OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4724

Core Mathematics 4

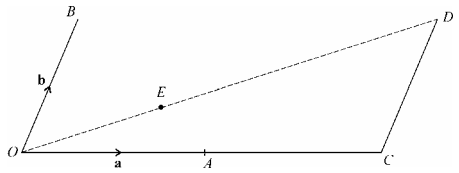
MARK SCHEME

Specimen Paper

MAXIMUM MARK	72

This mark scheme consists of 4 printed pages.

© OCR 2004
Registered Charity Number: 1066969
[Turn over

1	$\dfrac{x^4+1}{x^2+1} = x^2 - 1 + \dfrac{2}{x^2+1}$	B1		For correct leading term x^2 in quotient
		M1		For evidence of correct division process
		A1		For correct quotient $x^2 - 1$
		A1	**4**	For correct remainder 2
			4	
2	**(i)** $(1-2x)^{-\frac{1}{2}} = 1 + (-\frac{1}{2})(-2x) + \dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-2x)^2 +$			
	$+ \dfrac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(-2x)^3 + \ldots$	M1		For 2nd, 3rd or 4th term OK (unsimplified)
	$= 1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3$	A1		For $1 + x$ correct
		A1		For $+\frac{3}{2}x^2$ correct
		A1	**4**	For $+\frac{5}{2}x^3$ correct
	(ii) Valid for $\lvert x \rvert < \frac{1}{2}$	B1	**1**	For any correct expression(s)
			5	
3	$\int_0^1 x\,\mathrm{e}^{-2x}\,\mathrm{d}x = \left[-\frac{1}{2}x\,\mathrm{e}^{-2x}\right]_0^1 - \int_0^1 -\frac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x$	M1		For attempt at 'parts' going the correct way
		A1		For correct terms $-\frac{1}{2}x\,\mathrm{e}^{-2x} - \int -\frac{1}{2}\mathrm{e}^{-2x}\,\mathrm{d}x$
	$= \left[-\frac{1}{2}x\,\mathrm{e}^{-2x} - \frac{1}{4}\mathrm{e}^{-2x}\right]_0^1$	M1		For consistent attempt at second integration
		M1		For correct use of limits throughout
	$= \frac{1}{4} - \frac{3}{4}\mathrm{e}^{-2}$	A1	**5**	For correct (exact) answer in any form
			5	
4	**(i)** 	B1		For *C* correctly located on sketch
		B1		For *D* correctly located on sketch
		B1√	**3**	For *E* correctly located wrt *O* and *D*
	(ii) $\overrightarrow{AE} = \frac{1}{3}(2\mathbf{a}+\mathbf{b}) - \mathbf{a} = \frac{1}{3}(\mathbf{b}-\mathbf{a})$	M1		For relevant subtraction involving $\overrightarrow{OE}$
		A1		For correct expression for $(\pm)\overrightarrow{AE}$ or $\overrightarrow{EB}$
	Hence *AE* is parallel to *AB*	A1		For correct recognition of parallel property
	i.e. *E* lies on the line joining *A* to *B*	A1	**4**	For complete proof of required result
			7	
5	**(i)** $4x + x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y + 2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$	B1		For correct terms $x\dfrac{\mathrm{d}y}{\mathrm{d}x} + y$
		B1		For correct term $2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$
	Hence $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{4x+y}{x+2y}$	M1		For solving for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$
		A1	**4**	For any correct form of expression
	(ii) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow y = -4x$	M1		For stating or using their $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$
	Hence $2x^2 + (-4x^2) + (-4x)^2 = 14$	M1		For solving simultaneously with curve equn
	i.e. $x^2 = 1$	A1		For correct value of x^2 (or y^2)
	So the two points are $(1, -4)$ and $(-1, 4)$	A1	**4**	For both correct points identified
			8	

