

OCR Maths FP3

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<p>1 Directions $[1, 1, -1]$ and $[2, -3, 1]$</p> $\theta = \cos^{-1} \frac{ [1, 1, -1] \cdot [2, -3, 1] }{\sqrt{3} \sqrt{14}}$ $= \cos^{-1} \frac{ -2 }{\sqrt{42}}$ $= 72.0^\circ, 72^\circ \text{ or } 1.26 \text{ rad}$	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1 4</p> <p>4</p>	<p>For identifying both directions (may be implied by working)</p> <p>For using scalar product of their direction vectors</p> <p>For completely correct process for their angle</p> <p>For correct answer</p>
<p>2 (i) Identities $b, 6$ Subgroups $\{b, d\}, \{6, 4\}$</p>	<p>B1 B1</p> <p>B1 B1</p> <p>4</p>	<p>For correct identities</p> <p>For correct subgroups</p>
<p>(ii) $\{a, b, c, d\} \leftrightarrow \{2, 6, 8, 4\}$ or $\{8, 6, 2, 4\}$</p>	<p>B1 B1</p> <p>B1 3</p> <p>7</p>	<p>For $b \leftrightarrow 6, d \leftrightarrow 4$</p> <p>For $a, c \leftrightarrow 2, 8$ in either order</p> <p>SR If B0 B0 B0 then M1 A1 may be awarded for stating the orders of all elements in G and H</p>
<p>3 (i) $3y^2 \frac{dy}{dx} = \frac{dz}{dx}$</p> $\Rightarrow \frac{dz}{dx} + 2xz = e^{-x^2}$ <p>Integrating factor $\left(e^{\int 2x dx} = \right) e^{x^2}$</p> $\Rightarrow \frac{d}{dx} \left(ze^{x^2} \right) \text{ OR } \frac{d}{dx} \left(y^3 e^{x^2} \right) = 1$ $\Rightarrow ze^{x^2} \text{ OR } y^3 e^{x^2} = x + c$ $\Rightarrow y = (x+c)^{\frac{1}{3}} e^{-\frac{1}{3}x^2}$	<p>M1</p> <p>A1</p> <p>B1 \checkmark</p> <p>M1</p> <p>A1</p> <p>A1 6</p>	<p>For differentiating substitution</p> <p>For resulting equation in z and x</p> <p>For correct IF f.t. for an equation in suitable form</p> <p>For using IF correctly</p> <p>For correct integration ($+c$ not required here)</p> <p>For correct answer AEF</p>
<p>(ii) As $x \rightarrow \infty, y \rightarrow 0$</p>	<p>B1 1</p> <p>7</p>	<p>For correct statement</p>
<p>4 (i) $\cos \theta = \frac{1}{2} (e^{i\theta} + e^{-i\theta}),$ $\sin \theta = \frac{1}{2i} (e^{i\theta} - e^{-i\theta})$</p> $\Rightarrow \cos^2 \theta \sin^4 \theta = \frac{1}{4} (e^{i\theta} + e^{-i\theta})^2 \frac{1}{16} (e^{i\theta} - e^{-i\theta})^4$ $= \frac{1}{4} (e^{2i\theta} + 2 + e^{-2i\theta}) \cdot \frac{1}{16} (e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta})$ $= \frac{1}{64} \left((e^{6i\theta} + e^{-6i\theta}) - 2(e^{4i\theta} + e^{-4i\theta}) - (e^{2i\theta} + e^{-2i\theta}) + 4 \right)$ $= \frac{1}{32} (\cos 6\theta - 2 \cos 4\theta - \cos 2\theta + 2) \text{ AG}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1 6</p>	<p>For either expression, seen or implied z may be used for $e^{i\theta}$ throughout</p> <p>For expanding terms</p> <p>For the 2 correct expansions</p> <p>SR Allow A1 A0 for $k(e^{2i\theta} + 2 + e^{-2i\theta})(e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}), k \neq \frac{1}{64}$</p> <p>For grouping terms and using multiple angles</p> <p>For answer obtained correctly</p>

<p>(ii) $\int_0^{\frac{1}{3}\pi} \cos^2 \theta \sin^4 \theta \, d\theta =$ $= \frac{1}{32} \left[\frac{1}{6} \sin 6\theta - \frac{1}{2} \sin 4\theta - \frac{1}{2} \sin 2\theta + 2\theta \right]_0^{\frac{1}{3}\pi}$ $= \frac{1}{32} \left[0 + \frac{1}{4} \sqrt{3} - \frac{1}{4} \sqrt{3} + \frac{2}{3} \pi - 0 \right] = \frac{1}{48} \pi$</p>	<p>M1 A1 A1 3 9</p>	<p>For integrating answer to (i) For all terms correct For correct answer</p>
<p>5 (i) <i>EITHER</i> $z = \sqrt{8} \operatorname{cis}(2k+1)\frac{\pi}{4}, k = 0, 1, 2, 3$ <i>OR</i> $z = \sqrt{8} e^{(2k+1)\frac{\pi}{4}i}, k = 0, 1, 2, 3$</p>	<p>B1 B1 2</p>	<p>For correct modulus AEF For correct arguments AEF</p>
<p>(ii) $z = 2\sqrt{2} \left\{ \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i, -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i, -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i, \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right\}$ $z = 2 + 2i, -2 + 2i, -2 - 2i, 2 - 2i$ $(z - \alpha), (z - \beta), (z - \gamma), (z - \delta)$</p>	<p>B1 B1 B1 B1 $\sqrt{4}$</p>	<p>For any of $\pm \frac{1}{\sqrt{2}} \pm \frac{1}{\sqrt{2}}i$ For any one value of z correct For all values of z correct AEFcartesian (may be implied from symmetry or factors) f.t., where $\alpha, \beta, \gamma, \delta$ are answers above</p>
<p>(iii) <i>EITHER</i> $(z - (2 + 2i))(z - (-2 - 2i))$ $\times (z - (-2 + 2i))(z - (2 - 2i))$ $= (z^2 + 4z + 8)(z^2 - 4z + 8)$</p>	<p>M1 M1 A1</p>	<p>For combining factors from (ii) in pairs Use of complex conjugate pairs For correct answer</p>
<p><i>OR</i> $z^4 + 64 = (z^2 + az + b)(z^2 + cz + d)$ $\Rightarrow a + c = 0, b + ac + d = 0, ad + bc = 0, bd = 64$ Obtain $(z^2 + 4z + 8)(z^2 - 4z + 8)$</p>	<p>M1 M1 A1 3 9</p>	<p>For equating coefficients For solving equations For correct answer</p>
<p>6 (i) MB = [2, 1, -2], OF = [4, 1, 2] MB \times OF = [4, -12, -2] <i>OR</i> $k[2, -6, -1]$</p>	<p>B1 M1 A1 3</p>	<p>For either vector correct (allow multiples) For finding vector product of their MB and OF For correct vector</p>
<p>(ii) <i>EITHER</i> Find vector product of any two of $\pm[2, -1, 2], \pm[0, 0, 2], \pm[2, -1, 0]$ and any two of $\pm[4, 0, 2], \pm[4, -1, 0], \pm[0, 1, 2]$ Obtain $k[1, 2, 0]$ Obtain $k[1, 4, -2]$ $x + 2y = 2$ and $x + 4y - 2z = 0$</p>	<p>M1 A1 A1 M1 A1</p>	<p>For finding two relevant vector products For correct LHS of plane <i>CMG</i> For correct LHS of plane <i>OEG</i> For substituting a point into each equation For both equations correct AEF</p>
<p><i>OR</i> Use $ax + by + cz = d$ with coordinates of <i>C, M, G</i> <i>OR</i> <i>O, E, G</i> substituted Obtain $a : b : c = 1 : 2 : 0$ for <i>CMG</i> Obtain $a : b : c = 1 : 4 : -2$ for <i>OEG</i> $x + 2y = 2$ and $x + 4y - 2z = 0$</p>	<p>M1 A1 A1 M1 A1 5</p>	<p>For use of cartesian equation of plane For correct ratio For correct ratio For substituting a point into each equation For both equations correct AEF</p>

<p>(iii) EITHER Put x, y OR $z = t$ in planes OR evaluate $k[1, 2, 0] \times k[1, 4, -2]$</p> <p>Obtain $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ where $\mathbf{a} = [0, 1, 2], [2, 0, 1]$ OR $[4, -1, 0]$ $\mathbf{b} = k[-2, 1, 1]$</p>	<p>M1</p> <p>A1</p> <p>A1 3</p> <p>11</p>	<p>For solving plane equations in terms of a parameter OR for finding vector product of normals to planes from (ii)</p> <p>Obtain a correct point AEF</p> <p>Obtain correct direction AEF</p>
<p>7 (i) $(x^{-1}ax)^m = (x^{-1}ax)(x^{-1}ax)\dots(x^{-1}ax)$ $= x^{-1}a a \dots a x$, associativity, $xx^{-1} = e$</p> <p>$= x^{-1}a^m x = x^{-1}ex$ when $m = n$, not $m < n$ $= x^{-1}x$ $= e \Rightarrow$ order n</p>	<p>M1</p> <p>A1 A1</p> <p>B1</p> <p>A1</p> <p>A1 6</p>	<p>For considering powers of $x^{-1}ax$</p> <p>For using associativity and inverse properties</p> <p>For using order of a correctly</p> <p>For using property of identity</p> <p>For correct conclusion</p>
<p>(ii) EITHER $(x^{-1}ax)z = e$ $\Rightarrow axz = xe = x \Rightarrow xz = a^{-1}x$ $\Rightarrow z = x^{-1}a^{-1}x$</p>	<p>M1</p> <p>A1</p> <p>A1</p>	<p>For attempt to solve for z AEF</p> <p>For using pre- or post multiplication</p> <p>For correct answer</p>
<p>OR Use $(pq)^{-1} = q^{-1}p^{-1}$ OR $(pqr)^{-1} = r^{-1}q^{-1}p^{-1}$</p> <p>State $(x^{-1})^{-1} = x$</p> <p>Obtain $x^{-1}a^{-1}x$</p>	<p>M1</p> <p>A1</p> <p>A1 3</p>	<p>For applying inverse of a product of elements</p> <p>For stating this property</p> <p>For correct answer with no incorrect working SR correct answer with no working scores B1 only</p>
<p>(iii) $ax = xa \Rightarrow x = a^{-1}xa$ $\Rightarrow xa^{-1} = a^{-1}x$</p>	<p>M1</p> <p>A1 2</p> <p>11</p>	<p>Start from commutative property for ax</p> <p>Obtain commutative property for $a^{-1}x$</p>
<p>8 (i) $m^2 + 2km + 4 = 0$ $\Rightarrow m = -k \pm \sqrt{k^2 - 4}$</p> <p>(a) $x = e^{-kt} \left(Ae^{\sqrt{k^2 - 4}t} + Be^{-\sqrt{k^2 - 4}t} \right)$</p>	<p>M1</p> <p>A1 2</p> <p>M1</p> <p>A1 2</p>	<p>For stating and attempting to solve auxiliary eqn</p> <p>For correct solutions, at any stage AEF</p> <p>For using $e^{f(t)}$ with distinct real roots of aux eqn</p> <p>For correct answer AEF</p>
<p>(b) $x = e^{-kt} \left(Ae^{i\sqrt{4 - k^2}t} + Be^{-i\sqrt{4 - k^2}t} \right)$</p> <p>$x = e^{-kt} \left(A' \cos \sqrt{4 - k^2}t + B' \sin \sqrt{4 - k^2}t \right)$</p> <p>OR $x = e^{-kt} \left(C' \cos \left(\sqrt{4 - k^2}t + \alpha \right) \right)$</p>	<p>M1</p> <p>A1 2</p>	<p>For using $e^{f(t)}$ with complex roots of aux eqn</p> <p>This form may not be seen explicitly but if stated as final answer earns M1 A0</p> <p>For correct answer</p>
<p>(c) $x = e^{-2t} (A'' + B''t)$</p>	<p>M1</p> <p>A1 2</p>	<p>For using $e^{f(t)}$ with equal roots of aux eqn</p> <p>For correct answer. Allow k for 2</p>

<p>(ii)(a) $x = B'e^{-t} \sin \sqrt{3}t$</p> $\dot{x} = B'e^{-t} (\sqrt{3} \cos \sqrt{3}t - \sin \sqrt{3}t)$ $t = 0, \dot{x} = 6 \Rightarrow B' = 2\sqrt{3}, x = 2\sqrt{3}e^{-t} \sin \sqrt{3}t$	<p>B1 \checkmark</p> <p>M1</p> <p>A1 \checkmark</p> <p>A1 4</p>	<p>For using $t = 0, x = 0$ correctly. f.t. from (b)</p> <p>For differentiating x</p> <p>For correct expression. f.t. from their x</p> <p>For correct solution AEF</p> <p>SR \checkmark and AEF OK for</p> $x = C'e^{-t} \cos\left(\sqrt{3}t + \frac{1}{2}\pi\right)$
<p>(b) $x \rightarrow 0$</p> $e^{-t} \rightarrow 0 \text{ and } \sin(\) \text{ is bounded}$	<p>B1</p> <p>B1 2</p> <p>14</p>	<p>For correct statement</p> <p>For both statements</p>

<p>1 (a) Identity = $1+0i$</p> <p>Inverse = $\frac{1}{1+2i}$</p> <p>$= \frac{1}{1+2i} \times \frac{1-2i}{1-2i} = \frac{1}{5} - \frac{2}{5}i$</p>	<p>B1</p> <p>B1</p> <p>B1 3</p>	<p>For correct identity. Allow 1</p> <p>For $\frac{1}{1+2i}$ seen or implied</p> <p>For correct inverse AEFcartesian</p>
<p>(b) Identity = $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$</p> <p>Inverse = $\begin{pmatrix} -3 & 0 \\ 0 & 0 \end{pmatrix}$</p>	<p>B1</p> <p>B1 2</p> <p>5</p>	<p>For correct identity</p> <p>For correct inverse</p>
<p>2 (a) $(z_1 z_2 =) 6e^{\frac{5}{12}\pi i}$</p> <p>$\left(\frac{z_1}{z_2} = \frac{2}{3} e^{-\frac{1}{12}\pi i} \right) = \frac{2}{3} e^{\frac{23}{12}\pi i}$</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1 4</p>	<p>For modulus = 6</p> <p>For argument = $\frac{5}{12}\pi$</p> <p>For subtracting arguments</p> <p>For correct answer</p>
<p>(b) $(w^{-5} =) 2^{-5} \operatorname{cis}\left(-\frac{5}{8}\pi\right)$</p> <p>$= \frac{1}{32} \left(\cos \frac{11}{8}\pi + i \sin \frac{11}{8}\pi \right)$</p>	<p>M1</p> <p>A1</p> <p>A1 3</p> <p>7</p>	<p>For use of de Moivre</p> <p>For $-\frac{5}{8}\pi$ seen or implied</p> <p>For correct answer (allow 2^{-5} and $\operatorname{cis} \frac{11}{8}\pi$)</p>

<p>3 EITHER $\mathbf{c} - \mathbf{a} = \pm[11, 3, -2]$ $(\mathbf{c} - \mathbf{a}) \times [8, 3, -6]$</p> <p>$\mathbf{n} = \pm[-12, 50, 9]$</p> $d = \frac{ \mathbf{n} }{ [8, 3, -6] }$ $= \frac{\sqrt{2725}}{\sqrt{109}}$ <p>$(d =) 5$</p>	<p>B1 M1* A1 √ M1 (dep*) A1 A1</p>	<p>For vector joining lines For attempt at vector product of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For obtaining \mathbf{n}. f.t. from incorrect $\mathbf{c} - \mathbf{a}$ For dividing \mathbf{n} by magnitude of $[8, 3, -6]$ For either magnitude correct For correct distance CAO</p>
<p>OR $\mathbf{c} - \mathbf{a} = \pm[11, 3, -2]$ $(\mathbf{c} - \mathbf{a}) \cdot [8, 3, -6]$</p> $\cos \theta = \pm \frac{109}{\sqrt{134}\sqrt{109}} = \pm \frac{\sqrt{109}}{\sqrt{134}}$ $d = \sqrt{134} \sin \theta$ <p>$(d =) 5$</p>	<p>B1 M1* A1 √ M1 (dep*) A1 A1</p>	<p>For vector joining lines For attempt at scalar product of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For correct $\cos \theta$ AEF. f.t. from incorrect $\mathbf{c} - \mathbf{a}$ For using trigonometry for perpendicular distance For correct expression for d in terms of θ For correct distance CAO</p>
<p>OR $\mathbf{c} - \mathbf{a} = \pm[11, 3, -2]$ $(\mathbf{c} - \mathbf{a}) \cdot [8, 3, -6]$</p> $x = \frac{109}{\sqrt{109}} = \sqrt{109}$ $d = \sqrt{134 - 109}$ <p>$(d =) 5$</p>	<p>B1 M1* A1 √ M1 (dep*) A1 A1</p>	<p>For vector joining lines For attempt at scalar product of $\mathbf{c} - \mathbf{a}$ and $[8, 3, -6]$ For finding projection of $\mathbf{c} - \mathbf{a}$ onto $[8, 3, -6]$ f.t. from incorrect $\mathbf{c} - \mathbf{a}$ For using Pythagoras for perpendicular distance For correct expression for d For correct distance CAO</p>
<p>OR $\mathbf{CP} = \pm[-11 + 8t, -3 + 3t, 2 - 6t]$ $\mathbf{CP} \cdot [8, 3, -6] = 0$</p> <p>$t = \pm 1$ OR $P = (9, 5, -1)$</p> $d = \sqrt{3^2 + 0^2 + 4^2}$ <p>$(d =) 5$</p>	<p>B1 M1* A1 √ M1 (dep*) A1 A1 6</p>	<p>For finding a vector from $C(12, 5, 3)$ to a point on the line For using scalar product for perpendicularity For correct point. f.t. from incorrect CP For finding magnitude of CP For correct expression for d For correct distance CAO SR Obtain $\mathbf{CP} = [11, 3, -2] - [8, 3, -6] = \pm[3, 0, 4]$ B1 Verify $[3, 0, 4] \cdot [8, 3, -6] = 0$ M1* $d = \sqrt{3^2 + 0^2 + 4^2} = 5$ M1(dep*) A1 A1 (maximum 5 / 6)</p>

<p>4 Integrating factor $e^{\int -\frac{x^2}{1+x^3} dx}$</p> $= e^{-\frac{1}{3}\ln(1+x^3)} = (1+x^3)^{-\frac{1}{3}}$ $\Rightarrow \frac{d}{dx} \left(y(1+x^3)^{-\frac{1}{3}} \right) = \frac{x^2}{(1+x^3)^{\frac{1}{3}}}$ $\Rightarrow y(1+x^3)^{-\frac{1}{3}} = \frac{1}{2}(1+x^3)^{\frac{2}{3}} (+c)$ $\Rightarrow 1 = \frac{1}{2} + c \Rightarrow c = \frac{1}{2}$ $\Rightarrow y = \frac{1}{2}(1+x^3) + \frac{1}{2}(1+x^3)^{\frac{1}{3}}$	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 √</p> <p>A1</p> <p>8</p>	<p>For correct process for finding integrating factor</p> <p>For correct IF, simplified (here or later)</p> <p>For multiplying through by their IF</p> <p>For integrating RHS to obtain $A(1+x^3)^k$ OR $\ln A(1+x^3)^k$</p> <p>For correct integration (+c not required here)</p> <p>For substituting (0, 1) into GS (including + c)</p> <p>For correct c. f.t. from their GS</p> <p>For correct solution. AEF in form $y = f(x)$</p>
<p>5 (i) EITHER $\mathbf{a} = [2, 3, 5]$, $\mathbf{b} = \pm[2, 2, 0]$</p> $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \pm k[-10, 10, -2]$ <p>Use (2, 1, 5) OR (0, -1, 5)</p> $\Rightarrow 5x - 5y + z = 10$	<p>B1</p> <p>M1</p> <p>A1 √</p> <p>M1</p> <p>A1</p>	<p>For stating 2 vectors in the plane</p> <p>For finding perpendicular to plane</p> <p>For correct \mathbf{n}. f.t. from incorrect \mathbf{b}</p> <p>For substituting a point into equation $ax + by + cz = d$ where $[a, b, c] = \text{their } \mathbf{n}$</p> <p>For correct cartesian equation AEF</p>
<p>OR $\mathbf{a} = [2, 3, 5]$, $\mathbf{b} = \pm[2, 2, 0]$</p> <p>e.g. $\mathbf{r} = [2, 1, 5] + \lambda[2, 2, 0] + \mu[2, 3, 5]$</p> $[x, y, z] = [2 + 2\lambda + 2\mu, 1 + 2\lambda + 3\mu, 5 + 5\mu]$ $\Rightarrow 5x - 5y + z = 10$	<p>B1</p> <p>M1</p> <p>A1 √</p> <p>M1</p> <p>A1</p> <p>5</p>	<p>For stating 2 vectors in the plane</p> <p>For stating parametric equation of plane</p> <p>For writing 3 equations in x, y, z f.t. from incorrect \mathbf{b}</p> <p>For eliminating λ and μ</p> <p>For correct cartesian equation AEF</p>
<p>(ii) $[2t, 3t - 4, 5t - 9]$</p>	<p>B1</p> <p>1</p>	<p>For stating a point A on l_1 with parameter t AEF</p>
<p>(iii) $\pm[2t + 5, 3t - 7, 5t - 13]$</p> $\pm[2t + 5, 3t - 7, 5t - 13] \cdot [2, 3, 5] = 0$ $\Rightarrow t = 2$ $\frac{x+5}{9} = \frac{y-3}{-1} = \frac{z-4}{-3} \text{ OR}$ $\frac{x-4}{9} = \frac{y-2}{-1} = \frac{z-1}{-3}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>4</p> <p>10</p>	<p>For finding direction of l_2 from A and (-5, 3, 4)</p> <p>For using scalar product for perpendicularity with any vector involving t</p> <p>For correct value of t</p> <p>For a correct equation AEFcartesian</p> <p>SR For $2p + 3q + 5r = 0$ and no further progress award B1</p>

