OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4723

Core Mathematics 3

MARK SCHEME

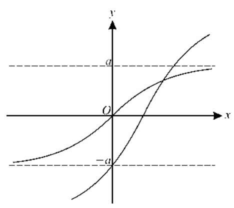
Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

1 *EITHER*: $4x^2+4x+1>x^2-2x+1$	M1	For squaring both sides
i.e. $3x^2+6x>0$	A1	For reduction to correct quadratic
So $x(x+2)>0$	M1	For factorising, or equivalent
Hence $x<-2$ or $x>0$	A1	For both critical values correct
	A1	For completely correct solution set
OR: Critical values where $2x+1=\pm(x-1)$	M1	For considering both cases, or from graphs
i.e. where $x=-2$ and $x=0$	B1	For the correct value –2
	A1	For the correct value 0
Hence $x<-2$ or $x>0$	M1	For any correct method for solution set using two critical values
	A1 **5** **[5]**	For completely correct solution set
2 **(i)** $\sin x(\tfrac{1}{2}\sqrt{3})+\cos x(\tfrac{1}{2})+(\sqrt{3})(\cos x(\tfrac{1}{2}\sqrt{3})-\sin x(\tfrac{1}{2}))$	M1	For expanding both compound angles
	A1	For completely correct expansion
	M1	For using exact values of $\sin 30°$ and $\cos 30°$
$=\tfrac{1}{2}\cos x+\tfrac{3}{2}\cos x=2\cos x$, as required	A1 **4**	For showing given answer correctly
(ii) $\sin 45°+(\sqrt{3})\cos 45°=2\cos 15°$	M1	For letting $x=15°$ throughout
Hence $\cos 15°=\dfrac{1+\sqrt{3}}{2\sqrt{2}}$	A1 **2** **[6]**	For any correct exact form
3 **(i)** $x_2=\sqrt[3]{7}=1.9129...$	B1	For 1.91… seen or implied
$x_3=1.9517...,\quad x_4=1.9346...$	M1	For continuing the correct process
$\alpha=1.94$ to 2dp	A1 **3**	For correct value reached, following x_5 and x_6 both 1.94 to 2dp
(ii) $x=\sqrt[3]{(17-5x)}\ \Rightarrow x^3+5x-17=0$	M1	For letting $x_n=x_{n+1}=x$ (or α)
	A1 **2**	For correct equation stated
(iii) *EITHER*: Graphs of $y=x^3$ and $y=17-5x$ only cross once	M1	For argument based on sketching a pair of graphs, or a sketch of the cubic by calculator
Hence there is only one real root	A1√	For correct conclusion for a valid reason
OR: $\dfrac{d}{dx}(x^3+5x-17)=3x^2+5>0$	M1	For consideration of the cubic's gradient
Hence there is only one real root	A1√ **2** **[7]**	For correct conclusion for a valid reason
4 **(i)** $\int_0^2(4x+1)^{-\frac{1}{2}}\,dx=\left[\tfrac{1}{2}(4x+1)^{\frac{1}{2}}\right]_0^2=\tfrac{1}{2}(3-1)=1$	M1	For integral of the form $k(4x+1)^{\frac{1}{2}}$
	A1	For correct indefinite integral
	M1	For correct use of limits
	A1 **4**	For given answer correctly shown
(ii) $\pi\int_0^2\dfrac{1}{4x+1}\,dx=\pi\left[\tfrac{1}{4}\ln(4x+1)\right]_0^2=\tfrac{1}{4}\pi\ln 9$	M1	For integral of the form $k\ln(4x+1)$
	A1	For correct $\tfrac{1}{4}\ln(4x+1)$, with or without π
	M1	Correct use of limits and π
	A1 **4** **[8]**	For correct (simplified) exact value