6	**(i)**	$\theta = 0$ at the origin	B1	For the correct value
		A is $(0, a\pi)$	B1	For the correct y-coordinate at A
		B is $(a, 0)$	B1 **3**	For the correct x-coordinate at B
	(ii)	$\frac{dx}{d\theta} = a\cos\theta$	B1	For correct differentiation of x
		$\frac{dy}{d\theta} = a(\cos\theta - \theta\sin\theta)$	M1	For differentiating y using product rule
		Hence $\frac{dy}{dx} = \frac{\cos\theta - \theta\sin\theta}{\cos\theta} = 1 - \theta\tan\theta$	M1	For use of $\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta}$
			A1	For given result correctly obtained
		Gradient of tangent at the origin is 1	M1	For using $\theta = 0$
		Hence equation is $y = x$	A1 **6**	For correct equation
			9	
6	**(i)**	$L_1 : \mathbf{r} = 3\mathbf{i} + 6\mathbf{j} + \mathbf{k} + s(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$	M1	For correct RHS structure for either line
		$L_2 : \mathbf{r} = 3\mathbf{i} - \mathbf{j} + 4\mathbf{k} + t(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	A1 **2**	For both lines correct
	(ii)	$3 + 2s = 3 + t, 6 + 3s = -1 - 2t, 1 - s = 4 + t$	M1	For at least 2 equations with two parameters
		First pair of equations give $s = -1, t = -2$	M1	For solving any relevant pair of equations
			A1	For both parameters correct
		Third equation checks: $1 + 1 = 4 - 2$	A1	For explicit check in unused equation
		Point of intersection is $(1, 3, 2)$	A1 **5**	For correct coordinates
	(iii)	$2\times1 + 3\times(-2) + (-1)\times1 = (\sqrt{14})(\sqrt{6})\cos\theta$	B1	For scalar product of correct direction vectors
			B1	For correct magnitudes $\sqrt{14}$ and $\sqrt{6}$
			M1	For correct process for $\cos\theta$ with *any* pair of vectors relevant to these lines
		Hence acute angle is $56.9°$	A1 **4**	For correct acute angle
			11	
8	**(i)**	$I = \int \frac{1}{u^2(1+u)^2} \times 2u\, du = \int \frac{2}{u(1+u)^2}\, du$	M1	For any attempt to find $\frac{dx}{du}$ or $\frac{du}{dx}$
			A1	For '$dx = 2u\, du$' or equivalent correctly used
			A1 **3**	For showing the given result correctly
	(ii)	$2 \equiv A(1+u)^2 + Bu(1+u) + Cu$	M1	For correct identity stated
		$A = 2$	B1	For correct value stated
		$C = -2$	B1	For correct value stated
		$0 = A + B$ (e.g.)	A1	For any correct equation involving B
		$B = -2$	A1 **5**	For correct value
	(iii)	$2\ln u - 2\ln(1+u) + \frac{2}{1+u}$	B1√	For $A\ln u + B\ln(1+u)$ with their values
			B1√	For $-C(1+u)^{-1}$ with their value
		Hence $I = \ln x - 2\ln(1+\sqrt{x}) + \frac{2}{1+\sqrt{x}} + c$	M1	For substituting back
			A1 **4**	For completely correct answer (excluding c)
			12	

9	$\frac{dx}{dt} = -k\sqrt{x}$	M1		For use of derivative for rate of change
		A1		For correct equation (neg sign optional here)
	$x = 100$ and $\frac{dx}{dt} = -1 \Rightarrow k = 0.1$	M1		For use of data and their DE to find k
	Hence equation is $\frac{dx}{dt} = -0.1\sqrt{x}$	A1		For any form of correct DE
	$\int x^{-\frac{1}{2}}\,dx = -0.1\int dt \Rightarrow 2x^{\frac{1}{2}} = -0.1t + c$	M1		For separation and integration of both sides
		A1		For $2x^{\frac{1}{2}}$ correct
		A1√		For $(\pm)kt$ correct (the numerical evaluation of k may be delayed until after the DE is solved)
		B1		For one arbitrary constant included (or equivalent statement of both pairs of limits)
	$x = 200, t = 0 \Rightarrow c = 2\sqrt{200}$	M1		For evaluation of c
	So when $x = 100$, $2\sqrt{100} = -0.1t + 2\sqrt{200}$	M1		For evaluation of t
	i.e. $t = 82.8$	A1	**11**	For correct value 82.8 (minutes)
			11	