<p>6 (i) $(m^2 + 4 = 0 \Rightarrow) m = \pm 2i$</p> <p>CF = $A \cos 2x + B \sin 2x$</p> <p>PI = $p \sin x (+ q \cos x)$</p> <p>$-p \sin x (-q \cos x) + 4p \sin x (+4q \cos x) = \sin x$</p> <p>$\Rightarrow p = \frac{1}{3}, q = 0$</p> <p>$\Rightarrow y = A \cos 2x + B \sin 2x + \frac{1}{3} \sin x$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1 $\sqrt{6}$</p>	<p>For correct solutions of auxiliary equation (may be implied by correct CF)</p> <p>For correct CF (AEtrig but not $Ae^{2ix} + Be^{-2ix}$ only)</p> <p>State a trial PI with at least $p \sin x$</p> <p>For substituting PI into DE</p> <p>For correct p and q (which may be implied)</p> <p>For using GS = CF + PI, with 2 arbitrary constants in CF and none in PI</p>
<p>(ii) $(0, 0) \Rightarrow A = 0$</p> <p>$\frac{dy}{dx} = 2B \cos 2x + \frac{1}{3} \cos x \Rightarrow \frac{4}{3} = 2B + \frac{1}{3}$</p> <p>$A = 0, B = \frac{1}{2}$</p> <p>$\Rightarrow y = \frac{1}{2} \sin 2x + \frac{1}{3} \sin x$</p>	<p>B1 $\sqrt{}$</p> <p>M1</p> <p>A1</p> <p>A1 4</p> <p>10</p>	<p>For correct equation in A and/or B f.t. from their GS</p> <p>For differentiating their GS and substituting values for x and $\frac{dy}{dx}$</p> <p>For correct A and B Allow $A = -\frac{1}{4}i, B = \frac{1}{4}i$ from CF $Ae^{2ix} + Be^{-2ix}$</p> <p>For stating correct solution CAO</p>
<p>7 (i) $C + iS = 1 + e^{i\theta} + e^{2i\theta} + e^{3i\theta} + e^{4i\theta} + e^{5i\theta}$</p> <p>$= \frac{e^{6i\theta} - 1}{e^{i\theta} - 1}$</p> <p>$= \frac{e^{3i\theta} - e^{-3i\theta}}{e^{\frac{1}{2}i\theta} - e^{-\frac{1}{2}i\theta}} \cdot \frac{e^{3i\theta}}{e^{\frac{1}{2}i\theta}} = \frac{e^{3i\theta} - e^{-3i\theta}}{e^{\frac{1}{2}i\theta} - e^{-\frac{1}{2}i\theta}} e^{\frac{5}{2}i\theta}$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1 4</p>	<p>For using de Moivre, showing at least 3 terms</p> <p>For recognising GP</p> <p>For correct GP sum</p> <p>For obtaining correct expression AG</p>
<p>(ii) $C + iS = \frac{2i \sin 3\theta}{2i \sin \frac{1}{2}\theta} \cdot e^{\frac{5}{2}i\theta}$</p> <p>Re $\Rightarrow C = \sin 3\theta \cos \frac{5}{2}\theta \operatorname{cosec} \frac{1}{2}\theta$</p> <p>Im $\Rightarrow S = \sin 3\theta \sin \frac{5}{2}\theta \operatorname{cosec} \frac{1}{2}\theta$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>B1 4</p>	<p>For expressing numerator and denominator in terms of sines</p> <p>For $k \sin 3\theta$ and $k \sin \frac{1}{2}\theta$</p> <p>For correct expression AG</p> <p>For correct expression</p>
<p>(iii) $C = S \Rightarrow \sin 3\theta = 0, \tan \frac{5}{2}\theta = 1$</p> <p>$\theta = \frac{1}{3}\pi, \frac{2}{3}\pi$</p> <p>$\theta = \frac{1}{10}\pi, \frac{1}{2}\pi, \frac{9}{10}\pi$</p>	<p>M1</p> <p>A1</p> <p>A2 4</p> <p>12</p>	<p>For either equation deduced AEF</p> <p>Ignore values outside $0 < \theta < \pi$</p> <p>For both values correct and no extras</p> <p>For all values correct and no extras. Allow A1 for any 1 value OR all correct with extras</p>

8 (i) $r^4 \cdot a \neq a \cdot r^4$	B1 1	For stating the non-commutative product in the given table, or justifying another correct one																									
(ii) Possible subgroups order 2, 5	B1 2	For either order stated For both orders stated, and no more (Ignore 1)																									
(iii) (a) $\{e, a\}$ (b) $\{e, r, r^2, r^3, r^4\}$	B1 2	For correct subgroup For correct subgroup																									
(iv) order of $r^3 = 5$ $(ar)^2 = ar \cdot ar = r^4 a \cdot ar = e$ \Rightarrow order of $ar = 2$ $(ar^2)^2 = ar^2 ar \cdot r = ar^2 r^4 a \cdot r = ara \cdot r = e$ \Rightarrow order of $ar^2 = 2$	B1 M1 A1 A1 4	For correct order For attempt to find $(ar)^m = e$ OR $(ar^2)^m = e$ For correct order For correct order																									
(v) <table border="1" style="display: inline-table; vertical-align: middle;"> <thead> <tr> <th></th> <th>ar</th> <th>ar^2</th> <th>ar^3</th> <th>ar^4</th> </tr> </thead> <tbody> <tr> <th>ar</th> <td>e</td> <td>r</td> <td>r^2</td> <td>r^3</td> </tr> <tr> <th>ar^2</th> <td>r^4</td> <td>e</td> <td>r</td> <td>r^2</td> </tr> <tr> <th>ar^3</th> <td>r^3</td> <td>r^4</td> <td>e</td> <td>r</td> </tr> <tr> <th>ar^4</th> <td>r^2</td> <td>r^3</td> <td>r^4</td> <td>e</td> </tr> </tbody> </table>		ar	ar^2	ar^3	ar^4	ar	e	r	r^2	r^3	ar^2	r^4	e	r	r^2	ar^3	r^3	r^4	e	r	ar^4	r^2	r^3	r^4	e	B1 B1 B1 B1 B1 5 14	If the border elements $ar ar^2 ar^3 ar^4$ are not written, it will be assumed that the products arise from that order For all 16 elements of the form e or r^m For all 4 elements in leading diagonal = e For no repeated elements in any completed row or column For any two rows or columns correct For all elements correct
	ar	ar^2	ar^3	ar^4																							
ar	e	r	r^2	r^3																							
ar^2	r^4	e	r	r^2																							
ar^3	r^3	r^4	e	r																							
ar^4	r^2	r^3	r^4	e																							

1 (i) Attempt to show no closure $3 \times 3 = 1, 5 \times 5 = 1$ OR $7 \times 7 = 1$	M1 A1	For showing operation table or otherwise For a convincing reason
OR Attempt to show no identity Show $a \times e = a$ has no solution	M1 A1 2	For attempt to find identity OR for showing operation table For showing identity is not 3, not 5, and not 7 by reference to operation table or otherwise
(ii) ($a =$) 1	B1 1	For value of a stated
(iii) EITHER: $\{e, r, r^2, r^3\}$ is cyclic, (ii) group is not cyclic	B1*	For a pair of correct statements
OR: $\{e, r, r^2, r^3\}$ has 2 self-inverse elements, (ii) group has 4 self-inverse elements	B1*	For a pair of correct statements
OR: $\{e, r, r^2, r^3\}$ has 1 element of order 2 (ii) group has 3 elements of order 2	B1*	For a pair of correct statements
OR: $\{e, r, r^2, r^3\}$ has element(s) of order 4 (ii) group has no element of order 4	B1*	For a pair of correct statements
Not isomorphic	B1 (dep*) 2 5	For correct conclusion
2 EITHER: $[3, 1, -2] \times [1, 5, 4]$ $\Rightarrow \mathbf{b} = k[1, -1, 1]$ e.g. put x OR y OR $z = 0$ and solve 2 equations in 2 unknowns Obtain $[0, 2, -1]$ OR $[2, 0, 1]$ OR $[1, 1, 0]$	M1 A1 M1 M1 A1	For attempt to find vector product of both normals For correct vector identified with \mathbf{b} For giving a value to one variable For solving the equations in the other variables For a correct vector identified with \mathbf{a}
OR: Solve $3x + y - 2z = 4, x + 5y + 4z = 6$ e.g. $y + z = 1$ OR $x - z = 1$ OR $x + y = 2$ Put x OR y OR $z = t$ $[x, y, z] = [t, 2 - t, -1 + t]$ OR $[2 - t, t, 1 - t]$ OR $[1 + t, 1 - t, t]$ Obtain $[0, 2, -1]$ OR $[2, 0, 1]$ OR $[1, 1, 0]$ Obtain $k[1, -1, 1]$	M1 M1 M1 A1 A1 5 5	For eliminating one variable between 2 equations For solving in terms of a parameter For obtaining a parametric solution for x, y, z For a correct vector identified with \mathbf{a} For correct vector identified with \mathbf{b}
3 (i) $z = \frac{6 \pm \sqrt{36 - 144}}{2}$ $z = 3 \pm 3\sqrt{3}i$ Obtain ($r =$) 6 Obtain ($\theta =$) $\frac{1}{3}\pi$	M1 A1 A1 A1 4	For using quadratic equation formula or completing the square For obtaining cartesian values AEF For correct modulus For correct argument
(ii) EITHER: 6^{-3} OR $\frac{1}{216}$ seen $Z^{-3} = 6^{-3}(\cos(-\pi) \pm i \sin(-\pi))$ Obtain $-\frac{1}{216}$	B1√ M1 A1	f.t. from their r^{-3} For using de Moivre with $n = \pm 3$ For correct value
OR: $z^3 = 6z^2 - 36z = 6(6z - 36) - 36z$ 216 seen Obtain $-\frac{1}{216}$	M1 B1 A1 3 7	For using equation to find z^3 Ignore any remaining z terms For correct value

<p>4 (i) $(y = xz \Rightarrow) \frac{dy}{dx} = x \frac{dz}{dx} + z$</p> $x \frac{dz}{dx} + z = \frac{x^2(1-z^2)}{x^2 z} = \frac{1}{z} - z$ $x \frac{dz}{dx} = \frac{1}{z} - 2z = \frac{1-2z^2}{z}$	<p>B1</p> <p>M1</p> <p>A1 3</p>	<p>For a correct statement</p> <p>For substituting into differential equation and attempting to simplify to a variables separable form</p> <p>For correct equation AG</p>
<p>(ii) $\int \frac{z}{1-2z^2} dz = \int \frac{1}{x} dx \Rightarrow -\frac{1}{4} \ln(1-2z^2) = \ln cx$</p> $1-2z^2 = (cx)^{-4}$ $\frac{x^2-2y^2}{x^2} = \frac{c^{-4}}{x^4}$ $x^2(x^2-2y^2) = k$	<p>M1</p> <p>M1*</p> <p>A1</p> <p>A1√</p> <p>M1 (dep*)</p> <p>A1 6</p> <p>9</p>	<p>For separating variables and writing integrals</p> <p>For integrating both sides to ln forms</p> <p>For correct result (c not required here)</p> <p>For exponentiating their ln equation including a constant (this may follow the next M1)</p> <p>For substituting $z = \frac{y}{x}$</p> <p>For correct solution properly obtained, including dealing with any necessary change of constant to k as given AG</p>
<p>5 (i) (a) e, p, p^2</p> <p>(b) e, q, q^2</p>	<p>B1</p> <p>B1 2</p>	<p>For correct elements</p> <p>For correct elements</p> <p>SR If the answers to parts (i) and (iv) are reversed, full credit may be earned for both parts</p>
<p>(ii) $p^3 = q^3 = e \Rightarrow (pq)^3 = p^3 q^3 = e$</p> <p>$\Rightarrow$ order 3</p> <p>$(pq^2)^3 = p^3 q^6 = p^3 (q^3)^2 = e \Rightarrow$ order 3</p>	<p>M1</p> <p>A1</p> <p>A1 3</p>	<p>For finding $(pq)^3$ or $(pq^2)^3$</p> <p>For correct order</p> <p>For correct order</p> <p>SR For answer(s) only allow B1 for either or both</p>
<p>(iii) 3</p>	<p>B1 1</p>	<p>For correct order and no others</p>
<p>(iv)</p> <p>$e, pq, p^2 q^2$ OR $e, pq, (pq)^2$</p> <p>$e, pq^2, p^2 q$ OR $e, pq^2, (pq^2)^2$</p> <p>OR $e, p^2 q, (p^2 q)^2$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1 4</p> <p>10</p>	<p>For stating e and either pq or $p^2 q^2$</p> <p>For all 3 elements and no more</p> <p>For stating e and either $p q^2$ or $p^2 q$</p> <p>For all 3 elements and no more</p>

6 (i) (CF $m = -3 \Rightarrow$) Ae^{-3x}	B1 1	For correct CF
(ii) $(y =) px + q$ $\Rightarrow p + 3(px + q) = 2x + 1$ $\Rightarrow p = \frac{2}{3}, q = \frac{1}{9}$ \Rightarrow GS $y = Ae^{-3x} + \frac{2}{3}x + \frac{1}{9}$	B1 M1 A1 A1 A1√	For stating linear form for PI (may be implied) For substituting PI into DE (needs y and $\frac{dy}{dx}$) For correct values For correct GS. f.t. from their CF + PI
I.F. $e^{3x} \Rightarrow \frac{d}{dx}(ye^{3x}) = (2x + 1)e^{3x}$ $\Rightarrow ye^{3x} = \frac{1}{3}e^{3x}(2x + 1) - \int \frac{2}{3}e^{3x} dx$ $\Rightarrow ye^{3x} = \frac{2}{3}xe^{3x} + \frac{1}{3}e^{3x} - \frac{2}{9}e^{3x} + A$ \Rightarrow GS $y = Ae^{-3x} + \frac{2}{3}x + \frac{1}{9}$	B1 M1 A2 * A1√ 5	SR Integrating factor method may be used, but CF must be stated somewhere to earn the mark in (i) For stating integrating factor For attempt at integrating by parts the right way round For correct integration, including constant Award A1 for any 2 algebraic terms correct For correct GS. f.t. from their * with constant
(iii) EITHER $\frac{dy}{dx} = -3Ae^{-3x} + \frac{2}{3}$ $\Rightarrow -3A + \frac{2}{3} = 0$ $y = \frac{2}{9}e^{-3x} + \frac{2}{3}x + \frac{1}{9}$	M1 M1 A1	For differentiating their GS For putting $\frac{dy}{dx} = 0$ when $x = 0$ For correct solution
OR $\frac{dy}{dx} = 0, x = 0 \Rightarrow 3y = 1$ $\Rightarrow \frac{1}{3} = A + \frac{1}{9}$ $y = \frac{2}{9}e^{-3x} + \frac{2}{3}x + \frac{1}{9}$	M1 M1 A1 3	For using original DE with $\frac{dy}{dx} = 0$ and $x = 0$ to find y For using their GS with y and $x = 0$ to find A For correct solution
(iv) $y = \frac{2}{3}x + \frac{1}{9}$	B1√ 1 10	For correct function. f.t. from linear part of (iii)

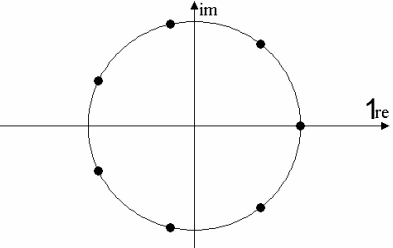
<p>7 (i) EITHER: (\mathbf{AG} is $\mathbf{r} =$) $[6, 4, 8] + tk[1, 0, 1]$ or $[3, 4, 5] + tk[1, 0, 1]$</p> <p>Normal to BCD is</p> <p>$\mathbf{n} = k[1, 1, -3]$</p> <p>Equation of BCD is $\mathbf{r} \cdot [1, 1, -3] = -6$</p> <p>Intersect at $(6+t) + 4 + (-3)(8+t) = -6$ $t = -4$ ($t = -1$ using $[3, 4, 5]$) $\Rightarrow \mathbf{OM} = [2, 4, 4]$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>For a correct equation</p> <p>For finding vector product of any two of $\pm[1, -4, -1], \pm[2, 1, 1], \pm[1, 5, 2]$</p> <p>For correct \mathbf{n}</p> <p>For correct equation (or in cartesian form)</p> <p>For substituting point on AG into plane</p> <p>For correct position vector of M \mathbf{AG}</p>
<p>OR: (\mathbf{AG} is $\mathbf{r} =$) $[6, 4, 8] + tk[1, 0, 1]$ or $[3, 4, 5] + tk[1, 0, 1]$</p> <p>$\mathbf{r} = \mathbf{u} + \lambda\mathbf{v} + \mu\mathbf{w}$, where</p> <p>$\mathbf{u} = [2, 1, 3]$ or $[1, 5, 4]$ or $[3, 6, 5]$</p> <p>$\mathbf{v}, \mathbf{w} =$ two of $[1, -4, -1], [1, 5, 2], [2, 1, 1]$</p> <p>($x =$) $6+t = 2 + \lambda + \mu$ e.g. ($y =$) $4 = 1 - 4\lambda + 5\mu$ ($z =$) $8+t = 3 - \lambda + 2\mu$</p> <p>$t = -4$ or $\lambda = -\frac{1}{3}, \mu = \frac{1}{3}$ $\Rightarrow \mathbf{OM} = [2, 4, 4]$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>6</p>	<p>For a correct equation</p> <p>For a correct parametric equation of BCD</p> <p>For forming 3 equations in t, λ, μ from line and plane, and attempting to solve them</p> <p>For correct value of t or λ, μ</p> <p>For correct position vector of M \mathbf{AG}</p>
<p>(ii)</p> <p>A, G, M have $t = 0, -3, -4$ OR</p> <p>$AG = 3\sqrt{2}, AM = 4\sqrt{2}$ OR</p> <p>$\mathbf{AG} = [-3, 0, -3], \mathbf{AM} = [-4, 0, -4]$ } $\Rightarrow AG : AM = 3 : 4$</p>	<p>B1</p> <p>1</p>	<p>For correct ratio \mathbf{AEF}</p>
<p>(iii) $\mathbf{OP} = \mathbf{OC} + \frac{4}{3}\mathbf{CG}$</p> <p>$= \left[\frac{11}{3}, \frac{11}{3}, \frac{16}{3} \right]$</p>	<p>M1</p> <p>A1</p> <p>2</p>	<p>For using given ratio to find position vector of P</p> <p>For correct vector</p>
<p>(iv) EITHER: Normal to ABD is</p> <p>$\mathbf{n} = k[19, 3, -17]$</p> <p>Equation of ABD is $\mathbf{r} \cdot [19, 3, -17] = -10$</p> <p>$19 \cdot \frac{11}{3} + 3 \cdot \frac{11}{3} - 17 \cdot \frac{16}{3} = -10$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>For finding vector product of any two of $\pm[4, 3, 5], \pm[1, 5, 2], \pm[3, -2, 3]$</p> <p>For correct \mathbf{n}</p> <p>For finding equation (or in cartesian form)</p> <p>For verifying that P satisfies equation</p>
<p>OR: Equation of ABD is</p> <p>$\mathbf{r} = [6, 4, 8] + \lambda[4, 3, 5] + \mu[1, 5, 2]$ (etc.)</p> <p>$\left[\frac{11}{3}, \frac{11}{3}, \frac{16}{3} \right] = [6, 4, 8] + \lambda[4, 3, 5] + \mu[1, 5, 2]$</p> <p>$\lambda = -\frac{2}{3}, \mu = \frac{1}{3}$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>For finding equation in parametric form</p> <p>For substituting P and solving 2 equations for λ, μ</p> <p>For correct λ, μ</p> <p>For verifying 3rd equation is satisfied</p>
<p>OR: $\mathbf{AP} = \left[-\frac{7}{3}, -\frac{1}{3}, -\frac{8}{3} \right]$</p> <p>$\mathbf{AB} = [-4, -3, -5], \mathbf{AD} = [-3, 2, -3]$</p> <p>$\Rightarrow \mathbf{AB} + \mathbf{AD} = [-7, -1, -8]$</p> <p>$\Rightarrow \mathbf{AP} = \frac{1}{3}(\mathbf{AB} + \mathbf{AD})$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>4</p> <p>13</p>	<p>For finding 3 relevant vectors in plane $ABDP$</p> <p>For correct \mathbf{AP} or \mathbf{BP} or \mathbf{DP}</p> <p>For finding \mathbf{AB}, \mathbf{AD} or \mathbf{BA}, \mathbf{BD} or \mathbf{DB}, \mathbf{DA}</p> <p>For verifying linear relationship</p>