5	**(i)**	200 °C	B1 **1**	For value 200
	(ii)	$150 = 200 - 180\mathrm{e}^{-0.1t} \Rightarrow \mathrm{e}^{-0.1t} = \frac{50}{180}$	M1	For isolating the exponential term
		Hence $-0.1t = \ln\frac{5}{18} \Rightarrow t = 12.8$	M1	For taking logs correctly
			A1 **3**	For correct value 12.8 (minutes)
	(iii)	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = 18\mathrm{e}^{-0.1t}$	M1	For differentiation attempt
			A1	For correct derivative
		Hence rate is $18\mathrm{e}^{-0.1\times 12.8} = 5.0$ °C per minute	M1	For using their value from **(ii)** in their $\dot{\theta}$
			A1 **4**	For value 5.0(0)
			8	
6	**(i)**	Domain of f^{-1} is $x \geqslant 1$	B1	For the correct set, in any notation
		Range is $x \geqslant 0$	B1 **2**	Ditto
	(ii)	If $y = 1 + \sqrt{x}$, then $x = (y-1)^2$	M1	For changing the subject, or equivalent
		Hence $\mathrm{f}^{-1}(x) = (x-1)^2$	A1 **2**	For correct expression in terms of x
	(iii)	The graphs intersect on the line $y = x$	B1	For stating or using this fact
		Hence x satisfies $x = (x-1)^2$	B1	For either $x = \mathrm{f}(x)$ or $x = \mathrm{f}^{-1}(x)$
		i.e. $x^2 - 3x + 1 = 0 \Rightarrow x = \frac{3 \pm \sqrt{5}}{2}$	M1	For solving the relevant quadratic equation
		So $x = \frac{1}{2}(3 + \sqrt{5})$ as x must be greater than 1	A1 **4**	For showing the given answer fully
			8	
7	**(i)**	$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$	B1 **1**	For correct RHS stated
	(ii)	$\frac{8t}{1-t^2} + 3 \times \frac{1}{t} \times (1 + t^2) = 0$	B1	For $\cot x = \frac{1}{t}$ seen
			B1	For $\sec^2 x = 1 + t^2$ seen
		Hence $8t^2 + 3(1-t^2)(1+t^2) = 0$	M1	For complete substitution in terms of t
		i.e. $3t^4 - 8t^2 - 3 = 0$, as required	A1 **4**	For showing given equation correctly
	(iii)	$(3t^2 + 1)(t^2 - 3) = 0$	M1	For factorising or other solution method
		Hence $t = \pm\sqrt{3}$	A1	For $t^2 = 3$ found correctly
		So $x = \frac{1}{3}\pi, \frac{2}{3}\pi, \frac{4}{3}\pi, \frac{5}{3}\pi$	A1	For any two correct angles
			A1 **4**	For all four correct and no others
			9	

8	**(i)**	$\frac{dy}{dx} = \frac{2\ln x}{x}$	M1		For relevant attempt at the chain rule
			A1		For correct result, in any form
		$\frac{d^2y}{dx^2} = \frac{x(2/x) - 2\ln x}{x^2} = \frac{2 - 2\ln x}{x^2}$	M1		For relevant attempt at quotient rule
			A1	**4**	For correct simplified answer
	(ii)	For maximum gradient, $2 - 2\ln x = 0 \Rightarrow x = \mathrm{e}$	M1		For equating second derivative to zero
			A1		For correct value e
		Hence P is $(\mathrm{e}, 1)$	A1√		For stating or using the y-coordinate
		The gradient at P is $\frac{2}{\mathrm{e}}$	A1√		For stating or using the gradient at P
		Tangent at P is $y - 1 = \frac{2}{\mathrm{e}}(x - \mathrm{e})$	M1		For forming the equation of the tangent
		Hence, when $x = 0$, $y = -1$ as required	A1	**6**	For correct verification of $(0, -1)$
				10	
9	**(i)**	$a = \frac{1}{2}\pi$	B1	**1**	For correct exact value stated
	(ii)	$x = \tan(\frac{1}{4}\pi) = 1$	M1		For use of $x = \tan(\frac{1}{2}a)$
			A1√	**2**	For correct answer, following their a
	(iii)		B1		For x-translation of (approx) $+1$
			B1		For y-stretch with (approx) factor 2
		Asymptotes are $y = \pm 2a$	B1	**3**	For correct statement of asymptotes
	(iv)	x — $\tan^{-1} x$ — $2\tan^{-1}(x-1)$ 1.535 — 0.993 — 0.983 1.545 — 0.996 — 0.998	M1		For relevant evaluations at 1.535, 1.545
		Hence graphs cross between 1.535 and 1.545	A1	**2**	For correct details and explanation
	(v)	Relevant values of $(\tan^{-1} x)^2$ are (approximately) 0, 0.0600, 0.2150, 0.4141, 0.6169	M1		For the relevant function values seen or implied; must be radians, not degrees
		$\frac{1}{12}\{0 + 4(0.0600 + 0.4141) + 2\times 0.2150 + 0.6169\}$	M1		For use of correct formula with $h = \frac{1}{4}$
		Hence required approximation is 0.245	A1	**3**	For correct (2 or 3sf) answer
				4	