x^2 - 1 = \frac{4}{x}$ $2x^2 = \frac{4+x}{x}$ $x^2 = \frac{4+x}{2x}$ $x = \sqrt{\frac{2}{x} + \frac{1}{2}} \quad \mathbf{AG}$ Alternative 1: $2x^2 - 1 - \frac{4}{x} = 0$ $2x^2 = 1 + \frac{4}{x}$ $x^2 = \frac{1}{2} + \frac{4}{2x}$ $x\sqrt{\frac{1}{2} + \frac{2}{x}} \quad \mathbf{AG}$ Alternative 2: $x^2 = \frac{2}{x} + \frac{1}{2}$ $2x^3 = 4 + x$ $2x^2 - x - 4 = 0$	M1 M1 A1 (3) M1 A1 M1 M1 A1

	(b) 1.41, 1.39, 1.39 (1.40765, 1.38593, 1.393941) (c) $f(1.3915) = -3 \times 10^{-3}$ $f(1.3925) = 7 \times 10^{-3}$ change in sign means root between 1.3915 & 1.3925 $\therefore$ 1.392 to 3dp	B1,B1,B1 (3) M1 A1 B1 (3)
6.	(a) $-2x + 4 = \frac{3}{2x}$ $4x^2 - 8x + 3 = 0$ $(2x - 3)(2x - 1) = 0$ $x = 0.5, 1.5$ (b) $\int_{0.5}^{1.5} -2x + 4 dx = \left[-x^2 + 4x \right]_{0.5}^{1.5}$ or $\frac{1}{2} \times (3+1) \times 1$ $= 2$ $\int_{0.5}^{1.5} \frac{3}{2x} dx = \left[\frac{3}{2} \ln x \right]_{0.5}^{1.5}$ $= \frac{3}{2} \ln 3$ $\therefore \text{Area} = 2 - \frac{3}{2} \ln 3$ Alternative solution: $\text{Area} = \int_{0.5}^{1.5} -2x + 4 - \frac{3}{2x} dx$ $= \left[-x^2 + 4x - \frac{3}{2} \ln x \right]_{0.5}^{1.5}$ $= \frac{-9}{4} + \frac{1}{4} + 6 - 2 - \frac{3}{2} \ln 3$ o.e. $= 2 - \frac{3}{2} \ln 3$	M1 A1 M1 A1 A1ft A1 (6) M1 M1A1A1 A1ft A1

7.	(a)	<table><tr><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>0</td><td>0.062</td><td>0.271</td><td>0.716</td><td>1.612</td><td>3.482</td></tr></table>	0	1	2	3	4	5	0	0.062	0.271	0.716	1.612	3.482	B1, B1, B1 (3)
0	1	2	3	4	5										
0	0.062	0.271	0.716	1.612	3.482										
	(b)	$1 \times \frac{1}{2} (0 + 3.482 + 2) \times (0.062 + 0.271 + 0.716 + 1.612)$ $= 4.402 \text{ m}^2$	B1, M1, A1ft A1 (4)												
	(c)	$6 \times 4.402 = 26.4 \text{ m}^3$	B1 ft (1)												
	(d)	trapezium rule overestimates $\therefore$ will be enough	B1B1 (2)												
8.	(a)	$gf(x) = e^{2(2x + \ln 2)}$ $= e^{4x} e^{2 \ln 2}$ $= e^{4x} e^{\ln 4}$ $= 4e^{4x} \quad \text{AG}$	M1 M1 M1 A1 (4)												
	(b)		B1 shape & (0,4) (1)												
	(c)	$gf(x) > 0$	B1 (1)												
	(d)	$\frac{d}{dx} gf(x) = 16e^{4x}$ $e^{4x} = \frac{3}{16}$ $4x = \ln \frac{3}{16}$ $x = -0.418$	M1 M1 attempt to solve A1 A1 (4)												