<p>8 (i) $\cos 4\theta + i \sin 4\theta =$ $c^4 + 4ic^3s - 6c^2s^2 - 4ics^3 + s^4$ $\Rightarrow \sin 4\theta = 4c^3s - 4cs^3$ and $\cos 4\theta = c^4 - 6c^2s^2 + s^4$ $\Rightarrow \tan 4\theta = \frac{4 \tan \theta - 4 \tan^3 \theta}{1 - 6 \tan^2 \theta + \tan^4 \theta}$</p>	<p>M1 A1 M1 A1 4</p>	<p>For using de Moivre with $n = 4$ For both expressions For expressing $\frac{\sin 4\theta}{\cos 4\theta}$ in terms of c and s For simplifying to correct expression</p>
<p>(ii) $\cot 4\theta = \frac{\cot^4 \theta - 6 \cot^2 \theta + 1}{4 \cot^3 \theta - 4 \cot \theta}$</p>	<p>B1 1</p>	<p>For inverting (i) and using $\cot \theta = \frac{1}{\tan \theta}$ or $\tan \theta = \frac{1}{\cot \theta}$. AG</p>
<p>(iii) $\cot 4\theta = 0$ Put $x = \cot^2 \theta$ $\theta = \frac{1}{8}\pi \Rightarrow x^2 - 6x + 1 = 0$ <i>OR</i> $x^2 - 6x + 1 = 0 \Rightarrow \theta = \frac{1}{8}\pi$</p>	<p>B1 B1 B1 3</p>	<p>For putting $\cot 4\theta = 0$ (can be awarded in (iv) if not earned here) For putting $x = \cot^2 \theta$ in the numerator of (ii) For deducing quadratic from (ii) and $\theta = \frac{1}{8}\pi$ <i>OR</i> For deducing $\theta = \frac{1}{8}\pi$ from (ii) and quadratic</p>
<p>(iv) $4\theta = \frac{3}{2}\pi$ <i>OR</i> $\frac{1}{2}(2n+1)\pi$ 2nd root is $x = \cot^2\left(\frac{3}{8}\pi\right)$ $\Rightarrow \cot^2\left(\frac{1}{8}\pi\right) + \cot^2\left(\frac{3}{8}\pi\right) = 6$ $\Rightarrow \operatorname{cosec}^2\left(\frac{1}{8}\pi\right) + \operatorname{cosec}^2\left(\frac{3}{8}\pi\right) = 8$</p>	<p>M1 A1 M1 M1 A1 5 13</p>	<p>For attempting to find another value of θ For the other root of the quadratic For using sum of roots of quadratic For using $\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$ For correct value</p>

1 (i) $zz^* = re^{i\theta} \cdot re^{-i\theta} = r^2 = z ^2$	B1 1	For verifying result AG
(ii) Circle Centre $0 (+0i)$ OR $(0, 0)$ OR O , radius 3	B1 B1 2 3	For stating circle For stating correct centre and radius
2 EITHER: $(\mathbf{r} \Rightarrow) [3+t, 1+4t, -2+2t]$ $8(3+t) - 7(1+4t) + 10(-2+2t) = 7$ $\Rightarrow (0t) + (-3) = 7 \Rightarrow$ contradiction l is parallel to Π , no intersection	M1 M1 A1 A1 B1 5	For parametric form of l seen or implied For substituting into plane equation For obtaining a contradiction For conclusion from correct working
OR: $[1, 4, 2] \cdot [8, -7, 10] = 0$ $\Rightarrow l$ is parallel to Π $(3, 1, -2)$ into Π $\Rightarrow 24 - 7 - 20 \neq 7$ l is parallel to Π , no intersection	M1 A1 M1 A1 B1	For finding scalar product of direction vectors For correct conclusion For substituting point into plane equation For obtaining a contradiction For conclusion from correct working
OR: Solve $\frac{x-3}{1} = \frac{y-1}{4} = \frac{z+2}{2}$ and $8x - 7y + 10z = 7$ eg $y - 2z = 3, 2y - 2 = 4z + 8$ eg $4z + 4 = 4z + 8$ l is parallel to Π , no intersection	M1 A1 M1 A1 B1 5	For eliminating one variable For eliminating another variable For obtaining a contradiction For conclusion from correct working
3 Aux. equation $m^2 - 6m + 8 (= 0)$ $m = 2, 4$ CF $(y \Rightarrow) Ae^{2x} + Be^{4x}$ PI $(y \Rightarrow) Ce^{3x}$ $9C - 18C + 8C = 1 \Rightarrow C = -1$ GS $y = Ae^{2x} + Be^{4x} - e^{3x}$	M1 A1 A1√ M1 A1 B1√ 6 6	For auxiliary equation seen For correct roots For correct CF. f.t. from their m For stating and substituting PI of correct form For correct value of C For GS. f.t. from their CF + PI with 2 arbitrary constants in CF and none in PI

4 (i) $q(st) = qp = s$ $(qs)t = tt = s$	B1 B1 2	For obtaining s For obtaining s
(ii) METHOD 1 Closed: see table Identity = r Inverses: $p^{-1} = s, q^{-1} = t, (r^{-1} = r),$ $s^{-1} = p, t^{-1} = q$	B1 B1 M1 A1 4	For stating closure with reason For stating identity r For checking for inverses For stating inverses <i>OR</i> For giving sufficient explanation to justify each element has an inverse eg r occurs once in each row and/or column
METHOD 2 Identity = r eg $p^2 = t, p^3 = q, p^4 = s$ $\Rightarrow p^5 = r$, so p is a generator	B1 M1 A1 A1	For stating identity r For attempting to establish a generator $\neq r$ For showing powers of p (<i>OR</i> q, s or t) are different elements of the set For concluding p^5 (<i>OR</i> q^5, s^5 or t^5) = r
(iii) e, d, d^2, d^3, d^4	B2 2 <u>8</u>	For stating all elements AEF eg d^{-1}, d^{-2}, dd

5 (i) $(\cos 6\theta =) \operatorname{Re}(c + is)^6$ $(\cos 6\theta =) c^6 - 15c^4s^2 + 15c^2s^4 - s^6$ $(\cos 6\theta =)$ $c^6 - 15c^4(1 - c^2) + 15c^2(1 - c^2)^2 - (1 - c^2)^3$ $(\cos 6\theta =) 32c^6 - 48c^4 + 18c^2 - 1$	M1 A1 M1 A1 4	For expanding (real part of) $(c + is)^6$ at least 4 terms and 1 evaluated binomial coefficient needed For correct expansion For using $s^2 = 1 - c^2$ For correct result AG
(ii) $64x^6 - 96x^4 + 36x^2 - 3 = 0 \Rightarrow \cos 6\theta = \frac{1}{2}$ $\Rightarrow (\theta =) \frac{1}{18}\pi, \frac{5}{18}\pi, \frac{7}{18}\pi$ etc. $\cos 6\theta = \frac{1}{2}$ has multiple roots largest x requires smallest θ \Rightarrow largest positive root is $\cos \frac{1}{18}\pi$	M1 A1 M1 A1 4 <u>8</u>	For obtaining a numerical value of $\cos 6\theta$ For any correct solution of $\cos 6\theta = \frac{1}{2}$ For stating or implying at least 2 values of θ For identifying $\cos \frac{1}{18}\pi$ AEF as the largest positive root from a list of 3 positive roots <i>OR</i> from general solution <i>OR</i> from consideration of the cosine function

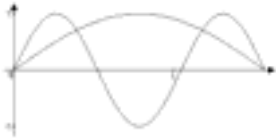
<p>6 (i) $\mathbf{n} = l_1 \times l_2$</p> $\mathbf{n} = [2, -1, 1] \times [4, 3, 2]$ $\mathbf{n} = k[-1, 0, 2]$ $[3, 4, -1] \cdot k[-1, 0, 2] = -5k$ $\mathbf{r} \cdot [-1, 0, 2] = -5$	<p>B1</p> <p>M1*</p> <p>A1</p> <p>M1 (*dep)</p> <p>A1 5</p>	<p>For stating or implying in (i) or (ii) that \mathbf{n} is perpendicular to l_1 and l_2</p> <p>For finding vector product of direction vectors</p> <p>For correct vector (any k)</p> <p>For substituting a point of l_1 into $\mathbf{r} \cdot \mathbf{n}$</p> <p>For obtaining correct p. AEF in this form</p>
<p>(ii) $[5, 1, 1] \cdot k[-1, 0, 2] = -3k$</p> $\mathbf{r} \cdot [-1, 0, 2] = -3$	<p>M1</p> <p>A1√ 2</p>	<p>For using same \mathbf{n} and substituting a point of l_2</p> <p>For obtaining correct p. AEF in this form f.t. on incorrect \mathbf{n}</p>
<p>(iii) $d = \frac{ -5+3 }{\sqrt{5}}$ OR $d = \frac{ [2, -3, 2] \cdot [-1, 0, 2] }{\sqrt{5}}$</p> <p>OR d from $(5, 1, 1)$ to $\Pi_1 = \frac{ 5(-1)+1(0)+1(2)+5 }{\sqrt{5}}$</p> <p>OR d from $(3, 4, -1)$ to $\Pi_2 = \frac{ 3(-1)+4(0)-1(2)+3 }{\sqrt{5}}$</p> <p>OR $[3-t, 4, -1+2t] \cdot [-1, 0, 2] = -3 \Rightarrow t = \frac{2}{5}$</p> <p>OR $[5-t, 1, 1+2t] \cdot [-1, 0, 2] = -5 \Rightarrow t = -\frac{2}{5}$</p> $d = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5} = 0.894427\dots$	<p>M1</p> <p>A1√ 2</p>	<p>For using a distance formula from their equations Allow omission of $$</p> <p>OR For finding intersection of \mathbf{n}_1 and Π_2 or \mathbf{n}_2 and Π_1</p> <p>For correct distance AEF f.t. on incorrect \mathbf{n}</p>
<p>(iv) d is the shortest OR perpendicular distance between l_1 and l_2</p>	<p>B1 1</p> <p>10</p>	<p>For correct statement</p>
<p>7 (i) $(z - e^{i\phi})(z - e^{-i\phi}) \equiv z^2 - (2)z \frac{(e^{i\phi} + e^{-i\phi})}{(2)} + 1$</p> $\equiv z^2 - (2 \cos \phi)z + 1$	<p>B1 1</p>	<p>For correct justification AG</p>
<p>(ii) $z = e^{\frac{2}{7}k\pi i}$</p> <p>for $k = 0, 1, 2, 3, 4, 5, 6$ OR $0, \pm 1, \pm 2, \pm 3$</p> 	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1 4</p>	<p>For general form OR any one non-real root</p> <p>For other roots specified ($k=0$ may be seen in any form, eg $1, e^0, e^{2\pi i}$)</p> <p>For answers in form $\cos \theta + i \sin \theta$ allow maximum B1 B0</p> <p>For any 7 points equally spaced round unit circle (circumference need not be shown)</p> <p>For 1 point on +ve real axis, and other points in correct quadrants</p>
<p>(iii) $(z^7 - 1) = (z - 1)(z - e^{\frac{2}{7}\pi i})(z - e^{\frac{4}{7}\pi i})$</p> $(z - e^{\frac{6}{7}\pi i})(z - e^{\frac{8}{7}\pi i})(z - e^{\frac{10}{7}\pi i})(z - e^{\frac{12}{7}\pi i})$ $= (z - e^{\frac{2}{7}\pi i})(z - e^{\frac{6}{7}\pi i}) \times (z - e^{\frac{4}{7}\pi i})(z - e^{\frac{8}{7}\pi i})$ $(z - e^{\frac{6}{7}\pi i})(z - e^{\frac{12}{7}\pi i}) \times (z - 1)$ $= (z^2 - (2 \cos \frac{2}{7}\pi)z + 1) \times$ $(z^2 - (2 \cos \frac{4}{7}\pi)z + 1) \times (z^2 - (2 \cos \frac{6}{7}\pi)z + 1) \times$ $\times (z - 1)$	<p>M1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>A1 5</p> <p>10</p>	<p>For using linear factors from (ii), seen or implied</p> <p>For identifying at least one pair of complex conjugate factors</p> <p>For linear factor seen</p> <p>For any one quadratic factor seen</p> <p>For the other 2 quadratic factors and expression written as product of 4 factors</p>

<p>8 (i) Integrating factor $e^{\int \tan x \, dx}$ $= e^{-\ln \cos x}$ $= (\cos x)^{-1}$ OR $\sec x$ $\Rightarrow \frac{d}{dx}(y(\cos x)^{-1}) = \cos^2 x$ $y(\cos x)^{-1} = \int \frac{1}{2}(1 + \cos 2x) \, dx$ $y(\cos x)^{-1} = \frac{1}{2}x + \frac{1}{4}\sin 2x + c$ $y = \left(\frac{1}{2}x + \frac{1}{4}\sin 2x + c\right)\cos x$</p>	<p>B1 M1 A1 B1√ M1 M1 A1 A1 8</p>	<p>For correct IF For integrating to ln form For correct simplified IF AEF For $\frac{d}{dx}(y \cdot \text{their IF}) = \cos^2 x \cdot \text{their IF}$ For integrating LHS For attempting to use $\cos 2x$ formula OR parts for $\int \cos^2 x \, dx$ For correct integration both sides AEF For correct general solution AEF</p>
<p>(ii) $2 = \left(\frac{1}{2}\pi + c\right) \cdot -1 \Rightarrow c = -2 - \frac{1}{2}\pi$ $y = \left(\frac{1}{2}x + \frac{1}{4}\sin 2x - 2 - \frac{1}{2}\pi\right)\cos x$</p>	<p>M1 A1 2 10</p>	<p>For substituting $(\pi, 2)$ into their GS and solve for c For correct solution AEF</p>
<p>9 (i) $3^n \times 3^m = 3^{n+m}$, $n + m \in \mathbb{Z}$ $(3^p \times 3^q) \times 3^r = (3^{p+q}) \times 3^r = 3^{p+q+r}$ $= 3^p \times (3^{q+r}) = 3^p \times (3^q \times 3^r) \Rightarrow$ associativity Identity is 3^0 Inverse is 3^{-n} $3^n \times 3^m = 3^{n+m} = 3^{m+n} = 3^m \times 3^n \Rightarrow$ commutativity</p>	<p>B1 M1 A1 B1 B1 B1 6</p>	<p>For showing closure For considering 3 distinct elements, seen bracketed 2+1 or 1+2 For correct justification of associativity For stating identity. Allow 1 For stating inverse For showing commutativity</p>
<p>(ii) (a) $3^{2n} \times 3^{2m} = 3^{2n+2m} (= 3^{2(n+m)})$ Identity, inverse OK</p>	<p>B1* B1 (*dep) 2</p>	<p>For showing closure For stating other two properties satisfied and hence a subgroup</p>
<p>(b) For 3^{-n}, $-n \notin$ subset</p>	<p>M1 A1 2</p>	<p>For considering inverse For justification of not being a subgroup 3^{-n} must be seen here or in (i)</p>
<p>(c) EITHER: eg $3^{1^2} \times 3^{2^2} = 3^5$ $\neq 3^{r^2} \Rightarrow$ not a subgroup OR: $3^{n^2} \times 3^{m^2} = 3^{n^2+m^2}$ $\neq 3^{r^2}$ eg $1^2 + 2^2 = 5 \Rightarrow$ not a subgroup</p>	<p>M1 A1 2 M1 A1 12</p>	<p>For attempting to find a specific counter-example of closure For a correct counter-example and statement that it is not a subgroup For considering closure in general For explaining why $n^2 + m^2 \neq r^2$ in general and statement that it is not a subgroup</p>

4727 Further Pure Mathematics 3

1 (a) (i) e.g. $ap \neq pa \Rightarrow$ not commutative	B1 1	For correct reason and conclusion
(ii) 3	B1 1	For correct number
(iii) e, a, b	B1 1	For correct elements
(b) c^3 has order 2 c^4 has order 3 c^5 has order 6	B1 B1 B1 3 6	For correct order For correct order For correct order
2 $m^2 - 8m + 16 = 0$ $\Rightarrow m = 4$ \Rightarrow CF ($y =$) $(A + Bx)e^{4x}$ For PI try $y = px + q$ $\Rightarrow -8p + 16(px + q) = 4x$ $\Rightarrow p = \frac{1}{4} \quad q = \frac{1}{8}$ \Rightarrow GS $y = (A + Bx)e^{4x} + \frac{1}{4}x + \frac{1}{8}$	M1 A1 A1√ M1 A1 A1 B1√ 7 7	For stating and attempting to solve auxiliary eqn For correct solution For CF of correct form. f.t. from m For using linear expression for PI For correct coefficients For GS = CF + PI. Requires $y =$. f.t. from CF and PI with 2 arbitrary constants in CF and none in PI
3 (i) line segment OA	B1 B1 2	For stating line through O OR A For correct description AEF
(ii) $(\mathbf{r} - \mathbf{a}) \times (\mathbf{r} - \mathbf{b}) = \vec{AP} \times \vec{BP}$ $= \vec{AP} \vec{BP} \sin \pi \cdot \hat{\mathbf{n}} = \mathbf{0}$	B1 B1 2	For identifying $\mathbf{r} - \mathbf{a}$ with \vec{AP} and $\mathbf{r} - \mathbf{b}$ with \vec{BP} Allow direction errors For using \times of 2 parallel vectors = 0 OR $\sin \pi = 0$ or $\sin 0 = 0$ in an appropriate vector expression
(iii) line through O parallel to AB	B1 B1 B1 3 7	For stating line For stating through O For stating correct direction SR For \vec{AB} or \vec{BA} allow B1 B0 B1
4 $(C + iS) = \int_0^{\frac{1}{2}\pi} e^{2x} (\cos 3x + i \sin 3x) (dx)$ $\cos 3x + i \sin 3x = e^{3ix}$ $\int_0^{\frac{1}{2}\pi} e^{(2+3i)x} (dx) = \frac{1}{2+3i} \left[e^{(2+3i)x} \right]_0^{\frac{1}{2}\pi}$ $= \frac{2-3i}{4+9} \left(e^{(2+3i)\frac{1}{2}\pi} - e^0 \right) = \frac{2-3i}{13} (-ie^\pi - 1)$ $= \left\{ \frac{1}{13} (-2 - 3e^\pi + i(3 - 2e^\pi)) \right\}$ $C = -\frac{1}{13} (2 + 3e^\pi)$ $S = \frac{1}{13} (3 - 2e^\pi)$	B1 M1* A1 A1 M1 (dep*) M1 (dep*) A1 A1 8	For using de Moivre, seen or implied For writing as a single integral in exp form For correct integration (ignore limits) For substituting limits correctly (unsimplified) (may be earned at any stage) For multiplying by complex conjugate of $2+3i$ For equating real and/or imaginary parts For correct expression AG For correct expression

<p>5 (i) IF $e^{\int \frac{1}{x} dx} = e^{\ln x} = x$ OR $x \frac{dy}{dx} + y = x \sin 2x$ $\Rightarrow \frac{d}{dx}(xy) = x \sin 2x$ $\Rightarrow xy = \int x \sin 2x (dx)$ $xy = -\frac{1}{2}x \cos 2x + \frac{1}{2} \int \cos 2x (dx)$ $xy = -\frac{1}{2}x \cos 2x + \frac{1}{4} \sin 2x (+c)$ $\Rightarrow y = -\frac{1}{2} \cos 2x + \frac{1}{4x} \sin 2x + \frac{c}{x}$</p>	<p>M1 A1 M1 A1 M1 A1 6</p>	<p>For correct process for finding integrating factor OR for multiplying equation through by x For writing DE in this form (may be implied) For integration by parts the correct way round For 1st term correct For their 1st term and attempt at integration of $\frac{\cos}{\sin} kx$ For correct expression for y</p>
<p>(ii) $(\frac{1}{4}\pi, \frac{2}{\pi}) \Rightarrow \frac{2}{\pi} = \frac{1}{\pi} + \frac{4c}{\pi} \Rightarrow c = \frac{1}{4}$ $\Rightarrow y = -\frac{1}{2} \cos 2x + \frac{1}{4x} \sin 2x + \frac{1}{4x}$</p>	<p>M1 A1 2</p>	<p>For substituting $(\frac{1}{4}\pi, \frac{2}{\pi})$ in solution For correct solution. Requires $\boxed{y=}$.</p>
<p>(iii) $(y \approx) -\frac{1}{2} \cos 2x$</p>	<p>B1√ 1 9</p>	<p>For correct function AEF f.t. from (ii)</p>
<p>6 (i)</p> <p>METHOD 1 State $B = (-1, -7, 2) + t(1, 2, -2)$ On plane $\Rightarrow (-1+t) + 2(-7+2t) - 2(2-2t) = -1$ $\Rightarrow t = 2 \Rightarrow B = (1, -3, -2)$ $AB = \sqrt{2^2 + 4^2 + 4^2}$ OR $2\sqrt{1^2 + 2^2 + 2^2} = 6$</p>	<p>M1 M1 M1 A1 A1 5</p>	<p>Either coordinates or vectors may be used Methods 1 and 2 may be combined, for a maximum of 5 marks For using vector normal to plane For substituting parametric form into plane For solving a linear equation in t For correct coordinates For correct length of AB</p>
<p>METHOD 2 $AB = \frac{ -1-14-4+1 }{\sqrt{1^2+2^2+2^2}} = 6$ OR $AB = \mathbf{AC} \cdot \frac{\mathbf{AB}}{ \mathbf{AB} } = \frac{[6, 7, 1] \cdot [1, 2, -2]}{\sqrt{1^2+2^2+2^2}} = 6$ $B = (-1, -7, 2) \pm 6 \frac{(1, 2, -2)}{\sqrt{1^2+2^2+2^2}}$ $B = (-1, -7, 2) \pm (2, 4, -4)$ $B = (1, -3, -2)$</p>	<p>M1 A1 M1 B1 A1</p>	<p>For using a correct distance formula For correct length of AB For using $B = A + \text{length of } AB \times \text{unit normal}$ For checking whether + or - is needed (substitute into plane equation) For correct coordinates (allow even if B0)</p>
<p>(ii) Find vector product of any two of $\pm[6, 7, 1], \pm[6, -3, 0], \pm(0, 10, 1)$ Obtain $k[1, 2, -20]$ $\theta = \cos^{-1} \frac{ [1, 2, -2] \cdot [1, 2, -20] }{\sqrt{1^2+2^2+2^2} \sqrt{1^2+2^2+20^2}}$ $\theta = \cos^{-1} \frac{45}{\sqrt{9} \sqrt{405}} = 41.8^\circ (41.810\dots^\circ, 0.72972\dots)$</p>	<p>M1 A1 M1* M1 (dep*) A1√ A1 6 11</p>	<p>For finding vector product of two relevant vectors For correct vector \mathbf{n} For using scalar product of two normal vectors For stating both moduli in denominator For correct scalar product. f.t. from \mathbf{n} For correct angle</p>

7 (i) (a) $\sin \frac{6}{8}\pi = \frac{1}{\sqrt{2}}$, $\sin \frac{2}{8}\pi = \frac{1}{\sqrt{2}}$	B1 1	For verifying $\theta = \frac{1}{8}\pi$
(b)  $\theta = \frac{3}{8}\pi$	M1 A1 2	For sketching $y = \sin 6\theta$ and $y = \sin 2\theta$ for $0 \leq \theta \leq \frac{1}{2}\pi$ <i>OR</i> any other correct method for solving $\sin 6\theta = \sin 2\theta$ for $\theta \neq k\frac{\pi}{2}$ <i>OR</i> appropriate use of symmetry <i>OR</i> attempt to verify a reasonable guess for θ For correct θ
(ii) $\text{Im}(c + is)^6 = 6c^5s - 20c^3s^3 + 6cs^5$ $\sin 6\theta = \sin \theta (6c^5 - 20c^3(1 - c^2) + 6c(1 - c^2)^2)$ $\sin 6\theta = \sin \theta (32c^5 - 32c^3 + 6c)$ $\sin 6\theta = 2 \sin \theta \cos \theta (16c^4 - 16c^2 + 3)$ $\sin 6\theta = \sin 2\theta (16 \cos^4 \theta - 16 \cos^2 \theta + 3)$	M1 A1 M1 A1 A1 5	For expanding $(c + is)^6$; at least 3 terms and 3 binomial coefficients needed For 3 correct terms For using $s^2 = 1 - c^2$ For any correct intermediate stage For obtaining this expression correctly AG
(iii) $16c^4 - 16c^2 + 3 = 1$ $\Rightarrow c^2 = \frac{2 \pm \sqrt{2}}{4}$ – sign requires larger $\theta = \frac{3}{8}\pi$	M1 A1 A1 3 11	For stating this equation AEF For obtaining both values of c^2 For stating and justifying $\theta = \frac{3}{8}\pi$ Calculator OK if figures seen

<p>8 (i) Group A: $e = 6$ Group B: $e = 1$ Group C: $e = 2^0$ OR 1 Group D: $e = 1$</p>	$\left. \begin{array}{l} \text{B1} \\ \text{B1} \\ \mathbf{2} \end{array} \right\}$	<p>For any two correct identities For two other correct identities AEF for D, but not “$m = n$”</p>
<p>(ii) EITHER OR</p> <p>A 2 4 6 8 2 4 8 2 6 orders of elements 4 8 6 4 2 1, 2, 4, 4 6 2 4 6 8 OR cyclic group 8 6 2 8 4</p> <p>B 1 5 7 11 1 1 5 7 11 orders of elements 5 5 1 11 7 1, 2, 2, 2 7 7 11 1 5 OR non-cyclic group 11 11 7 5 1 OR Klein group</p> <p>C 2^0 2^1 2^2 2^3 2^0 2^0 2^1 2^2 2^3 orders of elements 2^1 2^1 2^2 2^3 2^0 1, 2, 4, 4 2^2 2^2 2^3 2^0 2^1 OR cyclic group 2^3 2^3 2^0 2^1 2^2</p> <p>$A \not\cong B$ $B \not\cong C$ $A \cong C$</p>	<p>B1* B1* B1 (dep*) B1 (dep*) B1 (dep*) 5</p>	<p>For showing group table OR sufficient details of orders of elements OR stating cyclic / non-cyclic / Klein group (as appropriate)</p> <p>for one of groups A, B, C for another of groups A, B, C</p> <p>For stating non-isomorphic } with sufficient detail For stating non-isomorphic } relating to the first 2 marks For stating isomorphic }</p>
<p>(iii) $\frac{1+2m}{1+2n} \times \frac{1+2p}{1+2q} = \frac{1+2m+2p+4mp}{1+2n+2q+4nq}$</p> <p>$= \frac{1+2(m+p+2mp)}{1+2(n+q+2nq)} \equiv \frac{1+2r}{1+2s}$</p>	<p>M1* M1 (dep*) A1 A1 4</p>	<p>For considering product of 2 distinct elements of this form For multiplying out For simplifying to form shown For identifying as correct form, so closed</p> <p>SR $\frac{\text{odd}}{\text{odd}} \times \frac{\text{odd}}{\text{odd}} = \frac{\text{odd}}{\text{odd}}$ earns full credit SR If clearly attempting to prove commutativity, allow at most M1</p>
<p>(iv) Closure not satisfied Identity and inverse not satisfied</p>	<p>B1 B1 2 13</p>	<p>For stating closure For stating identity and inverse SR If associativity is stated as not satisfied, then award at most B1 B0 OR B0 B1</p>

4727 Further Pure Mathematics 3

1 (a)(i)	e, r^3, r^6, r^9	M1	For stating e, r^m (any $m \geq 2$), and 2 other different elements in terms of e and r
		A1	2 For all elements correct
(ii)	r generates G	B1	1 For this or any statement equivalent to: all elements of G are included in a group with e and r OR order of $r >$ order of all possible proper subgroups
(b)	m, n, p, mn, np, pm	B1	For any 3 orders correct
		B1	2 For all 6 correct and no extras (Ignore 1 and mnp)
5			
2	METHOD 1		
	$[1, 3, 2] \times [1, 2, -1]$	M1	For attempt to find normal vector, e.g. by finding vector product of correct vectors, or Cartesian equation
	$\mathbf{n} = k[-7, 3, -1]$ OR $7x - 3y + z = c$ ($= 17$)	A1	For correct vector OR LHS of equation
	$\theta = \sin^{-1} \frac{ [1, 4, -1] \cdot [-7, 3, -1] }{\sqrt{1^2 + 4^2 + 1^2} \sqrt{7^2 + 3^2 + 1^2}}$	M1√	For using correct vectors for line and plane f.t. from normal
		M1*	For using scalar product of line and plane vectors
		M1	For calculating both moduli in denominator
	$\theta = \sin^{-1} \frac{6}{\sqrt{18}\sqrt{59}} = 10.6^\circ$	A1√	For scalar product. f.t. from their numerator
	(10.609...°, 0.18517...)	(*dep)	
		A1	7 For correct angle
	METHOD 2		
	$[1, 3, 2] \times [1, 2, -1]$	M1	For attempt to find normal vector, e.g. by finding vector product of correct vectors, or Cartesian equation
	$\mathbf{n} = k[-7, 3, -1]$ OR $7x - 3y + z = c$	A1	For correct vector OR LHS of equation
	$7x - 3y + z = 17$	M1√	For attempting to find RHS of equation f.t. from \mathbf{n} or LHS of equation
	$d = \frac{ 21 - 12 + 2 - 17 }{\sqrt{7^2 + 3^2 + 1^2}} = \frac{6}{\sqrt{59}}$	M1	For using distance formula from a point on the line, e.g.
		A1√	(3, 4, 2), to the plane
			For correct distance. f.t. from equation
	$\theta = \sin^{-1} \frac{\frac{6}{\sqrt{59}}}{\sqrt{1^2 + 4^2 + 1^2}} = 10.6^\circ$	M1	For using trigonometry
	(10.609...°, 0.18517...)	A1	For correct angle
7			
3 (i)	$\frac{dz}{dx} = 1 + \frac{dy}{dx}$	M1	For differentiating substitution (seen or implied)
	$\frac{dz}{dx} - 1 = \frac{z+3}{z-1} \Rightarrow \frac{dz}{dx} = \frac{2z+2}{z-1} = \frac{2(z+1)}{z-1}$	A1	For correct equation in z AEF
		A1	3 For correct simplification to AG
(ii)	$\int \frac{z-1}{z+1} dz = 2 \int dx$	B1	For $\int \frac{z-1}{z+1} (dz)$ and $\int (1) (dx)$ seen or implied
	$\Rightarrow \int 1 - \frac{2}{z+1} dz$ OR $\int 1 - \frac{2}{u} du = 2x (+c)$	M1	For rearrangement of LHS into integrable form OR substitution e.g. $u = z+1$ or $u = z-1$
	$\Rightarrow z - 2 \ln(z+1)$ OR $z+1 - 2 \ln(z+1)$	A1	For correct integration of LHS as $f(z)$
	$= 2x (+c)$		
	$\Rightarrow -2 \ln(x+y+1) = x - y + c$	A1	4 For correct general solution AEF

4 (i)	$\cos^5 \theta = \left(\frac{e^{i\theta} + e^{-i\theta}}{2} \right)^5$	B1	For $\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$ seen or implied z may be used for $e^{i\theta}$ throughout
	$\cos^5 \theta = \frac{1}{32} (e^{i\theta} + e^{-i\theta})^5$	M1	For expanding $(e^{i\theta} + e^{-i\theta})^5$. At least 3 terms and 2 binomial coefficients required <i>OR</i> reasonable attempt at expansion in stages
	$\cos^5 \theta = \frac{1}{32} (e^{5i\theta} + e^{-5i\theta} + 5(e^{3i\theta} + e^{-3i\theta}) + 10(e^{i\theta} + e^{-i\theta}))$	A1	For correct binomial expansion
	$\cos^5 \theta = \frac{1}{16} (\cos 5\theta + 5 \cos 3\theta + 10 \cos \theta)$	M1 A1	For grouping terms and using multiple angles For answer obtained correctly AG
<hr style="border-top: 1px dashed black;"/>			
(ii)	$\cos \theta = 16 \cos^5 \theta$	B1	For stating correct equation of degree 5 <i>OR</i> $1 = 16 \cos^4 \theta$ AEF
	$\Rightarrow \cos \theta = 0, \quad \cos \theta = \pm \frac{1}{2}$	M1	For obtaining at least one of the values of $\cos \theta$ from $\cos \theta = k \cos^5 \theta$ <i>OR</i> from $1 = k \cos^4 \theta$
	$\Rightarrow \theta = \frac{1}{2}\pi, \frac{1}{3}\pi, \frac{2}{3}\pi$	A1 A1	A1 for any two correct values of θ A1 4 A1 for the 3rd value and no more in $0, \theta, \pi$ Ignore values outside $0, \theta, \pi$

5 (i) METHOD 1

Lines meet where

$$(x =) k + 2\lambda = k + \mu$$

$$(y =) -1 - 5\lambda = -4 - 4\mu$$

$$(z =) 1 - 3\lambda = -2\mu$$

M1 For using parametric form to find where lines meet
A1 For at least 2 correct equations

$$\Rightarrow \lambda = -1, \mu = -2$$

M1 For attempting to solve any 2 equations
A1 For correct values of λ and μ
B1 For attempting a check in 3rd equation
OR verifying point of intersection is on both lines

$$\Rightarrow (k - 2, 4, 4)$$

A1 6 For correct point of intersection (allow vector)
SR For finding λ OR μ and point of intersection, but no check, award up to M1 A1 M1 A0 B0 A1

METHOD 2

$$d = \frac{|[0, 3, 1] \cdot [2, -5, -3] \times [1, -4, -2]|}{|\mathbf{b} \times \mathbf{c}|}$$

For using $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ with appropriate vectors (division by $|\mathbf{b} \times \mathbf{c}|$ is not essential)

$$d = c[0, 3, 1] \cdot [-2, 1, -3] = 0$$

B1 and showing $d = 0$ correctly

\Rightarrow lines intersect

Lines meet where

$$(x =) (k +) 2\lambda = (k +) \mu$$

$$(y =) -1 - 5\lambda = -4 - 4\mu$$

$$(z =) 1 - 3\lambda = -2\mu$$

M1 For using parametric form to find where lines meet
A1 For at least 2 correct equations

$$\Rightarrow \lambda = -1, \mu = -2$$

M1 For attempting to solve any 2 equations
A1 For correct value of λ OR μ

$$\Rightarrow (k - 2, 4, 4)$$

A1 For correct point of intersection (allow vector)

METHOD 3

e.g. $x - k = \frac{2(y+1)}{-5} = \frac{y+4}{-4}$

M1 For solving one pair of simultaneous equations

$$\Rightarrow y = 4$$

A1 For correct value of x, y or z

$$\frac{z-1}{-3} = \frac{y+1}{-5}$$

M1 For solving for the third variable

$$x = k - 2 \text{ OR } z = 4$$

A1 For correct values of 2 of x, y and z

$$x - k = \frac{z}{-2} \text{ checks with } x = k - 2, z = 4$$

B1 For attempting a check in 3rd equation

$$\Rightarrow (k - 2, 4, 4)$$

A1 For correct point of intersection (allow vector)

(ii) METHOD 1

$$\mathbf{n} = [2, -5, -3] \times [1, -4, -2]$$

M1 For finding vector product of 2 directions

$$\mathbf{n} = c[-2, 1, -3]$$

A1 For correct normal
SR Following Method 2 for (i), award M1 A1√ for \mathbf{n} , f.t. from their \mathbf{n}

$$(1, -1, 1) \text{ OR } (1, -4, 0) \text{ OR } (-1, 4, 4)$$

M1 For substituting a point in LHS

$$\Rightarrow 2x - y + 3z = 6$$

A1 4 For correct equation of plane **AEF cartesian**

METHOD 2

$$\mathbf{r} = [1, -1, 1] + \lambda[2, -5, -3] + \mu[1, -4, -2]$$

M1 For using vector equation of plane (OR $[1, -4, 0]$ for **a**)

$$x = 1 + 2\lambda + \mu$$

$$y = -1 - 5\lambda - 4\mu$$

$$z = 1 - 3\lambda - 2\mu$$

A1 For writing 3 linear equations

$$\Rightarrow 2x - y + 3z = 6$$

M1 For eliminating λ and μ
A1 For correct equation of plane **AEF cartesian**

6 (i)	<p>When a, b have opposite signs,</p> $a b = \pm ab, b a = \mp ba \Rightarrow a b \neq b a $	<p>M1 A1 2</p>	<p>For considering sign of $a b$ OR $b a$ in general or in a specific case For showing that $a b \neq b a$ Note that $x = \sqrt{x^2}$ may be used</p>
(ii)	$(a \circ b) \circ c = (a b) \circ c = a b c \text{ OR } a bc $ $a \circ (b \circ c) = a \circ (b c) = a b c = a b c \text{ OR } a bc $	<p>M1 A1 M1 A1 4</p>	<p>For using 3 distinct elements and simplifying $(a \circ b) \circ c$ OR $a \circ (b \circ c)$ For obtaining correct answer For simplifying the other bracketed expression For obtaining the same answer</p>
(iii)	<p><i>EITHER</i> $a \circ e = a e = a \Rightarrow e = \pm 1$</p> <p><i>OR</i> $e \circ a = e a = a$ $\Rightarrow e = 1$ for $a > 0$, $e = -1$ for $a < 0$ Not a group</p>	<p>B1* M1 A1 B1 (*dep) 4</p>	<p>For stating $e = \pm 1$ OR no identity For attempting algebraic justification of +1 and -1 for e For deducing no (unique) identity For stating not a group</p>

7 (i)		B1 1	Polar or cartesian values of ω and ω^2 may be used anywhere in this question
			For showing 3 points in approximately correct positions
			Allow ω and ω^2 interchanged, or unlabelled
(ii)	<p><i>EITHER</i> $1 + \omega + \omega^2$ = sum of roots of cubic = 0</p> <p><i>OR</i> $\omega^3 = 1 \Rightarrow (\omega - 1)(\omega^2 + \omega + 1) = 0$ $\Rightarrow 1 + \omega + \omega^2 = 0$ (for $\omega \neq 1$)</p> <p><i>OR</i> sum of G.P. $1 + \omega + \omega^2 = \frac{1 - \omega^3}{1 - \omega} \left(= \frac{0}{1 - \omega} \right) = 0$</p>	M1 A1 2	For result shown by any correct method AG
	<p><i>OR</i> shown on Argand diagram or explained in terms of vectors</p>		Reference to vectors in part (i) diagram may be made
	<p><i>OR</i> $1 + \text{cis } \frac{2}{3}\pi + \text{cis } \frac{4}{3}\pi = 1 + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 0$</p>		
(iii) (a)	$(2 + \omega)(2 + \omega^2) = 4 + 2(\omega + \omega^2) + \omega^3$ $= 4 - 2 + 1 = 3$	M1 A1 2	For using $1 + \omega + \omega^2 = 0$ <i>OR</i> values of ω , ω^2 For correct answer
(b)	$\frac{1}{2 + \omega} + \frac{1}{2 + \omega^2} = \frac{2 + (\omega + \omega^2) + 2}{3} = 1$	M1 A1√ 2	For combining fractions <i>OR</i> multiplying top and bottom of 2 fractions by complex conjugates For correct answer f.t. from (a)
(iv)	For the cubic $x^3 + px^2 + qx + r = 0$		
	METHOD 1		
	$\sum \alpha = 2 + 1 = 3 \Rightarrow p = -3$	M1	For calculating two of $\sum \alpha$, $\sum \alpha\beta$, $\alpha\beta\gamma$
	$\sum \alpha\beta = \frac{2}{2 + \omega} + \frac{2}{2 + \omega^2} + \frac{1}{3} = \frac{7}{3} (= q)$	M1	For calculating all of $\sum \alpha$, $\sum \alpha\beta$, $\alpha\beta\gamma$ <i>OR</i> all of p , q , r
	$\alpha\beta\gamma = \frac{2}{3} \left(\Rightarrow r = -\frac{2}{3} \right)$	A1	For at least two of $\sum \alpha$, $\sum \alpha\beta$, $\alpha\beta\gamma$ correct (or values of p , q , r)
	$\Rightarrow 3x^3 - 9x^2 + 7x - 2 = 0$	A1 4	For correct equation CAO
	METHOD 2		
	$\left(x - 2\right)\left(x - \frac{1}{2 + \omega}\right)\left(x - \frac{1}{2 + \omega^2}\right) = 0$		
	$x^3 + \left(-2 - \frac{1}{2 + \omega} - \frac{1}{2 + \omega^2}\right)x^2$	M1	For multiplying out LHS in terms of ω or $\text{cis } \frac{1}{3}k\pi$
	$+ \left(\frac{1}{(2 + \omega)(2 + \omega^2)} + \frac{2}{2 + \omega} + \frac{2}{2 + \omega^2}\right)x$		
	$-\frac{2}{(2 + \omega)(2 + \omega^2)} = 0$	M1	For simplifying, using parts (ii), (iii) or values of ω
	$\Rightarrow x^3 - 3x^2 + \frac{7}{3}x - \frac{2}{3} = 0$	A1	For at least two of p , q , r correct
	$\Rightarrow 3x^3 - 9x^2 + 7x - 2 = 0$	A1	For correct equation CAO

8 (i)	$m^2 + 1 = 0 \Rightarrow m = \pm i$	M1	For stating and attempting to solve correct auxiliary equation
	\Rightarrow C.F.	A1	2 For correct C.F. (must be in trig form)
	$(y =) Ce^{ix} + De^{-ix} = A \cos x + B \sin x$		SR If some or all of the working is omitted, award full credit for correct answer
(ii)(a)	$y = p(\ln \sin x) \sin x + qx \cos x$	M1	For attempting to differentiate P.I. (product rule needed at least once)
	$\frac{dy}{dx} = p \frac{\cos x}{\sin x} \sin x + p(\ln \sin x) \cos x + q \cos x - qx \sin x$	A1	For correct (unsimplified) result AEF
	$\frac{d^2y}{dx^2} = -p \sin x - p(\ln \sin x) \sin x + \frac{p \cos^2 x}{\sin x} - 2q \sin x - qx \cos x$	A1	For correct (unsimplified) result AEF
	$-p \sin x + \frac{p \cos^2 x}{\sin x} - 2q \sin x \equiv \frac{1}{\sin x}$	M1	For substituting their $\frac{d^2y}{dx^2}$ and y into D.E.
		M1	For using $\sin^2 x + \cos^2 x = 1$
	$\Rightarrow p - 2(p+q) \sin^2 x \equiv 1$	A1	6 For simplifying to AG (\equiv may be =)
(b)		M1	For attempting to find p and q by equating coefficients of constant and $\sin^2 x$ AND/OR giving value(s) to x (allow any value for x , including 0)
	$p = 1, q = -1$	A1	2 For both values correct
(iii)	G.S. $y = A \cos x + B \sin x + (\ln \sin x) \sin x - x \cos x$	B1 $\sqrt{}$	For correct G.S. f.t. from their C.F. and P.I. with 2 arbitrary constants in C.F. (allow given form of P.I. if p and q have not been found)
	$\operatorname{cosec} x$ undefined at $x = 0, \pi, 2\pi$	M1	For considering domain of $\operatorname{cosec} x$ OR $\sin x \neq 0$ OR $\ln \sin x$ term
	OR $\sin x > 0$ in $\ln \sin x$		
	$\Rightarrow 0 < x < \pi$	A1	3 For stating correct range CAO SR Award B1 for correct answer with justification omitted or incorrect

4727 Further Pure Mathematics 3

1 (i) (a)	$(n =) 3$	B1	1	For correct n
(b)	$(n =) 6$	B1	1	For correct n
(c)	$(n =) 4$	B1	1	For correct n
(ii)	$(n =) 4, 6$	B1		For <i>either</i> 4 or 6
		B1	2	For both 4 and 6 and no extras Ignore all $n \dots 8$ SR B0 B0 if more than 3 values given, even if they include 4 or 6
5				
2 (i)	$\frac{\sqrt{3}+i}{\sqrt{3}-i} \times \frac{\sqrt{3}+i}{\sqrt{3}+i} = \frac{1}{2} + \frac{1}{2}i\sqrt{3}$	M1		For multiplying top and bottom by complex conjugate
	$\text{OR } \frac{\sqrt{3}+i}{\sqrt{3}-i} = \frac{2e^{\frac{1}{6}\pi i}}{2e^{-\frac{1}{6}\pi i}}$			<i>OR</i> for changing top and bottom to polar form
	$= (1)e^{\frac{1}{3}\pi i}$	A1		For $(r =) 1$ (may be implied)
		A1	3	For $(\theta =) \frac{1}{3}\pi$ SR Award maximum A1 A0 if $e^{i\theta}$ form is not seen
(ii)	$\left(e^{\frac{1}{3}\pi i} \right)^6 = e^{2\pi i} = 1 \Rightarrow (n =) 6$	M1		For use of $e^{2\pi i} = 1$, $e^{i\pi} = -1$, $\sin k\pi = 0$ or $\cos k\pi = \pm 1$ (may be implied)
		A1	2	For $(n =) 6$ SR For $(n =) 3$ only, award M1 A0
5				
3 (i)	$\mathbf{n} = [2, 1, 3] \times [3, 1, 5]$ $= [2, -1, -1]$	M1		For using direction vectors and attempt to find vector product
		A1	2	For correct direction (allow multiples)
(ii)	$d = \frac{ [5, 2, 1] \cdot [2, -1, -1] }{\sqrt{6}}$	B1		For $(\mathbf{AB} =) [5, 2, 1]$ or any vector joining lines
		M1		For attempt at evaluating $\mathbf{AB} \cdot \mathbf{n}$
		M1		For $ \mathbf{n} $ in denominator
	$= \frac{7}{\sqrt{6}} = \frac{7}{6}\sqrt{6} = 2.8577$	A1	4	For correct distance
6				