9. (a) (i)	$\frac{\cos 2x}{\cos x + \sin x} = \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x}$ $= \frac{(\cos x + \sin x)(\cos x - \sin x)}{\cos x + \sin x}$ $= \cos x - \sin x \quad \mathbf{AG}$	M1 A1 (2)
(ii)	$\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(2\cos^2 x - 1 - 2\sin x \cos x)$ $= \cos^2 x - \frac{1}{2} - \sin x \cos x \quad \mathbf{AG}$	M1, M1 A1 (3)
(b)	$\cos \theta \left(\frac{\cos 2\theta}{\cos \theta + \sin \theta} \right) = \frac{1}{2}$ $\cos \theta (\cos \theta - \sin \theta) = \frac{1}{2}$ $\cos^2 \theta - \cos \theta \sin \theta = \frac{1}{2}$ $\frac{1}{2}(\cos 2\theta + 1) - \frac{1}{2}\sin 2\theta = \frac{1}{2}$ $\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$ $\sin 2\theta = \cos 2\theta \quad \mathbf{AG}$	M1 M1 A1 (3)
(c)	$\sin 2\theta = \cos 2\theta$ $\tan 2\theta = 1$ $2\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$ $\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$	M1 A1 for 1 M1(4 solns) A1 (4)

Question Number	Scheme	Marks
1 (a)	$\left(1 + \left(-\frac{1}{2}\right)(-2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2}(-2x)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1.2.3}(-2x)^3 + \dots\right)$ $= 1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3 + \dots$	M1 (corr bin coeffs) M1 (powers of $-2x$) A1, A1 (4)
Alternative	May use McLaurin $f(0)=1$ and $f'(0)=1$ to obtain 1 st two terms $1 + x$ Differentiates two further times and uses formula with correct factorials to give $\frac{3}{2}x^2 + \frac{5}{2}x^3$	M1 A1 M1 A1 (4)
(b)	$(100 - 200x)^{-\frac{1}{2}} = 100^{-\frac{1}{2}}(1 - 2x)^{-\frac{1}{2}}$. So series is $\frac{1}{10}$ (previous series)	M1A1 ft (2)
2	Uses $f(2) = 0$ to give $16 - 4 + 2a + b = 0$ Uses $f(-1) = 6$ to give $-2 - 1 - a + b = 6$ Solves simultaneous equations to give $a = -7$, and $b = 2$	M1 A1 M1 A1 M1 A1 A1 (7)
3 (a)	Uses circle equation $(x - 4)^2 + (y - 3)^2 = (\sqrt{5})^2$ Multiplies out to give $x^2 - 8x + 16 + y^2 - 6y + 9 = 5$ and thus $x^2 + y^2 - 8x - 6y + 20 = 0$ (*)	M1 A1 A1 (3)
Alternative	Or states equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$ has centre $(-g, -f)$ and so $g = -4$ and $f = -3$ Uses $g^2 + f^2 - c = r^2$ to give $c = 3^2 + 4^2 - \sqrt{5}^2$, i.e. $c = 20$ $x^2 + y^2 - 8x - 6y + 20 = 0$	M1 A1 A1 (3)
(b)	$y = 2x$ meets the circle when $x^2 + (2x)^2 - 8x - 6(2x) + 20 = 0$ $5x^2 - 20x + 20 = 0$ Solves and substitutes to obtain $x = 2$ and $y = 4$. Coordinates are $(2, 4)$ Or Implicit differentiation attempt, $2x + 2y \frac{dy}{dx} - 8 - 6 \frac{dy}{dx} = 0$ Uses $y = 2x$ and $\frac{dy}{dx} = 2$ to give $10x - 20 = 0$. Thus $x = 2$ and $y = 4$	M1 A1 M1 A1 (4) M1 A1 M1 A1 (4)

Question Number	Scheme	Marks
4.(a)	$f'(x) = (x^2 + 1) \times \frac{1}{x} + \ln x \times 2x$ $f'(e) = (e^2 + 1) \times \frac{1}{e} + 2e = 3e + \frac{1}{e}$	M1 A1 M1 A1 (4)
(b)	$\left(\frac{x^3}{3} + x\right) \ln x - \int \left(\frac{x^3}{3} + x\right) \frac{1}{x} dx$ $= \left(\frac{x^3}{3} + x\right) \ln x - \int \left(\frac{x^2}{3} + 1\right) dx$ $= \left[\left(\frac{x^3}{3} + x\right) \ln x - \left(\frac{x^3}{9} + x\right) \right]_1^e$ $= \frac{2}{9}e^3 + \frac{10}{9}$	M1 A1 A1 M1 A1 (5)
5. (a)	$\frac{9+4x^2}{9-4x^2} = -1 + \frac{18}{(3+2x)(3-2x)}, \text{ so } A = -1$ Uses $18 = B(3-2x) + C(3+2x)$ and attempts to find B and C $B = 3$ and $C = 3$ Or Uses $9+4x^2 = A(9-4x^2) + B(3-2x) + C(3+2x)$ and attempts to find A, B and C $A = -1, B = 3$ and $C = 3$	B1 M1 A1 A1 (4)
(b)	Obtains $Ax + \frac{B}{2} \ln(3+2x) - \frac{C}{2} \ln(3-2x)$ Substitutes limits and subtracts to give $2A + \frac{B}{2} \ln(5) - \frac{C}{2} \ln\left(\frac{1}{5}\right)$ $= -2 + 3\ln 5 \quad \text{or} \quad -2 + \ln 125$	M1 A1 M1 A1ft A1 (5)