4	$m^2 + 4m + 5 (= 0) \Rightarrow m = \frac{-4 \pm \sqrt{16 - 20}}{2}$	M1	For attempt to solve correct auxiliary equation
	$= -2 \pm i$	A1	For correct roots
	CF = $e^{-2x}(C \cos x + D \sin x)$	A1√	For correct CF (here or later). f.t. from m AEtrig but not forms including e^{ix}
	PI = $p \sin 2x + q \cos 2x$	B1	For stating a trial PI of the correct form
	$y' = 2p \cos 2x - 2q \sin 2x$	M1	For differentiating PI twice and substituting into the DE
	$y'' = -4p \sin 2x - 4q \cos 2x$		
	$\cos 2x(-4q + 8p + 5q)$	A1	For correct equation
	$+ \sin 2x(-4p - 8q + 5p) = 65 \sin 2x$		
	$\left. \begin{matrix} 8p + q = 0 \\ p - 8q = 65 \end{matrix} \right\} p = 1, q = -8$	M1	For equating coefficients of $\cos 2x$ and $\sin 2x$ and attempting to solve for p and/or q
	PI = $\sin 2x - 8 \cos 2x$	A1	For correct p and q
$\Rightarrow y = e^{-2x}(C \cos x + D \sin x) + \sin 2x - 8 \cos 2x$	B1√	For using GS = CF + PI, with 2 arbitrary constants in CF and none in PI	
9			

5 (i)	$y = u - \frac{1}{x} \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{1}{x^2}$	M1	For differentiating substitution
	$x^3 \left(\frac{du}{dx} + \frac{1}{x^2} \right) = x \left(u - \frac{1}{x} \right) + x + 1$	A1	For correct expression
	$\Rightarrow x^2 \frac{du}{dx} = u$	M1	For substituting y and $\frac{dy}{dx}$ into DE
		A1	4 For obtaining correct equation AG

(ii)	METHOD 1	M1	For separating variables and attempt at integration
	$\int \frac{1}{u} du = \int \frac{1}{x^2} dx \Rightarrow \ln ku = -\frac{1}{x}$	A1	For correct integration (k not required here)
	$ku = e^{-1/x} \Rightarrow k \left(y + \frac{1}{x} \right) = e^{-1/x}$	M1	For any 2 of $\left. \begin{matrix} k \text{ seen,} \\ \text{exponentiating,} \\ \text{substituting for } u \end{matrix} \right\}$
		M1	
$\Rightarrow y = Ae^{-1/x} - \frac{1}{x}$	A1	5 For correct solution AEF in form $y = f(x)$	

	METHOD 2		
	$\frac{du}{dx} - \frac{1}{x^2}u = 0 \Rightarrow \text{I.F. } e^{\int -1/x^2 dx} = e^{1/x}$	M1	For attempt to find I.F.
	$\Rightarrow \frac{d}{dx}(ue^{1/x}) = 0$	A1	For correct result
	$ue^{1/x} = k \Rightarrow y + \frac{1}{x} = ke^{-1/x}$	M1	From $\boxed{u \times \text{I.F.} =}$, for k seen for substituting for u } in either order
$\Rightarrow y = ke^{-1/x} - \frac{1}{x}$	M1		
	A1	For correct solution AEF in form $y = f(x)$	

9

6 (i)	METHOD 1		
	Use 2 of [−4, 2, 0], [0, 0, 3], [−4, 2, 3], [4, −2, 3] or multiples	M1	For finding vector product of 2 appropriate vectors in plane <i>ACGE</i>
	$\mathbf{n} = k [1, 2, 0]$	A1	For correct \mathbf{n}
	Use <i>A</i> [4, 0, 0], <i>C</i> [0, 2, 0], <i>G</i> [0, 2, 3] OR <i>E</i> [4, 0, 3]	M1	For substituting a point in the plane
	$\mathbf{r} \cdot [1, 2, 0] = 4$	A1	4 For correct equation. AEF in this form
(ii)	METHOD 2		
	$\mathbf{r} = [4, 0, 0] + \lambda[-4, 2, 0] + \mu[0, 0, 3]$	M1	For writing plane in 2-parameter form
	$\Rightarrow x = 4 - 4\lambda, y = 2\lambda, z = 3\mu$	A1	For 3 correct equations
	$x + 2y = 4$	M1	For eliminating λ (and μ)
	$\Rightarrow \mathbf{r} \cdot [1, 2, 0] = 4$	A1	For correct equation. AEF in this form
	$\theta = \cos^{-1} \frac{ [3, 0, -4] \cdot [1, 2, 0] }{\sqrt{3^2 + 0^2 + 4^2} \sqrt{1^2 + 2^2 + 0^2}}$	B1√	For using correct vectors (allow multiples). f.t. from \mathbf{n}
	$\theta = \cos^{-1} \frac{3}{5\sqrt{5}} = 74.4^\circ$ (74.435...°, 1.299...)	A1	4 For correct angle
(iii)	<i>AM</i> : $(\mathbf{r} =) [4, 0, 0] + t[-2, 2, 3]$ (or $[2, 2, 3] + t[-2, 2, 3]$)	M1	For obtaining parametric expression for <i>AM</i>
	$3(4 - 2t) - 4(3t) = 0$ (or $3(2 - 2t) - 4(3 + 3t) = 0$)	A1	For correct expression seen or implied
	$t = \frac{2}{3}$ (or $t = -\frac{1}{3}$) OR $\mathbf{w} = [\frac{8}{3}, \frac{4}{3}, 2]$	M1	For finding intersection of <i>AM</i> with <i>ACGE</i>
	<i>AW</i> : <i>WM</i> = 2 : 1	A1	For correct <i>t</i> OR position vector
		A1	5 For correct ratio
13			
7 (i) (a)	$x + y - a \in \mathbb{R}$	B1	For stating closure is satisfied
	$(x * y) * z = (x + y - a) * z = x + y + z - 2a$	M1	For using 3 distinct elements bracketed both ways
	$x * (y * z) = x * (y + z - a) = x + y + z - 2a$	A1	For obtaining the same result twice for associativity
	$x + e - a = x \Rightarrow e = a$	B1	SR 3 distinct elements bracketed once, expanded, and symmetry noted scores M1 A1 For stating identity = <i>a</i>
$x + x^{-1} - a = a \Rightarrow x^{-1} = 2a - x$	M1	For attempting to obtain inverse of <i>x</i>	
	A1	6 For obtaining inverse = $2a - x$ OR for showing that inverses exist, where $x + x^{-1} = 2a$	
(b)	$x + y - a = y + x - a \Rightarrow$ commutative	B1	1 For stating commutativity is satisfied, with justification
(c)	x order 2 $\Rightarrow x * x = e \Rightarrow 2x - a = e$	M1	For obtaining equation for an element of order 2
	$\Rightarrow 2x - a = a \Rightarrow x = a = e$ OR $x = x^{-1} \Rightarrow x = 2a - x \Rightarrow x = a = e$ \Rightarrow no elements of order 2	A1	2 For solving and showing that the only solution is the identity (which has order 1) OR For proving that there are no self-inverse elements (other than the identity)

(ii)	e.g. $2+1-5 = -2 \notin \mathbb{R}^+$	M1	For attempting to disprove closure
	\Rightarrow not closed	A1	For stating closure is not necessarily satisfied ($0 < x + y$, 5 required)
	e.g. $2 \times 5 - 11 = -1 \notin \mathbb{R}^+$	M1	For attempting to find an element with no inverse
	\Rightarrow no inverse	A1 4	For stating inverse is not necessarily satisfied ($x \dots 10$ required)

13

8 (i)	$\sin \theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$	B1	z may be used for $e^{i\theta}$ throughout For expression for $\sin \theta$ seen or implied
	$\sin^6 \theta =$	M1	For expanding $(e^{i\theta} - e^{-i\theta})^6$ At least 4 terms and 3 binomial coefficients required.
	$-\frac{1}{64}(e^{6i\theta} - 6e^{4i\theta} + 15e^{2i\theta} - 20 + 15e^{-2i\theta} - 6e^{-4i\theta} + e^{-6i\theta})$	A1	For correct expansion. Allow $\frac{\pm(i)}{64}(\dots)$
	$= -\frac{1}{64}(2 \cos 6\theta - 12 \cos 4\theta + 30 \cos 2\theta - 20)$	M1	For grouping terms and using multiple angles
	$\sin^6 \theta = -\frac{1}{32}(\cos 6\theta - 6 \cos 4\theta + 15 \cos 2\theta - 10)$	A1 5	For answer obtained correctly AG

(ii)	$\cos^6 \theta = \text{OR } \sin^6\left(\frac{1}{2}\pi - \theta\right) =$	M1	For substituting $\left(\frac{1}{2}\pi - \theta\right)$ for θ throughout
	$-\frac{1}{32}(\cos(3\pi - 6\theta) - 6 \cos(2\pi - 4\theta) + 15 \cos(\pi - 2\theta) - 10)$	A1	For correct unsimplified expression
	$\cos^6 \theta = \frac{1}{32}(\cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10)$	A1 3	For correct expression with $\cos n\theta$ terms AEF

(iii)	$\int_0^{\frac{1}{4}\pi} \frac{1}{32}(-2 \cos 6\theta - 30 \cos 2\theta) d\theta$	B1√	For correct integral. f.t. from $\sin^6 \theta - \cos^6 \theta$
	$= -\frac{1}{16} \left[\frac{1}{6} \sin 6\theta + \frac{15}{2} \sin 2\theta \right]_0^{\frac{1}{4}\pi}$	M1	For integrating $\cos n\theta, \sin n\theta$ or $e^{in\theta}$
	$= -\frac{11}{24}$	A1√ 4	For correct integration. f.t. from integrand For correct answer WWW

12

4727 Further Pure Mathematics 3

1	$\left(\frac{1}{2}\sqrt{3} + \frac{1}{2}i\right)^{\frac{1}{3}} = \left(\cos\frac{1}{6}\pi + i\sin\frac{1}{6}\pi\right)^{\frac{1}{3}}$	B1	For $\arg z = \frac{1}{6}\pi$ seen or implied
	$= \cos\frac{1}{18}\pi + i\sin\frac{1}{18}\pi,$	M1	For dividing $\arg z$ by 3
	$\cos\frac{13}{18}\pi + i\sin\frac{13}{18}\pi,$	A1	For any one correct root
	$\cos\frac{25}{18}\pi + i\sin\frac{25}{18}\pi$	A1 4	For 2 other roots and no more in range $0, \theta < 2\pi$
4			
2 (i)	$\frac{1}{5}e^{-\frac{1}{3}\pi i}$	B1 1	For stating correct inverse in the form $re^{i\theta}$
	(ii) $r_1e^{i\theta} \times r_2e^{i\phi} = r_1r_2e^{i(\theta+\phi)}$	M1	For stating 2 distinct elements multiplied
		A1 2	For showing product of correct form
(iii)	$Z^2 = e^{2i\gamma}$	B1	For $e^{2i\gamma}$ seen or implied
	$\Rightarrow e^{2i\gamma-2\pi i}$	B1 2	For correct answer. aef
5			
3 (i)	$[6-4\lambda, -7+8\lambda, -10+7\lambda]$ on l	B1	For point on l seen or implied
	$\Rightarrow 3(6-4\lambda) - 4(-7+8\lambda) - 2(-10+7\lambda) = 8$	M1	For substituting into equation of p
	$\Rightarrow \lambda = 1 \Rightarrow (2, 1, -3)$	A1 3	For correct point. Allow position vector
(ii)	METHOD 1		
	$\mathbf{n} = [-4, 8, 7] \times [3, -4, -2]$	M1*	For direction of l and normal of p seen
		M1	For attempting to find $\mathbf{n}_1 \times \mathbf{n}_2$
		(*dep)	
	$\mathbf{n} = k[12, 13, -8]$	A1	For correct vector
	$(2, 1, -3)$ OR $(6, -7, -10)$	M1	For finding scalar product of their point on l with their attempt at \mathbf{n} , or equivalent
	$\Rightarrow 12x + 13y - 8z = 61$	A1 5	For correct equation, aef cartesian
	METHOD 2		
	$\mathbf{r} = [2, 1, -3]$ OR $[6, -7, -10]$	M1	For stating eqn of plane in parametric form (may be implied by next stage), using $[2, 1, -3]$ (ft from
	$\quad + \lambda[-4, 8, 7] + \mu[3, -4, -2]$	A1√	(i) Or $[6, -7, -10]$, \mathbf{n}_1 and \mathbf{n}_2 (as above)
	$x = 2 - 4\lambda + 3\mu$	M1	For writing as 3 linear equations
	$y = 1 + 8\lambda - 4\mu$	M1	For attempting to eliminate λ and μ
	$z = -3 + 7\lambda - 2\mu$		
	$\Rightarrow 12x + 13y - 8z = 61$	A1	For correct equation aef cartesian
	METHOD 3		
	$3(6+3\mu) - 4(-7-4\mu) - 2(-10-2\mu) = 8$	M1	For finding foot of perpendicular from point on l to p
	$\Rightarrow \mu = -2 \Rightarrow (0, 1, -6)$	A1	For correct point or position vector
	From 3 points $(2, 1, -3)$, $(6, -7, -10)$, $(0, 1, -6)$,		
	$\mathbf{n} =$ vector product of 2 of		
	$[2, 0, 3]$, $[6, -8, -4]$, $[-4, 8, 7]$	M1	Use vector product of 2 vectors in plane
	$\Rightarrow \mathbf{n} = k[12, 13, -8]$		
	$(2, 1, -3)$ OR $(6, -7, -10)$	M1	For finding scalar product of their point on l with their attempt at \mathbf{n} , or equivalent
	$\Rightarrow 12x + 13y - 8z = 61$	A1	For correct equation aef cartesian
8			

4 (i)	IF $e^{\int \frac{1}{1-x^2} dx} = e^{\frac{1}{2} \ln \frac{1+x}{1-x}} = \left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$	M1	For IF stated or implied. Allow \pm and omission of
		A1	dx For integration and simplification to AG (intermediate step must be seen)
(ii)	$\frac{d}{dx} \left(y \left(\frac{1+x}{1-x} \right)^{\frac{1}{2}} \right) = (1+x)^{\frac{1}{2}}$	M1*	For multiplying both sides by IF
	$y \left(\frac{1+x}{1-x} \right)^{\frac{1}{2}} = \frac{2}{3} (1+x)^{\frac{3}{2}} + c$	M1	For integrating RHS to $k(1+x)^n$
		A1	For correct equation (including + c)
	$(0, 2) \Rightarrow 2 = \frac{2}{3} + c \Rightarrow c = \frac{4}{3}$	M1	In either order:
		(*dep)	For substituting (0, 2) into their GS (including +c)
		M1	For dividing solution through by IF, including dividing c or their numerical value for c
		(*dep)	
	$y = \frac{2}{3} (1+x) (1-x)^{\frac{1}{2}} + \frac{4}{3} \left(\frac{1-x}{1+x} \right)^{\frac{1}{2}}$	A1	For correct solution aef (even unsimplified) in form $y = f(x)$
8			
5 (i)	$m^2 - 6m + 9 (= 0) \Rightarrow m = 3$	M1	For attempting to solve correct auxiliary equation
		A1	For correct m
	CF = $(A + Bx)e^{3x}$	A1	For correct CF
(ii)	ke^{3x} and kxe^{3x} both appear in CF	B1	For correct statement
(iii)	$y = kx^2 e^{3x} \Rightarrow y' = 2kx e^{3x} + 3kx^2 e^{3x}$	M1	For differentiating $kx^2 e^{3x}$ twice
		A1	For correct y' aef
	$\Rightarrow y'' = 2ke^{3x} + 12kx e^{3x} + 9kx^2 e^{3x}$	A1	For correct y'' aef
	$\Rightarrow ke^{3x} (2 + 12x + 9x^2 - 12x - 18x^2 + 9x^2) = e^{3x}$	M1	For substituting y'', y', y into DE
	$\Rightarrow k = \frac{1}{2}$	A1	For correct k
9			

6 (i)	METHOD 1			
	$\mathbf{n}_1 = [1, 1, 0] \times [1, -5, -2]$	M1	For attempting to find vector product of the pair of direction vectors	
	$= [-2, 2, -6] = k[1, -1, 3]$	A1	For correct \mathbf{n}_1	
	Use $(2, 2, 1)$	M1	For substituting a point into equation	
	$\Rightarrow \mathbf{r} \cdot [-2, 2, -6] = -6 \Rightarrow \mathbf{r} \cdot [1, -1, 3] = 3$	A1	4 For correct equation. aef in this form	
METHOD 2				
	$x = 2 + \lambda + \mu$	M1	For writing as 3 linear equations	
	$y = 2 + \lambda - 5\mu$	M1	For attempting to eliminate λ and μ	
	$z = 1 - 2\mu$			
	$\Rightarrow x - y + 3z = 3$	A1	For correct cartesian equation	
	$\Rightarrow \mathbf{r} \cdot [1, -1, 3] = 3$	A1	For correct equation. aef in this form	
<hr/>				
(ii)	For $\mathbf{r} = \mathbf{a} + t\mathbf{b}$			
	METHOD 1			
	$\mathbf{b} = [1, -1, 3] \times [7, 17, -3]$	M1	For attempting to find $\mathbf{n}_1 \times \mathbf{n}_2$	
	$= k[2, -1, -1]$	A1√	For a correct vector. ft from \mathbf{n}_1 in (i)	
	e.g. x, y or $z = 0$ in $\begin{cases} x - y + 3z = 3 \\ 7x + 17y - 3z = 21 \end{cases}$	M1	For attempting to find a point on the line	
	$\Rightarrow \mathbf{a} = \left[0, \frac{3}{2}, \frac{3}{2}\right]$ OR $[3, 0, 0]$ OR $[1, 1, 1]$	A1√	For a correct vector. ft from equation in (i) SR a correct vector may be stated without working	
	Line is (e.g.) $\mathbf{r} = [1, 1, 1] + t[2, -1, -1]$	A1√	5 For stating equation of line ft from \mathbf{a} and \mathbf{b} SR for $\mathbf{a} = [2, 2, 1]$ stated award M0	
	METHOD 2			
	Solve $\begin{cases} x - y + 3z = 3 \\ 7x + 17y - 3z = 21 \end{cases}$	M1	In either order: For attempting to solve equations	
	by eliminating one variable (e.g. z)			
Use parameter for another variable (e.g. x) to find other variables in terms of t	M1	For attempting to find parametric solution		
(eg) $y = \frac{3}{2} - \frac{1}{2}t, z = \frac{3}{2} - \frac{1}{2}t$	A1√	For correct expression for one variable		
	A1√	For correct expression for the other variable ft from equation in (i) for both		
Line is (eg) $\mathbf{r} = \left[0, \frac{3}{2}, \frac{3}{2}\right] + t[2, -1, -1]$	A1√	For stating equation of line. ft from parametric solutions		
METHOD 3				
eg x, y or $z = 0$ in $\begin{cases} x - y + 3z = 3 \\ 7x + 17y - 3z = 21 \end{cases}$	M1	For attempting to find a point on the line		
$\Rightarrow \mathbf{a} = \left[0, \frac{3}{2}, \frac{3}{2}\right]$ OR $[3, 0, 0]$ OR $[1, 1, 1]$	A1√	For a correct vector. ft from equation in (i) SR a correct vector may be stated without working SR for $\mathbf{a} = [2, 2, 1]$ stated award M0		
eg $[3, 0, 0] - [1, 1, 1]$	M1	For finding another point on the line and using it with the one already found to find \mathbf{b}		
$\mathbf{b} = k[2, -1, -1]$	A1√	For a correct vector. ft from equation in (i)		
Line is (eg) $\mathbf{r} = [1, 1, 1] + t[2, -1, -1]$	A1√	For stating equation of line. ft from \mathbf{a} and \mathbf{b}		

6 (ii) contd	METHOD 4		
A point on Π_1 is [2 + λ + μ , 2 + λ - 5 μ , 1 - 2 μ]	M1	For using parametric form for Π_1 and substituting into Π_2	
On $\Pi_2 \Rightarrow$ [2 + λ + μ , 2 + λ - 5 μ , 1 - 2 μ] · [7, 17, -3] = 21	A1	For correct unsimplified equation	
$\Rightarrow \lambda - 3\mu = -1$	A1	For correct equation	
Line is (e.g.) $\mathbf{r} = [2, 2, 1] + (3\mu - 1)[1, 1, 0] + \mu[1, -5, -2]$	M1	For substituting into Π_1 for λ or μ	
$\Rightarrow \mathbf{r} = [1, 1, 1]$ or $\left[\frac{7}{3}, \frac{1}{3}, \frac{1}{3}\right] + t[2, -1, -1]$	A1	For stating equation of line	

9

7 (i)	$\cos 3\theta + i \sin 3\theta = c^3 + 3ic^2s - 3cs^2 - is^3$	M1	For using de Moivre with $n = 3$
	$\Rightarrow \cos 3\theta = c^3 - 3cs^2$ and $\sin 3\theta = 3c^2s - s^3$	A1	For both expressions in this form (seen or implied) SR For expressions found without de Moivre M0 A0
	$\Rightarrow \tan 3\theta = \frac{3c^2s - s^3}{c^3 - 3cs^2}$	M1	For expressing $\frac{\sin 3\theta}{\cos 3\theta}$ in terms of c and s
	$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} = \frac{\tan \theta (3 - \tan^2 \theta)}{1 - 3 \tan^2 \theta}$	A1	4 For simplifying to AG

(ii) (a) $\theta = \frac{1}{12}\pi \Rightarrow \tan 3\theta = 1$

$\Rightarrow 1 - 3t^2 = t(3 - t^2) \Rightarrow$

$t^3 - 3t^2 - 3t + 1 = 0$

B1 **1** For both stages correct **AG**

(b) $(t+1)(t^2 - 4t + 1) = 0$

M1 For attempt to factorise cubic
A1 For correct factors

$\Rightarrow (t = -1), t = 2 \pm \sqrt{3}$

A1 For correct roots of quadratic

- sign for smaller root \Rightarrow

A1 **4** For choice of - sign and correct root **AG**

$\tan \frac{1}{12}\pi = 2 - \sqrt{3}$

(iii)

$dt = (1 + t^2) d\theta$

B1 For differentiation of substitution
and use of $\sec^2 \theta = 1 + \tan^2 \theta$

$\Rightarrow \int_0^{\frac{1}{12}\pi} \tan 3\theta d\theta$

B1 For integral with correct θ limits seen

$= \left[\frac{1}{3} \ln(\sec 3\theta) \right]_0^{\frac{1}{12}\pi} = \frac{1}{3} \ln(\sec \frac{1}{4}\pi)$

M1 For integrating to $k \ln(\sec 3\theta)$ OR $k \ln(\cos 3\theta)$

$= \frac{1}{3} \ln \sqrt{2} = \frac{1}{6} \ln 2$

M1 For substituting limits
and $\sec \frac{1}{4}\pi = \sqrt{2}$ OR $\cos \frac{1}{4}\pi = \frac{1}{\sqrt{2}}$ seen

A1 **5** For correct answer aef

14

8 (i)	$a^2 = (ap)^2 = apap \Rightarrow a = pap$	B1	For use of given properties to obtain AG																									
	$p^2 = (ap)^2 = apap \Rightarrow p = apa$	B1 2	For use of given properties to obtain AG SR allow working from AG to obtain relevant properties																									
(ii)	$(p^2)^2 = p^4 = e \Rightarrow \text{order } p^2 = 2$	B1	For correct order with no incorrect working seen																									
	$(a^2)^2 = (p^2)^2 = e \Rightarrow \text{order } a = 4$	B1	For correct order with no incorrect working seen																									
	$(ap)^4 = a^4 = e \Rightarrow \text{order } ap = 4$	B1	For correct order with no incorrect working seen																									
	$(ap^2)^2 = ap^2ap^2 = ap \cdot a \cdot p = a^2$	M1	For relevant use of (i) or given properties																									
	OR $ap^2 = a \cdot a^2 = a^3 \Rightarrow$ $(ap^2)^2 = a^6 = a^2$ $\Rightarrow \text{order } ap^2 = 4$	A1 5	For correct order with no incorrect working seen																									
(iii)	METHOD 1 $p^2 = a^2, ap^2 = a^3$	M2	For use of the given properties to simplify p^2 and ap^2																									
	$\Rightarrow \{e, a, p^2, ap^2\} = \{e, a, a^2, a^3\}$	A1	For obtaining a^2 and a^3																									
	which is a cyclic group	A1 4	For justifying that the set is a group																									
	METHOD 2																											
	<table border="1"> <thead> <tr> <th></th> <th>e</th> <th>a</th> <th>p^2</th> <th>ap^2</th> </tr> </thead> <tbody> <tr> <th>e</th> <td>e</td> <td>a</td> <td>p^2</td> <td>ap^2</td> </tr> <tr> <th>a</th> <td>a</td> <td>p^2</td> <td>ap^2</td> <td>e</td> </tr> <tr> <th>p^2</th> <td>p^2</td> <td>ap^2</td> <td>e</td> <td>a</td> </tr> <tr> <th>ap^2</th> <td>ap^2</td> <td>e</td> <td>a</td> <td>p^2</td> </tr> </tbody> </table>		e	a	p^2	ap^2	e	e	a	p^2	ap^2	a	a	p^2	ap^2	e	p^2	p^2	ap^2	e	a	ap^2	ap^2	e	a	p^2	M1	For attempting closure with all 9 non-trivial products seen
	e	a	p^2	ap^2																								
e	e	a	p^2	ap^2																								
a	a	p^2	ap^2	e																								
p^2	p^2	ap^2	e	a																								
ap^2	ap^2	e	a	p^2																								
		A1	For all 16 products correct																									
	Completed table is a cyclic group	B2	For justifying that the set is a group																									
	METHOD 3																											
	<table border="1"> <thead> <tr> <th></th> <th>e</th> <th>a</th> <th>p^2</th> <th>ap^2</th> </tr> </thead> <tbody> <tr> <th>e</th> <td>e</td> <td>a</td> <td>p^2</td> <td>ap^2</td> </tr> <tr> <th>a</th> <td>a</td> <td>p^2</td> <td>ap^2</td> <td>e</td> </tr> <tr> <th>p^2</th> <td>p^2</td> <td>ap^2</td> <td>e</td> <td>a</td> </tr> <tr> <th>ap^2</th> <td>ap^2</td> <td>e</td> <td>a</td> <td>p^2</td> </tr> </tbody> </table>		e	a	p^2	ap^2	e	e	a	p^2	ap^2	a	a	p^2	ap^2	e	p^2	p^2	ap^2	e	a	ap^2	ap^2	e	a	p^2	M1	For attempting closure with all 9 non-trivial products seen
	e	a	p^2	ap^2																								
e	e	a	p^2	ap^2																								
a	a	p^2	ap^2	e																								
p^2	p^2	ap^2	e	a																								
ap^2	ap^2	e	a	p^2																								
		A1	For all 16 products correct																									
	Identity = e	B1	For stating identity																									
	Inverses exist since EITHER: e is in each row/column OR: p^2 is self-inverse; a, ap^2 form an inverse pair	B1	For justifying inverses ($e^{-1} = e$ may be assumed)																									

(iv)	METHOD 1	M1	For attempting to find a non-commutative pair of elements, at least one involving a (may be embedded in a full or partial table)
	e.g. $a \cdot ap = a^2 p = p^3$	M1	For simplifying elements both ways round
	$ap \cdot a = p$	B1	For a correct pair of non-commutative elements
	commutative	A1	For stating Q non-commutative, with a clear argument
		4	
	METHOD 2		
	Assume commutativity, so (eg) $ap = pa$	M1	For setting up proof by contradiction
	(i) \Rightarrow		
	$p = ap \cdot a \Rightarrow p = pa \cdot a = pa^2 = pp^2 = p^3$	M1	For using (i) and/or given properties
	But p and p^3 are distinct	B1	For obtaining and stating a contradiction
	$\Rightarrow Q$ is non-commutative	A1	For stating Q non-commutative, with a clear argument
		15	

4727 Further Pure Mathematics 3

1	METHOD 1 line segment between l_1 and $l_2 = \pm[4, -3, -9]$ $\mathbf{n} = [1, -1, 2] \times [2, 3, 4] = (\pm)[-2, 0, 1]$ distance = $\frac{ [4, -3, -9] \cdot [-2, 0, 1] }{(\sqrt{2^2 + 0^2 + 1^2})} = \frac{17}{(\sqrt{5})}$ $\neq 0$, so skew	B1 M1* A1 M1 (*dep) A1	For correct vector For finding vector product of direction vectors For using numerator of distance formula For correct scalar product and correct conclusion
	METHOD 2 lines would intersect where $\begin{cases} 1 + s = -3 + 2t \\ -2 - s = 1 + 3t \\ -4 + 2s = 5 + 4t \end{cases} \Rightarrow \begin{cases} s - 2t = -4 \\ s + 3t = -3 \\ 2s - 4t = 9 \end{cases}$ \Rightarrow contradiction, so skew	B1 M1* A1 M1 (*dep) A1	For correct parametric form for either line For 3 equations using 2 different parameters For attempting to solve to show (in)consistency For correct conclusion
5			
2 (i)	$(a + b\sqrt{5})(c + d\sqrt{5})$ $= ac + 5bd + (bc + ad)\sqrt{5} \in H$	M1 A1	For using product of 2 distinct elements For correct expression
(ii)	$(e =) 1 \text{ OR } 1 + 0\sqrt{5}$	B1	For correct identity
(iii)	<i>EITHER</i> $\frac{1}{a + b\sqrt{5}} \times \frac{a - b\sqrt{5}}{a - b\sqrt{5}}$ <i>OR</i> $(a + b\sqrt{5})(c + d\sqrt{5}) = 1 \Rightarrow \begin{cases} ac + 5bd = 1 \\ bc + ad = 0 \end{cases}$ inverse = $\frac{a}{a^2 - 5b^2} - \frac{b}{a^2 - 5b^2}\sqrt{5}$	M1 A1	For correct inverse as $(a + b\sqrt{5})^{-1}$ and multiplying top and bottom by $a - b\sqrt{5}$ <i>OR</i> for using definition and equating parts For correct inverse. Allow as a single fraction
(iv)	5 is prime <i>OR</i> $\sqrt{5} \notin \mathbb{Q}$	B1	For a correct property (or equivalent)
6			
3	Integrating factor = $e^{\int 2dx} = e^{2x}$ $\Rightarrow \frac{d}{dx}(ye^{2x}) = e^{-x}$ $\Rightarrow ye^{2x} = -e^{-x} + c$ $(0, 1) \Rightarrow c = 2$ $\Rightarrow y = -e^{-3x} + 2e^{-2x}$	B1 M1 A1 M1 A1 A1	For correct IF For $\frac{d}{dx}(y \cdot \text{their IF}) = e^{-3x}$. their IF For correct integration both sides For substituting (0, 1) into their GS and solving for c For correct c f.t. from their GS For correct solution
6			
4 (i)	$(z =) 2, -2, 2i, -2i$	M1 A1	For at least 2 roots of the form $k\{1, i\}$ AEF For correct values

<p>(ii) $\frac{w}{1-w} = 2, -2, 2i, -2i$</p> $w = \frac{z}{1+z}$ $w = \frac{2}{3}, 2$ $w = \frac{4}{5} \pm \frac{2}{5}i$	<p>M1 M1 B1 A1 A1 5</p>	<p>For $\frac{w}{1-w} =$ any one solution from (i)</p> <p>For attempting to solve for w, using any solution or in general</p> <p>For any one of the 4 solutions</p> <p>For both real solutions</p> <p>For both complex solutions</p> <p>SR Allow B1√ and one A1√ from $k \neq 2$</p>
7		
<p>5 (i) $\mathbf{AB} = k\left[\frac{2}{3}\sqrt{3}, 0, -\frac{2}{3}\sqrt{6}\right],$ $\mathbf{BC} = k\left[-\sqrt{3}, 1, 0\right], \mathbf{CA} = k\left[\frac{1}{3}\sqrt{3}, -1, \frac{2}{3}\sqrt{6}\right]$ $\mathbf{n} = k_1\left[\frac{2}{3}\sqrt{6}, \frac{2}{3}\sqrt{18}, \frac{2}{3}\sqrt{3}\right] = k_2\left[1, \sqrt{3}, \frac{1}{2}\sqrt{2}\right]$ substitute A, B or $C \Rightarrow x + \sqrt{3}y + \frac{1}{2}\sqrt{2}z = \frac{2}{3}\sqrt{3}$</p>	<p>B1 B1 M1 M1 A1 5</p>	<p>For any one edge vector of $\triangle ABC$</p> <p>For any other edge vector of $\triangle ABC$</p> <p>For attempting to find vector product of any two edges</p> <p>For substituting A, B or C into $\mathbf{r} \cdot \mathbf{n}$</p> <p>For correct equation AG</p> <p>SR For verification only allow M1, then A1 for 2 points and A1 for the third point</p>
<p>(ii) Symmetry in plane OAB or Oxz or $y = 0$</p>	<p>B1* B1 (*dep)2</p>	<p>For quoting symmetry or reflection</p> <p>For correct plane</p> <p>Allow “in y coordinates” or “in y axis”</p> <p>SR For symmetry implied by reference to opposite signs in y coordinates of C and D, award B1 only</p>
<p>(iii) $\cos \theta = \frac{\left[1, \sqrt{3}, \frac{1}{2}\sqrt{2}\right] \cdot \left[1, -\sqrt{3}, \frac{1}{2}\sqrt{2}\right]}{\sqrt{1+3+\frac{1}{2}}\sqrt{1+3+\frac{1}{2}}}$ $= \frac{\left 1-3+\frac{1}{2}\right }{\frac{9}{2}} = \frac{\frac{3}{2}}{\frac{9}{2}} = \frac{1}{3}$</p>	<p>M1 A1 M1 A1 4</p>	<p>For using scalar product of normal vectors</p> <p>For correct scalar product</p> <p>For product of both moduli in denominator</p> <p>For correct answer. Allow $-\frac{1}{3}$</p>
11		
<p>6 (i) $(m^2 + 16 = 0 \Rightarrow) m = \pm 4i$</p> $\text{CF} = A \cos 4x + B \sin 4x$	<p>M1 A1 2</p>	<p>For attempt to solve correct auxiliary equation (may be implied by correct CF)</p> <p>For correct CF</p> <p>(AEtrig but not $Ae^{4ix} + Be^{-4ix}$ only)</p>
<p>(ii) $\frac{dy}{dx} = p \sin 4x + 4px \cos 4x$</p> $\frac{d^2y}{dx^2} = 8p \cos 4x - 16px \sin 4x$ $\Rightarrow 8p \cos 4x = 8 \cos 4x$ $\Rightarrow p = 1$ $\Rightarrow (y =) A \cos 4x + B \sin 4x + x \sin 4x$	<p>M1 A1 A1√ M1 A1 B1√ 6</p>	<p>For differentiating PI twice, using product rule</p> <p>For correct $\frac{dy}{dx}$</p> <p>For unsimplified $\frac{d^2y}{dx^2}$. f.t. from $\frac{dy}{dx}$</p> <p>For substituting into DE</p> <p>For correct p</p> <p>For using $\text{GS} = \text{CF} + \text{PI}$, with 2 arbitrary constants in CF and none in PI</p>

(iii)	$(0, 2) \Rightarrow A = 2$	B1√	For correct A. f.t. from their GS
	$\frac{dy}{dx} = -4A \sin 4x + 4B \cos 4x + \sin 4x + 4x \cos 4x$	M1	For differentiating their GS
	$x = 0, \frac{dy}{dx} = 0 \Rightarrow B = 0$	M1	For substituting values for x and $\frac{dy}{dx}$
	$\Rightarrow y = 2 \cos 4x + x \sin 4x$	A1 4	to find B For stating correct solution CAO including $y =$
12			
7 (i)	$\cos 6\theta = 0 \Rightarrow 6\theta = k \times \frac{1}{2}\pi$	M1	For multiples of $\frac{1}{2}\pi$ seen or implied
	$\Rightarrow \theta = \frac{1}{12}\pi \{1, 3, 5, 7, 9, 11\}$	A1	A1 for any 3 correct
		A1 3	A1 for the rest, and no extras in $0 < \theta < \pi$
(ii)	METHOD 1		
	$\operatorname{Re}(c + is)^6 = \cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	M1	For expanding $(c + is)^6$ at least 4 terms and 2 binomial coefficients needed
		A1	For 4 correct terms
	$\cos 6\theta = c^6 - 15c^4(1 - c^2) + 15c^2(1 - c^2)^2 - (1 - c^2)^3$	M1	For using $s^2 = 1 - c^2$
	$\Rightarrow \cos 6\theta = 32c^6 - 48c^4 + 18c^2 - 1$	A1	For correct expression for $\cos 6\theta$
	$\Rightarrow \cos 6\theta = (2c^2 - 1)(16c^4 - 16c^2 + 1)$	A1 5	For correct result AG (may be written down from correct $\cos 6\theta$)
	METHOD 2		
	$\operatorname{Re}(c + is)^3 = \cos 3\theta = \cos^3 \theta - 3\cos \theta \sin^2 \theta$	M1	For expanding $(c + is)^3$ at least 2 terms and 1 binomial coefficient needed
		A1	For 2 correct terms
	$\Rightarrow \cos 6\theta = \cos 2\theta (\cos^2 2\theta - 3\sin^2 2\theta)$	M1	For replacing θ by 2θ
	$\Rightarrow \cos 6\theta = (2\cos^2 \theta - 1) \left(4(2\cos^2 \theta - 1)^2 - 3 \right)$	A1	For correct expression in $\cos \theta$ (unsimplified)
	$\Rightarrow \cos 6\theta = (2c^2 - 1)(16c^4 - 16c^2 + 1)$	A1	For correct result AG
(iii)	METHOD 1		
	$\cos 6\theta = 0$	M1	For putting $\cos 6\theta = 0$
	$\Rightarrow 6$ roots of $\cos 6\theta = 0$ satisfy	A1	For association of roots with quartic and quadratic
	$16c^4 - 16c^2 + 1 = 0$ and $2c^2 - 1 = 0$	B1	For correct association of roots with quadratic
	But $\theta = \frac{1}{4}\pi, \frac{3}{4}\pi$ satisfy $2c^2 - 1 = 0$	B1	For correct association of roots with quadratic
	<i>EITHER</i> Product of 4 roots <i>OR</i> $c = \pm \frac{1}{2}\sqrt{2 \pm \sqrt{3}}$	M1	For using product of 4 roots <i>OR</i> for solving quartic
	$\Rightarrow \cos \frac{1}{12}\pi \cos \frac{5}{12}\pi \cos \frac{7}{12}\pi \cos \frac{11}{12}\pi = \frac{1}{16}$	A1 5	For correct value (may follow A0 and B0)

METHOD 2

$\cos 6\theta = 0$	M1	For putting $\cos 6\theta = 0$
$\Rightarrow 6$ roots of $\cos 6\theta = 0$ satisfy	A1	For association of roots with sextic
$32c^6 - 48c^4 + 18c^2 - 1 = 0$		
Product of 6 roots \Rightarrow	M1	For using product of 6 roots
$\cos \frac{1}{12}\pi \cdot \frac{1}{\sqrt{2}} \cdot \cos \frac{5}{12}\pi \cos \frac{7}{12}\pi \cdot \frac{-1}{\sqrt{2}} \cdot \cos \frac{11}{12}\pi = -\frac{1}{32}$	B1	For using $\cos\left\{\frac{3}{12}\pi, \frac{9}{12}\pi\right\} = \left\{\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right\}$
$\cos \frac{1}{12}\pi \cos \frac{5}{12}\pi \cos \frac{7}{12}\pi \cos \frac{11}{12}\pi = \frac{1}{16}$	A1	For correct value

13

8 (i)	$g(x) = \frac{1}{2-2 \cdot \frac{1}{2-2x}} = \frac{2-2x}{2-4x} = \frac{1-x}{1-2x}$	M1	For use of $f f(x)$
		A1	For correct expression AG

	$gg(x) = \frac{1 - \frac{1-x}{1-2x}}{1 - 2 \cdot \frac{1-x}{1-2x}} = \frac{-x}{-1} = x$	M1	For use of $gg(x)$
		A1 4	For correct expression AG

(ii)	Order of $f = 4$	B1	For correct order
	order of $g = 2$	B1 2	For correct order

(iii) METHOD 1

	$y = \frac{1}{2-2x} \Rightarrow x = \frac{2y-1}{2y}$	M1	For attempt to find inverse
	$\Rightarrow f^{-1}(x) = h(x) = \frac{2x-1}{2x}$ OR $1 - \frac{1}{2x}$	A1 2	For correct expression

METHOD 2

	$f^{-1} = f^3 = f g$ or $g f$	M1	For use of $f g(x)$ or $g f(x)$
	$f g(x) = h(x) = \frac{1}{2-2\left(\frac{1-x}{1-2x}\right)} = \frac{1-2x}{-2x}$	A1	For correct expression

(iv)

	<table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td>e</td><td>f</td><td>g</td><td>h</td></tr> <tr><td>e</td><td>f</td><td>g</td><td>h</td></tr> <tr><td>f</td><td>g</td><td>h</td><td>e</td></tr> <tr><td>g</td><td>h</td><td>e</td><td>f</td></tr> <tr><td>h</td><td>e</td><td>f</td><td>g</td></tr> </table>	e	f	g	h	e	f	g	h	f	g	h	e	g	h	e	f	h	e	f	g	M1	For correct row 1 and column 1
e	f	g	h																				
e	f	g	h																				
f	g	h	e																				
g	h	e	f																				
h	e	f	g																				
		A1	For e, f, g, h in a latin square																				
		A1	For correct diagonal e - g - e - g																				
		A1 4	For correct table																				