Question Number	Scheme	Marks
6 (a)	$\frac{dC}{dt} = -kC$; rate of decrease/negative sign; k constant of proportionality/positive constant	B1 (1)
(b)	$\int \frac{dC}{C} = -k \int dt$ $\therefore \ln C = -kt + \ln A$ $\therefore C = Ae^{-kt}$	M1 M1 A1 (3)
(c)	At $t = 0$ $C = C_0$, $\therefore A = C_0$ and at $t = 4$ $C = \frac{1}{10}C_0$, $\therefore \frac{1}{10}C_0 = C_0e^{-4k}$, $\therefore \frac{1}{10} = e^{-4k}$ and $\therefore -4k = \ln \frac{1}{10}$, $\therefore k = \frac{1}{4} \ln 10$	B1 M1 M1, A1 (4)
7 (a)	Solves $9 + 2\lambda = 1$ or $7 + 2\lambda = -1$ to give $\lambda = -4$ so $p = 3$ Solves $9 + 2\lambda = 7$ or $7 + \lambda = 6$ to give $\lambda = -1$ so $q = 5$	M1 A1 M1 A1 (4)
(b)	$ 6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k} = 9$ so unit vector is $\frac{1}{9}(6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})$	M1 A1 (2)
(c)	$\cos \theta = \frac{2 \times 2 + 2 \times 1 + 1 \times 2}{3 \times 3}$ $\therefore \cos \theta = \frac{8}{9}$	M1 A1 A1 (3)
(d)	Write down two of $9 + 2\lambda = 3 + 2\mu$, $7 + 2\lambda = 2 + \mu$ or $7 + \lambda = 3 - 2\mu$ Solve to obtain $\mu = 1$ or $\lambda = -2$ Obtain coordinates (5, 3, 5)	B1 B1 M1 A1 A1 (5)

Question Number	Scheme	Marks
8(a)	$\frac{dx}{dt} = -3a \sin 3t, \quad \frac{dy}{dt} = a \cos t \quad \text{therefore} \quad \frac{dy}{dx} = \frac{\cos t}{-3 \sin 3t}$ When $x = 0, t = \frac{\pi}{6}$ Gradient is $-\frac{\sqrt{3}}{6}$ Line equation is $(y - \frac{1}{2}a) = -\frac{\sqrt{3}}{6}(x - 0)$	M1 A1 B1 M1 M1 A1 (6)
(b)	Area beneath curve is $\int a \sin t (-3a \sin 3t) dt$ $= -\frac{3a^2}{2} \int (\cos 2t - \cos 4t) dt$ $= -\frac{3a^2}{2} \left[\frac{1}{2} \sin 2t - \frac{1}{4} \sin 4t \right]$ Uses limits 0 and $\frac{\pi}{6}$ to give $\frac{3\sqrt{3}a^2}{16}$ Area of triangle beneath tangent is $\frac{1}{2} \times \frac{a}{2} \times \sqrt{3}a = \frac{\sqrt{3}a^2}{4}$ Thus required area is $\frac{\sqrt{3}a^2}{4} - \frac{3\sqrt{3}a^2}{16} = \frac{\sqrt{3}a^2}{16}$	M1 M1 M1 A1 M1 A1 M1 A1 A1 (9)
N.B.	The integration of the product of two sines is worth 3 marks (lines 2 and 3 of scheme to part (b)) If they use parts $\int \sin t \sin 3t dt = -\cos t \sin 3t + \int 3 \cos 3t \cos t dt$ $= -\cos t \sin 3t + 3 \cos 3t \sin t + \int 9 \sin 3t \sin t dt$ $8I = \cos t \sin 3t - 3 \cos 3t \sin t$	M1 M1 A1