12

1	Direction of $l_1 = k[7, 0, -10]$ } Direction of $l_2 = k[1, 3, -1]$ }	B1	For both directions
	<i>EITHER</i> $\mathbf{n} = [7, 0, -10] \times [1, 3, -1]$	M1	For finding vector product of directions of l_1 and l_2
	<i>OR</i> $\begin{cases} [x, y, z] \cdot [7, 0, -10] = 0 \Rightarrow 7x - 10z = 0 \\ [x, y, z] \cdot [1, 3, -1] = 0 \Rightarrow x + 3y - z = 0 \end{cases}$ $\Rightarrow \mathbf{n} = k[10, -1, 7]$	A1	<i>OR</i> for using 2 scalar products and obtaining equations For correct \mathbf{n}
METHOD 1			
Vector $(\mathbf{a} - \mathbf{b})$ from l_1 to $l_2 = \pm[4, 6, -10]$	B1	For a correct vector	
<i>OR</i> $\pm[-4, 3, 1]$ <i>OR</i> $\pm[3, 3, -9]$ <i>OR</i> $\pm[-3, 6, 0]$	M1*	For finding $(\mathbf{a} - \mathbf{b}) \cdot \mathbf{n}$	
$d = \frac{ (\mathbf{a} - \mathbf{b}) \cdot \mathbf{n} }{ \mathbf{n} } = \frac{36}{\sqrt{150}}$	M1 (*dep)	For $ \mathbf{n} $ in denominator <i>OR</i> for using $\hat{\mathbf{n}}$	
$d = \frac{6}{5}\sqrt{6} \approx 2.94$	A1	7 For correct distance AEF	
METHOD 2 Planes containing l_1 and l_2 perp. to \mathbf{n}			
are $\mathbf{r} \cdot [10, -1, 7] = p_1 = 70$, $\mathbf{r} \cdot [10, -1, 7] = p_2 = 34$	M1*	For finding planes and $p_1 - p_2$ seen	
$\Rightarrow d = \frac{ 70 - 34 }{\sqrt{150}} = \frac{36}{\sqrt{150}} = \frac{6}{5}\sqrt{6} \approx 2.94$	B1	For $p_1 = 70k$ and $p_2 = 34k$	
	M1 (*dep)	For $ \mathbf{n} $ in denominator <i>OR</i> for using $\hat{\mathbf{n}}$	
	A1	For correct distance AEF	
METHOD 3			
$\mathbf{r}_1 = [7\lambda, 0, 10 - 10\lambda]$ <i>OR</i> $[7 + 7\lambda, 0, -10\lambda]$	B1	For correct points on l_1 and l_2 using different parameters	
$\mathbf{r}_2 = [4 + \mu, 6 + 3\mu, -\mu]$ <i>OR</i> $[3 + \mu, 3 + 3\mu, 1 - \mu]$	M1*	For setting up 3 linear equations from $\mathbf{r}_1 + \alpha\mathbf{n} = \mathbf{r}_2$ and solving for α	
$\begin{array}{l} 7\lambda + 10\alpha - \mu = \begin{vmatrix} 4 & -3 & 3 & -4 \\ -\alpha - 3\mu & 6 & 3 & 3 \\ -10\lambda + 7\alpha + \mu & -10 & 0 & -9 & 1 \end{vmatrix} \\ \Rightarrow \alpha = -\frac{6}{25} \end{array}$	M1 (*dep)	For $ \mathbf{n} $ seen multiplying α	
$ \mathbf{n} = \sqrt{150}$	A1	For correct distance AEF	
$\Rightarrow d = \frac{6}{25}\sqrt{150} = \frac{6}{5}\sqrt{6} \approx 2.94$			
7			

2 (i)	$ar = r^5a \Rightarrow rar = r^6a$ $r^6 = e \Rightarrow rar = a$	M1 A1	Pre-multiply $ar = r^5a$ by r 2 Use $r^6 = e$ and obtain answer AG
(ii)	METHOD 1 For $n = 1$, $rar = a$ OR For $n = 0$, $r^0 ar^0 = a$ Assume $r^k ar^k = a$ <i>EITHER</i> Assumption $\Rightarrow r^{k+1} ar^{k+1} = rar = a$ OR $r^{k+1} ar^{k+1} = r \cdot r^k ar^k \cdot r = rar = a$ OR $r^{k+1} ar^{k+1} = r^k \cdot rar \cdot r^k = r^k ar^k = a$ Hence true for all $n \in \mathbb{Z}^+$	B1 M1 A1 A1	For stating true for $n = 1$ OR for $n = 0$ For attempt to prove true for $k + 1$ For obtaining correct form 4 For statement of induction conclusion
	METHOD 2 $r^2 ar^2 = r \cdot rar \cdot r = rar = a$, similarly for $r^3 ar^3 = a$ $r^4 ar^4 = r \cdot r^3 ar^3 \cdot r = rar = a$, similarly for $r^5 ar^5 = a$ $r^6 ar^6 = eae = a$ For $n > 6$, $r^n = r^{n \bmod 6}$, hence true for all $n \in \mathbb{Z}^+$	M1 A1 B1 A1	For attempt to prove for $n = 2, 3$ For proving true for $n = 2, 3, 4, 5$ For showing true for $n = 6$ For using $n \bmod 6$ and correct conclusion
	METHOD 3 $r^n ar^n = r^{n-1} \cdot rar \cdot r^{n-1}$ OR $r^n ar^n = r^n \cdot r^5 ar^{n-1} = r^{n+5} ar^{n-1}$ $= r^{n-1} ar^{n-1}$ $= r^{n-2} ar^{n-2} = \dots$ $= rar = a$	M1 A1 A1 B1	Starting from n , for attempt to prove true for $n - 1$ For proving true for $n - 1$ For continuation from $n - 2$ downwards For final use of $rar = a$ SR can be done in reverse
	METHOD 4 $ar = r^5a \Rightarrow ar^2 = r^5 ar = r^{10}a$ etc. $\Rightarrow ar^n = r^{5n}a$ $\Rightarrow r^n ar^n = r^{6n}a$ $= ea = a$	M1 A1 B1 A1	For attempt to derive $ar^n = r^{5n}a$ For correct equation SR may be stated without proof For pre-multiplication by r^n For obtaining a ($r^6 = e$ may be implied)

3

(i) $w^2 = \cos \frac{4}{5}\pi + i \sin \frac{4}{5}\pi$
 $w^3 = \cos \frac{6}{5}\pi + i \sin \frac{6}{5}\pi$
 $w^* = \cos \frac{2}{5}\pi - i \sin \frac{2}{5}\pi$
 $= \cos \frac{8}{5}\pi + i \sin \frac{8}{5}\pi$

Allow $\text{cis } \frac{k}{5}\pi$ and $e^{\frac{k}{5}\pi i}$ throughout

B1 For correct value

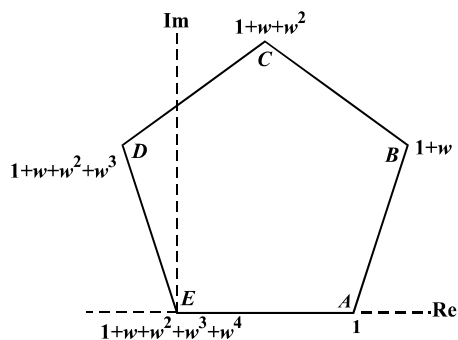
B1 For correct value

B1 For w^* seen or implied

B1 4 For correct value

SR For exponential form with i missing, award B0 first time, allow others

(ii)

B1* For $1+w$ in approximately correct positionB1 For $AB \approx BC \approx CD$

(*dep)

B1 For BC, CD equally inclined to Im axis

(*dep)

B1 4 For E at the originAllow points joined by arcs, or not joined
Labels not essential

(iii) $z^5 - 1 = 0$ OR $z^5 + z^4 + z^3 + z^2 + z = 0$

B1 1 For correct equation **AEF** (in any variable)
Allow factorised forms using w , \exp or trig

9

4 (i)

$$y = xz \Rightarrow \frac{dy}{dx} = z + x \frac{dz}{dx}$$

B1 For correct differentiation of substitution

$$\Rightarrow xz + x^2 \frac{dz}{dx} - xz = x \cos z \Rightarrow x \frac{dz}{dx} = \cos z$$

M1 For substituting into DE

A1 For DE in variables separable form

$$\Rightarrow \int \sec z \, dz = \int \frac{1}{x} \, dx$$

M1 For attempt at integration
to \ln form on LHS

$$\Rightarrow \ln(\sec z + \tan z) = \ln kx$$

A1

For correct integration (k not required here)

$$\text{OR } \ln \tan\left(\frac{1}{2}z + \frac{1}{4}\pi\right) = \ln kx$$

$$\Rightarrow \sec\left(\frac{y}{x}\right) + \tan\left(\frac{y}{x}\right) = kx$$

A1

6 For correct solution

AEF including $\text{RHS} = e^{(\ln x)+c}$

$$\text{OR } \tan\left(\frac{y}{2x} + \frac{1}{4}\pi\right) = kx$$

(ii) $(4, \pi) \Rightarrow \sec \frac{1}{4}\pi + \tan \frac{1}{4}\pi = 4k$

M1

For substituting $(4, \pi)$
into their solution (with k)

$$\text{OR } \tan\left(\frac{1}{8}\pi + \frac{1}{4}\pi\right) = 4k$$

$$\Rightarrow \sec\left(\frac{y}{x}\right) + \tan\left(\frac{y}{x}\right) = \frac{1}{4}(1+\sqrt{2})x$$

A1

2 For correct solution **AEF**Allow decimal equivalent 0.60355 x

$$\text{OR } \tan\left(\frac{y}{2x} + \frac{1}{4}\pi\right) = \left(\frac{1}{4} \tan \frac{3}{8}\pi\right)x \text{ or } \frac{1}{4}(1+\sqrt{2})x$$

Allow $e^{\ln x}$ for x

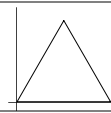

8

5 (i)	$C + iS = 1 + \frac{1}{2}e^{i\theta} + \frac{1}{4}e^{2i\theta} + \frac{1}{8}e^{3i\theta} + \dots$ $= \frac{1}{1 - \frac{1}{2}e^{i\theta}} = \frac{2}{2 - e^{i\theta}}$	M1	For using $\cos n\theta + i \sin n\theta = e^{in\theta}$ at least once for $n \geq 2$
		A1	For correct series
		M1	For using sum of infinite GP
		A1	4 For correct expression AG SR For omission of 1st stage award up to M0 A0 M1 A1 OEW
(ii)	$C + iS = \frac{2(2 - e^{-i\theta})}{(2 - e^{i\theta})(2 - e^{-i\theta})}$ $= \frac{4 - 2e^{-i\theta}}{4 - 2(e^{i\theta} + e^{-i\theta}) + 1} = \frac{4 - 2\cos\theta + 2i\sin\theta}{4 - 4\cos\theta + 1}$ $\Rightarrow C = \frac{4 - 2\cos\theta}{5 - 4\cos\theta}, \quad S = \frac{2\sin\theta}{5 - 4\cos\theta}$	M1	For multiplying top and bottom by complex conjugate
		M1	For reverting to $\cos\theta$ and $\sin\theta$ and equating Re <i>OR</i> Im parts
		A1	For correct expression for C AG
		A1	4 For correct expression for S
8			
6 (i)	<p>Aux. equation $m^2 + 2m + 17 = 0$</p> $\Rightarrow m = -1 \pm 4i$ <p>CF $(y =) e^{-x} (A \cos 4x + B \sin 4x)$</p>	M1	For attempting to solve correct auxiliary equation
		A1	For correct roots
		A1√	For correct CF (allow $A \frac{\cos}{\sin}(4x + \varepsilon)$) (trig terms required, not $e^{\pm 4ix}$) f.t. from their m with 2 arbitrary constants
	PI $(y =) px + q \Rightarrow 2p + 17(px + q) = 17x + 36$	M1	For stating and substituting PI of correct form
	$\Rightarrow p = 1$	A1	For correct value of p
	and $q = 2$	A1	For correct value of q
	GS $y = e^{-x} (A \cos 4x + B \sin 4x) + x + 2$	B1√	7 For GS. f.t. from their CF+PI with 2 arbitrary constants in CF and none in PI. Requires $\boxed{y =}$.
(ii)	<p>$x \gg 0 \Rightarrow e^{-x} \rightarrow 0$ <i>OR</i> very small</p> <p>$\Rightarrow y = x + 2$ approximately</p>	B1	For correct statement. Allow graph
		B1√	2 For correct equation Allow \approx , \rightarrow and in words Allow relevant f.t. from linear part of GS
9			

7 (i)	$(1, 3, 5)$ and $(5, 2, 5) \Rightarrow \pm[4, -1, 0]$ in Π	M1	For finding a vector in Π
	$\mathbf{n} = [2, -2, 3] \times [4, -1, 0] = k[1, 4, 2]$	M1	For finding vector product of direction vectors of l and a line in Π
	$\Rightarrow \mathbf{r} \cdot [1, 4, 2] = 23$	A1	For correct \mathbf{n}
		A1	4 For correct equation. Allow multiples
(ii)	METHOD 1		
	Perpendicular to Π through $(-7, -3, 0)$ meets Π	M1	For using perpendicular from point on l to Π Award mark for $k\mathbf{n}$ used
	where $(-7+k) + 4(-3+4k) + 2(2k) = 23$	M1	For substituting parametric line coords into Π
	$\Rightarrow k = 2 \Rightarrow d = 2\sqrt{1^2 + 4^2 + 2^2} = 2\sqrt{21} \approx 9.165$	M1	For normalising the \mathbf{n} used in this part
		A1	4 For correct distance AEF
	METHOD 2		
	Π is $x + 4y + 2z = 23$	M1	For attempt to use formula for perpendicular distance
	$\Rightarrow d = \frac{ (-7) + 4(-3) + 2(0) - 23 }{\sqrt{1^2 + 4^2 + 2^2}} = 2\sqrt{21} \approx 9.165$	M1	For substituting a point on l into plane equation
		M1	For normalising the \mathbf{n} used in this part
		A1	For correct distance AEF
	METHOD 3		
	$\mathbf{m} = [1, 3, 5] - [-7, -3, 0] = (\pm)[8, 6, 5]$	M1	For finding a vector from l to Π
$OR = [5, 2, 5] - [-7, -3, 0] = (\pm)[12, 5, 5]$			
$\Rightarrow d = \frac{\mathbf{m} \cdot [1, 4, 2]}{\sqrt{1^2 + 4^2 + 2^2}} = \frac{42}{\sqrt{21}} = 2\sqrt{21} \approx 9.165$	M1	For finding $\mathbf{m} \cdot \mathbf{n}$	
	M1	For normalising the \mathbf{n} used in this part	
	A1	For correct distance AEF	
METHOD 4			
$[-7, -3, 0] + k[1, 4, 2] = [1, 3, 5] + s[2, -2, 3] + t[4, -1, 0]$	M1	As Method 1, using parametric form of Π For using perpendicular from point on l to Π Award mark for $k\mathbf{n}$ used	
$\left. \begin{array}{l} k - 2s - 4t = 8 \\ 4k + 2s + t = 6 \\ 2k - 3s = 5 \end{array} \right\} \Rightarrow k = 2 \left(s = -\frac{1}{3}, t = -\frac{4}{3} \right)$	M1	For setting up and solving 3 equations	
$\Rightarrow d = 2\sqrt{1^2 + 4^2 + 2^2} = 2\sqrt{21} \approx 9.165$	M1	For normalising the \mathbf{n} used in this part	
	A1	For correct distance AEF	
METHOD 5			
$d_1 = \frac{23}{\sqrt{1^2 + 4^2 + 2^2}} = \frac{23}{\sqrt{21}}$	M1	For attempt to find distance from O to Π OR from O to parallel plane containing l	
$d_2 = \frac{[-7, -3, 0] \cdot [1, 4, 2]}{\sqrt{1^2 + 4^2 + 2^2}} = \frac{-19}{\sqrt{21}}$	M1	For normalising the \mathbf{n} used in this part	
$\Rightarrow d_1 - d_2 = d = \frac{23 - (-19)}{\sqrt{21}} = 2\sqrt{21} \approx 9.165$	M1	For finding $d_1 - d_2$	
	A1	For correct distance AEF	
(iii)	$(-7, -3, 0) + k(1, 4, 2)$	M1	State or imply coordinates of a point on the reflected line
	Use $k = 4$	M1	State or imply $2 \times$ distance from (ii) Allow $k = \pm 4$ OR $\pm 4\sqrt{21}$ f.t. from (ii)
	$\mathbf{b} = [2, -2, 3]$	B1	For stating correct direction
	$\mathbf{a} = [-3, 13, 8]$	A1	4 For correct point seen in equation $\mathbf{r} = \mathbf{a} + t\mathbf{b}$
	$\mathbf{r} = [-3, 13, 8] + t[2, -2, 3]$		AEF in this form

8 (i)	$\{A, D\}$ OR $\{A, E\}$ OR $\{A, F\}$	B1	1	For stating any one subgroup																																																	
(ii)	A is the identity 5 is not a factor of 6 OR elements can be only of order 1, 2, 3, 6	B1 B1	2	For identifying A as the identity For reference to factors of 6																																																	
(iii)	$BE = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = D$, $EB = \begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} = F$ D or $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, F or $\begin{pmatrix} 0 & \omega \\ \omega^2 & 0 \end{pmatrix} \in M$ \Rightarrow closure property satisfied	M1 A1 A1 A1	4	For finding BE and EB AND using $\omega^3 = 1$ For correct BE (D or matrix) For correct EB (F or matrix) For justifying closure																																																	
(iv)	$B^{-1} = \frac{1}{1} \begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix} = C$ $E^{-1} = \frac{1}{-1} \begin{pmatrix} 0 & -\omega^2 \\ -\omega & 0 \end{pmatrix} = E$	M1 A1 A1	3	For correct method of finding either inverse For correct $B^{-1} = C$ Allow $\begin{pmatrix} \omega^2 & 0 \\ 0 & \omega \end{pmatrix}$ For correct $E^{-1} = E$ Allow $\begin{pmatrix} 0 & \omega^2 \\ \omega & 0 \end{pmatrix}$																																																	
(v)	METHOD 1 M is not commutative e.g. from $BE \neq EB$ in part (iii) N is commutative (as $\times \text{ mod } 9$ is commutative) $\Rightarrow M$ and N not isomorphic	B1 B1 B1#	3	For justification of M being not commutative For statement that N is commutative For correct conclusion																																																	
	METHOD 2 Elements of M have orders 1, 3, 3, 2, 2, 2 Elements of N have orders 1, 6, 3, 2, 3, 6 Different orders OR self-inverse elements $\Rightarrow M$ and N not isomorphic	B1* B1 (*dep) B1#		For all orders of one group correct For sufficient orders of the other group correct For correct conclusion SR Award up to B1 B1 B1 if the self-inverse elements are sufficiently well identified for the groups to be non-isomorphic																																																	
	METHOD 3 M has no generator since there is no element of order 6 N has 2 OR 5 as a generator $\Rightarrow M$ and N not isomorphic	B1 B1 B1#		For all orders of M shown correctly For stating that N has generator 2 OR 5 For correct conclusion																																																	
	METHOD 4 <table border="1" style="display: inline-table; vertical-align: top;"> <thead> <tr> <th>M</th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>A</td> <td>B</td> <td>C</td> <td>D</td> <td>E</td> <td>F</td> </tr> <tr> <td>B</td> <td>B</td> <td>C</td> <td>A</td> <td>F</td> <td>D</td> <td>E</td> </tr> <tr> <td>C</td> <td>C</td> <td>A</td> <td>B</td> <td>E</td> <td>F</td> <td>D</td> </tr> <tr> <td>D</td> <td>D</td> <td>E</td> <td>F</td> <td>A</td> <td>B</td> <td>C</td> </tr> <tr> <td>E</td> <td>E</td> <td>F</td> <td>D</td> <td>C</td> <td>A</td> <td>B</td> </tr> <tr> <td>F</td> <td>F</td> <td>D</td> <td>E</td> <td>B</td> <td>C</td> <td>A</td> </tr> </tbody> </table> N 1 1 2 4 8 7 5 2 1 2 4 8 7 5 4 2 4 8 7 5 1 8 4 8 7 5 1 2 7 8 7 5 1 2 4 5 7 5 1 2 4 8 5 5 1 2 4 8 7 $\Rightarrow M$ and N not isomorphic	M	A	B	C	D	E	F	A	A	B	C	D	E	F	B	B	C	A	F	D	E	C	C	A	B	E	F	D	D	D	E	F	A	B	C	E	E	F	D	C	A	B	F	F	D	E	B	C	A	B1* B1 (*dep) B1#		For stating correctly all 6 squared elements of one group For stating correctly sufficient squared elements of the other group For correct conclusion
M	A	B	C	D	E	F																																															
A	A	B	C	D	E	F																																															
B	B	C	A	F	D	E																																															
C	C	A	B	E	F	D																																															
D	D	E	F	A	B	C																																															
E	E	F	D	C	A	B																																															
F	F	D	E	B	C	A																																															
				# In all Methods, the last B1 is dependent on at least one preceding B1																																																	

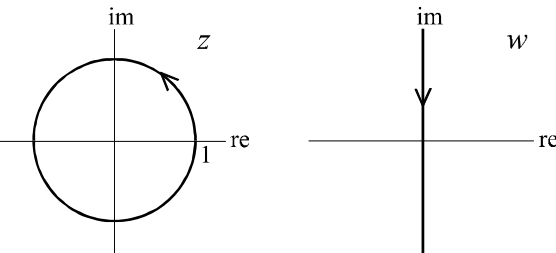
<p>1 (i) Integrating factor. $e^{\int x dx} = e^{\frac{1}{2}x^2}$ $\Rightarrow \frac{d}{dx} \left(y e^{\frac{1}{2}x^2} \right) = x e^{x^2}$ $\Rightarrow y e^{\frac{1}{2}x^2} = \frac{1}{2} e^{x^2} + c$ $\Rightarrow y = e^{-\frac{1}{2}x^2} \left(\frac{1}{2} e^{x^2} + c \right) = \frac{1}{2} e^{\frac{1}{2}x^2} + c e^{-\frac{1}{2}x^2}$</p>	<p>B1 For correct IF M1 For $\frac{d}{dx} (y \cdot \text{their IF}) = x e^{\frac{1}{2}x^2} \cdot \text{their IF}$ A1 For correct integration both sides A1 4 For correct solution AEF as $y = f(x)$</p>
<p>(ii) $(0, 1) \Rightarrow c = \frac{1}{2}$ $\Rightarrow y = \frac{1}{2} \left(e^{\frac{1}{2}x^2} + e^{-\frac{1}{2}x^2} \right)$</p>	<p>M1 For substituting $(0, 1)$ into their GS, solving for c and obtaining a solution of the DE A1 2 For correct solution AEF Allow $y = \cosh\left(\frac{1}{2}x^2\right)$</p>
6	
<p>2 (i) $\mathbf{n} = [2, 1, -3] \times [-1, 2, 4]$ $= [10, -5, 5] = k[2, -1, 1]$ $(1, 3, 4) \Rightarrow 2x - y + z = 3$</p>	<p>M1 For using \times of direction vectors A1 For correct \mathbf{n} A1 3 For substituting $(1, 3, 4)$ and obtaining AG (Verification only M0)</p>
<p>(ii) METHOD 1 distance = $\frac{21-3}{ \mathbf{n} }$ OR $\frac{[1, 3, 4] \cdot [2, -1, 1] - 21}{ \mathbf{n} }$ OR $\frac{ ([1, 3, 4] - [a, b, c]) \cdot [2, -1, 1] }{ \mathbf{n} }$ where (a, b, c) is on q $= \frac{18}{\sqrt{6}} = 3\sqrt{6}$</p>	<p>M1 For $21 - 3$ OR $[1, 3, 4] \cdot [2, -1, 1] - 21$ OR $([1, 3, 4] - [a, b, c]) \cdot [2, -1, 1]$ soi B1 For $\mathbf{n} = \sqrt{6}$ soi A1 3 For correct distance AEF</p>
<p>METHOD 2 $[1 + 2t, 3 - t, 4 + t]$ on q $\Rightarrow 2(1 + 2t) - (3 - t) + (4 + t) = 21 \Rightarrow t = 3$ $\Rightarrow \text{distance} = 3 \mathbf{n} = 3\sqrt{6}$</p>	<p>M1 For forming and solving an equation in t B1 For $\mathbf{n} = \sqrt{6}$ soi A1 For correct distance AEF</p>
<p>METHOD 3 As Method 2 to $t = 3 \Rightarrow (7, 0, 7)$ on q distance from $(1, 3, 4)$ $= \sqrt{(7-1)^2 + (0-3)^2 + (7-4)^2} = \sqrt{54} = 3\sqrt{6}$</p>	<p>M1* For finding point where normal meets q M1 For finding distance from $(1, 3, 4)$ (*dep) A1 For correct distance AEF</p>
6	
<p>3 (i) $\sin \theta = \frac{1}{2i} (e^{i\theta} - e^{-i\theta})$ $\sin^4 \theta = \frac{1}{16} (z^4 - 4z^2 + 6 - 4z^{-2} + z^{-4})$ $\Rightarrow \sin^4 \theta = \frac{1}{16} (2 \cos 4\theta - 8 \cos 2\theta + 6)$ $\Rightarrow \sin^4 \theta = \frac{1}{8} (\cos 4\theta - 4 \cos 2\theta + 3)$</p>	<p>B1 z or $e^{i\theta}$ may be used throughout For correct expression for $\sin \theta$ soi M1 For expanding $(e^{i\theta} - e^{-i\theta})^4$ (with at least 3 terms and 1 binomial coefficient) M1 For grouping terms and using multiple angles A1 4 For answer obtained correctly AG</p>
<p>(ii) $\int_0^{\frac{1}{6}\pi} \sin^4 \theta d\theta = \frac{1}{8} \left[\frac{1}{4} \sin 4\theta - 2 \sin 2\theta + 3\theta \right]_0^{\frac{1}{6}\pi}$ $= \frac{1}{8} \left(\frac{1}{8} \sqrt{3} - \sqrt{3} + \frac{1}{2} \pi \right) = \frac{1}{64} (4\pi - 7\sqrt{3})$</p>	<p>M1 For integrating (i) to $A \sin 4\theta + B \sin 2\theta + C\theta$ A1 For correct integration M1 For completing integration and substituting limits A1 4 For correct answer AEF(exact)</p>
8	

<p>4 (i)</p>	<p><i>EITHER</i> $1 + \omega + \omega^2$ $=$ sum of roots of $(z^3 - 1) = 0$</p> <hr/> <p>OR $\omega^3 = 1 \Rightarrow (\omega - 1)(\omega^2 + \omega + 1) = 0$ $\Rightarrow 1 + \omega + \omega^2 = 0$ (for $\omega \neq 1$)</p> <hr/> <p>OR sum of G.P. $1 + \omega + \omega^2 = \frac{1 - \omega^3}{1 - \omega} \left(= \frac{0}{1 - \omega} \right) = 0$</p> <hr/> <p>OR  shown on Argand diagram or explained in terms of vectors</p> <hr/> <p>OR $1 + \text{cis } \frac{2}{3}\pi + \text{cis } \frac{4}{3}\pi = 1 + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 0$</p>	<p>M1 A1 2</p>	<p>For result shown by any correct method AG</p>
<p>(ii)</p>	<p>Multiplication by $\omega \Rightarrow$ rotation through $\frac{2}{3}\pi$ </p> <p>$z_1 - z_3 = \vec{CA}$, $z_3 - z_2 = \vec{BC}$</p> <p>\vec{BC} rotates through $\frac{2}{3}\pi$ to direction of \vec{CA}</p> <p>ΔABC has $BC = CA$, hence result</p>	<p>B1 B1 M1 A1 4</p>	<p>For correct interpretation of \times by ω (allow 120° and omission of, or error in, \odot)</p> <p>For identification of vectors soi (ignore direction errors)</p> <p>For linking BC and CA by rotation of $\frac{2}{3}\pi$ OR ω</p> <p>For stating equal magnitudes \Rightarrow AG</p>
<p>(iii)</p>	<p>(ii) $\Rightarrow z_1 + \omega z_2 - (1 + \omega)z_3 = 0$</p> <p>$1 + \omega + \omega^2 = 0 \Rightarrow z_1 + \omega z_2 + \omega^2 z_3 = 0$</p>	<p>M1 A1 2</p>	<p>For using $1 + \omega + \omega^2 = 0$ in (ii)</p> <p>For obtaining AG</p>
<p>8</p>			
<p>5 (i)</p>	<p>Aux. equation $3m^2 + 5m - 2 (= 0)$</p> <p>$\Rightarrow m = \frac{1}{3}, -2$</p> <p>CF ($y =$) $Ae^{\frac{1}{3}x} + Be^{-2x}$</p> <p>PI ($y =$) $px + q \Rightarrow 5p - 2(px + q) = -2x + 13$</p> <p>$\Rightarrow p = 1, q = -4$</p> <p>GS ($y =$) $Ae^{\frac{1}{3}x} + Be^{-2x} + x - 4$</p>	<p>M1 A1 A1√ M1 A1 A1 B1√ 7</p>	<p>For correct auxiliary equation seen and solution attempted</p> <p>For correct roots</p> <p>For correct CF f.t. from m with 2 arbitrary constants</p> <p>For stating and substituting PI of correct form</p> <p>For correct value of p, and of q</p> <p>For GS f.t. from their CF+PI with 2 arbitrary constants in CF and none in PI</p>
<p>(ii)</p>	<p>$\left(0, -\frac{7}{2}\right) \Rightarrow A + B = \frac{1}{2}$</p> <p>$y' = \frac{1}{3}Ae^{\frac{1}{3}x} - 2Be^{-2x} + 1, (0, 0) \Rightarrow A - 6B = -3$</p> <p>$\Rightarrow A = 0, B = \frac{1}{2}$</p> <p>$\Rightarrow (y =) \frac{1}{2}e^{-2x} + x - 4$</p>	<p>M1 M1 M1 A1 B1√ 5</p>	<p>For substituting $\left(0, -\frac{7}{2}\right)$ in their GS and obtaining an equation in A and B</p> <p>For finding y', substituting $(0, 0)$ and obtaining an equation in A and B</p> <p>For solving their 2 equations in A and B</p> <p>For correct A and B CAO</p> <p>For correct solution f.t. with their A and B in their GS</p>
<p>(iii)</p>	<p>x large $\Rightarrow (y =) x - 4$</p>	<p>B1√ 1</p>	<p>For correct equation or function (allow \approx and \rightarrow) WWW f.t. from (ii) if valid</p>
<p>13</p>			

6 (i)	$a^4 = r^6 = e \Rightarrow a$ has order 4, a^2 has order 2 $(a^3)^4 = a^{12} = e \Rightarrow a^3$ has order 4 $(r^2)^3 = e \Rightarrow r^2$ has order 3	M1 A1 A1 B1	For considering powers of a For order of any one of a, a^2, a^3 correct For all correct For order of r^2 correct																		
(ii)	<p>G order 4</p> <table border="1" data-bbox="261 412 740 479"> <tr> <td>Order of element</td> <td>1</td> <td>2</td> <td>(4)</td> </tr> <tr> <td>Number of elements</td> <td>1</td> <td>3</td> <td>(0)</td> </tr> </table> <p>H order 6</p> <table border="1" data-bbox="261 506 815 573"> <tr> <td>Order of element</td> <td>1</td> <td>2</td> <td>3</td> <td>(6)</td> </tr> <tr> <td>Number of elements</td> <td>1</td> <td>3</td> <td>2</td> <td>(0)</td> </tr> </table> <p>G and H are the only non-cyclic groups of order which divides 12 Q has 1 element of order 2, G and H have 3, so no non-cyclic subgroups in Q</p>	Order of element	1	2	(4)	Number of elements	1	3	(0)	Order of element	1	2	3	(6)	Number of elements	1	3	2	(0)	M1 A1 A1 B1 B1	For top line in either table Allow inclusion of 4 and 6 respectively (and other orders if 0 appears below) For order 4 table For order 6 table For stating that only G and H need be considered AEF For argument completed by elements of order 2 AG SR Allow equivalent arguments for B1 B1
Order of element	1	2	(4)																		
Number of elements	1	3	(0)																		
Order of element	1	2	3	(6)																	
Number of elements	1	3	2	(0)																	
9																					
7 (i)	$[1, 1, -2] \times [1, -1, 3] = (\pm)[1, -5, -2]$ $[1, -1, 3] \times [1, 5, -12] = (\pm)[-3, 15, 6]$ $[-3, 15, 6] = k[1, -5, -2] \Rightarrow$ parallel	M1 A1 M1 A1 A1	For using \times of direction vectors For correct direction For using \times of direction vectors For correct direction For argument completed AG ($k = -3$ not essential)																		
(ii)	Line of intersection is parallel to l and m	B1	For correct statement																		
(iii)	<p>METHOD 1</p> $\left. \begin{array}{l} x + y - 2z = 5 \\ x - y + 3z = 6 \end{array} \right\} \text{e.g. } z = 0 \Rightarrow \left(\frac{11}{2}, -\frac{1}{2}, 0\right) \text{ on } l$ $\left. \begin{array}{l} x - y + 3z = 6 \\ x + 5y - 12z = 12 \end{array} \right\} \text{e.g. } z = 0 \Rightarrow (7, 1, 0) \text{ on } m$ $\left. \begin{array}{l} x + y - 2z = 5 \\ x + 5y - 12z = 12 \end{array} \right\} \text{e.g. } z = 0 \Rightarrow \left(\frac{13}{4}, \frac{7}{4}, 0\right) \text{ on } l_3$ <p>Different points \Rightarrow no common line of intersection</p>	M1 A1 A1 A1	For attempt to find points on 2 lines For a correct point on one line For a correct point on another line For correct answer																		
	<p>METHOD 2</p> $\left. \begin{array}{l} x + y - 2z = 5 \\ x - y + 3z = 6 \end{array} \right\} \text{e.g. } \Rightarrow z = 11 - 2x, y = 27 - 5x$ <p>LHS of eqn 3 = $x + (135 - 25x) - (132 - 24x) = 3 \neq 12$ \Rightarrow no common line of intersection</p>	M1 A1 A1 A1	For finding (e.g.) y and z in terms of x OR eliminating one variable For correct expressions OR equations For obtaining a contradiction from 3rd equation For correct answer																		
	<p>METHOD 3</p> <p>LHS $II_3 = 3II_1 - 2II_2$ RHS $3 \times 5 - 2 \times 6 = 3 \neq 12$ \Rightarrow no common line of intersection</p>	M2 A1 A1	For attempt to link 3 equations For obtaining a contradiction For correct answer																		
	SR Variations on all methods may gain full credit		SR f.t. may be allowed from relevant working																		
10																					

8 (i)	$((a,b)*(c,d))*(e,f) = (ac, ad+b)*(e,f)$	M1	For 3 distinct elements bracketed and attempt to expand
	$= (ace, acf + ad + b)$	A1	For correct expression
	$(a,b)*((c,d)*(e,f)) = (a,b)*(ce, cf + d)$ $= (ace, acf + ad + b)$	A1	3 For correct expression again
(ii)	$(a,b)*(1,1) = (a, a+b), (1,1)*(a,b) = (a, b+1)$	M1	For combining both ways round
	$a+b = b+1 \Rightarrow a = 1$	M1	For equating components (allow from incorrect pairs)
	$\Rightarrow (1, b) \forall b$	A1	3 For correct elements AEF
(iii)	$(mp, mq+n) \text{ OR } (pm, pn+q) = (1, 0)$	M1	For either element on LHS
	$\Rightarrow (p, q) = \left(\frac{1}{m}, -\frac{n}{m}\right)$	A1	2 For correct inverse
(iv)	$(a,b)*(a,b) = (a^2, ab+b) = (1, 0)$	M1	For attempt to find self-inverses
	$\text{OR } (a,b) = \left(\frac{1}{a}, -\frac{b}{a}\right) \Rightarrow a^2 = 1, ab = -b$	B1 A1	For $(1, 0)$. For $(-1, b)$ AEF
	\Rightarrow self-inverse elements $(1, 0)$ and $(-1, b) \forall b$	3	
(v)	$(0, y)$ has no inverse for any $y \Rightarrow$ not a group	B1	1 For stating any one element with no inverse. Allow $x \neq 0$ required, provided reference to inverse is made "Some elements have no inverse" B0

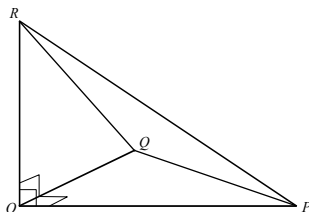
1 (i)	$\theta = \sin^{-1} \frac{ [5, 6, -7] \cdot [1, 2, -1] }{\sqrt{5^2 + 6^2 + (-7)^2} \sqrt{1^2 + 2^2 + (-1)^2}}$	M1*	For using scalar product of line and plane vectors
	$\theta = \sin^{-1} \frac{24}{\sqrt{110}\sqrt{6}} = 69.1^\circ (69.099\dots^\circ, 1.206)$	M1 (*dep) A1 A1	For both moduli seen For correct scalar product For correct angle
	$\phi = \sin^{-1} \frac{ [5, 6, -7] \times [1, 2, -1] }{\sqrt{5^2 + 6^2 + (-7)^2} \sqrt{1^2 + 2^2 + (-1)^2}}$	SR M1*	For vector product of line and plane vectors AND finding modulus of result
	$\phi = \sin^{-1} \frac{\sqrt{84}}{\sqrt{110}\sqrt{6}} = 20.9^\circ \Rightarrow \theta = 69.1^\circ$	M1 (*dep) A1 A1	For moduli of line and plane vectors seen For correct modulus $\sqrt{84}$ For correct angle
<hr/>			
1 (ii)	METHOD 1		
	$d = \frac{ 1+12+3-40 }{\sqrt{1^2+2^2+(-1)^2}} = \frac{24}{\sqrt{6}} = 4\sqrt{6} \approx 9.80$	M1 A1	For use of correct formula For correct distance
	METHOD 2		
	$(1+\lambda)+2(6+2\lambda)-(-3-\lambda)=40$	M1	For substituting parametric form into plane
	$\Rightarrow \lambda=4 \Rightarrow d=4\sqrt{6}$	A1	For correct distance
	OR distance from $(1, 6, -3)$ to $(5, 14, -7)$		
	$=\sqrt{4^2+8^2+(-4)^2}=\sqrt{96}$		
	METHOD 3		
	Plane through $(1, 6, -3)$ parallel to p is	M1	For finding parallel plane through $(1, 6, -3)$
	$x+2y-z=16 \Rightarrow d = \frac{40-16}{\sqrt{6}} = \frac{24}{\sqrt{6}}$	A1	For correct distance
METHOD 4			
e.g. $(0, 0, -40)$ on p	M1	For using any point on p to find vector and scalar product seen	
\Rightarrow vector to $(1, 6, -3) = \pm(1, 6, 37)$		e.g. $[1, 6, 37] \cdot [1, 2, -1]$	
$d = \frac{ [1, 6, 37] \cdot [1, 2, -1] }{\sqrt{6}} = \frac{24}{\sqrt{6}}$	A1	For correct distance	
METHOD 5			
l meets p where $(1+5t)+2(6+6t)-(-3-7t)=40$		For finding t where l meets p	
$\Rightarrow t=1 \Rightarrow d = [5, 6, -7] \sin \theta$	M1	and linking d with triangle	
$\Rightarrow d = \sqrt{110} \frac{24}{\sqrt{110}\sqrt{6}} = \frac{24}{\sqrt{6}}$	A1	For correct distance	
6			
2 (i)	METHOD 1		
	$\text{EITHER } \frac{1+e^{i\theta}}{1-e^{i\theta}} = \frac{e^{-\frac{1}{2}i\theta} + e^{\frac{1}{2}i\theta}}{e^{-\frac{1}{2}i\theta} - e^{\frac{1}{2}i\theta}}$	M1	<i>EITHER</i> For changing LHS terms to $e^{\pm\frac{1}{2}i\theta}$
	$= \frac{2\cos\frac{1}{2}\theta}{-2i\sin\frac{1}{2}\theta} = i\cot\frac{1}{2}\theta$	M1	<i>OR in reverse</i> For using $\cot\frac{1}{2}\theta = \frac{\cos\frac{1}{2}\theta}{\sin\frac{1}{2}\theta}$
OR in reverse with similar working	A1	For either of $\cos\frac{1}{2}\theta = \frac{e^{\frac{1}{2}i\theta} + e^{-\frac{1}{2}i\theta}}{2}$ so (2)(i) For fully correct proof to AG SR If factors of 2 or i are not clearly seen, award M1 M1 A0	

2 (i)	METHOD 2		
	$\text{EITHER } \frac{1+e^{i\theta}}{1-e^{i\theta}} \times \frac{1-e^{-i\theta}}{1-e^{-i\theta}} = \frac{e^{i\theta}-e^{-i\theta}}{2-(e^{i\theta}+e^{-i\theta})}$	M1	For multiplying top and bottom by complex conjugate in exp or trig form
	$\text{OR } \frac{1+\cos\theta+i\sin\theta}{1-\cos\theta-i\sin\theta} \times \frac{1-\cos\theta+i\sin\theta}{1-\cos\theta+i\sin\theta}$	M1	For using both double angle formulae correctly
	$= \frac{2i\sin\theta}{2-2\cos\theta} = \frac{2i\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}{2\sin^2\frac{1}{2}\theta} = i\cot\frac{1}{2}\theta$	A1	For fully correct proof to AG
	METHOD 3		
	$\frac{1+\cos\theta+i\sin\theta}{1-\cos\theta-i\sin\theta} = \frac{2\cos^2\frac{1}{2}\theta+2i\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}{2\sin^2\frac{1}{2}\theta-2i\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}$	M1	For using both double angle formulae correctly
	$= \frac{2\cos\frac{1}{2}\theta(\cos\frac{1}{2}\theta+i\sin\frac{1}{2}\theta)}{2\sin\frac{1}{2}\theta(\sin\frac{1}{2}\theta-i\cos\frac{1}{2}\theta)}$	M1	For appropriate factorisation
	$= i\cot\frac{1}{2}\theta \frac{(\sin\frac{1}{2}\theta-i\cos\frac{1}{2}\theta)}{(\sin\frac{1}{2}\theta-i\cos\frac{1}{2}\theta)} = i\cot\frac{1}{2}\theta$	A1	For fully correct proof to AG
	METHOD 4		
	$\frac{1+\cos\theta+i\sin\theta}{1-\cos\theta-i\sin\theta} = \frac{1+\frac{1-t^2}{1+t^2}+i\frac{2t}{1+t^2}}{1-\frac{1-t^2}{1+t^2}-i\frac{2t}{1+t^2}}$	M1	For substituting both t formulae correctly
	$= \frac{2+2it}{2t^2-2it} = \frac{1+i}{t-i} = \frac{i}{t-i} = i\cot\frac{1}{2}\theta$	M1 A1	For appropriate factorisation For fully correct proof to AG
	METHOD 5		
	$\frac{1+e^{i\theta}}{1-e^{i\theta}} \times \frac{1+e^{i\theta}}{1+e^{i\theta}} = \frac{1+2e^{i\theta}+e^{2i\theta}}{1-e^{2i\theta}}$	M1	For multiplying top and bottom by $1+e^{i\theta}$ and attempting to divide by $e^{i\theta}$
	$= \frac{2+e^{i\theta}+e^{-i\theta}}{e^{-i\theta}-e^{i\theta}}$	M1	OR multiplying top and bottom by $1+e^{-i\theta}$
	$= \frac{2(1+\cos\theta)}{-2i\sin\theta} = \frac{2\cos^2\frac{1}{2}\theta}{-2i\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta} = \frac{\cos\frac{1}{2}\theta}{-i\sin\frac{1}{2}\theta}$	M1	For using both double angle formulae correctly
	$= i\cot\frac{1}{2}\theta$	A1	For fully correct proof to AG
(ii)		M1 A1 B1	For a circle centre O For indication of radius = 1 and anticlockwise arrow shown For locus of w shown as imaginary axis described downwards
6			

3 (i)	METHOD 1 $m + 4 (= 0) \Rightarrow \text{CF } (y =) Ae^{-4x}$	M1 A1	For correct auxiliary equation (soi) 2 For correct CF
	METHOD 2 Separating variables on $\frac{dy}{dx} + 4y = 0$ $\Rightarrow \ln y = -4x$ $\Rightarrow \text{CF } (y =) Ae^{-4x}$	M1 A1	For integration to this stage For correct CF
(ii)	PI $(y =) p \cos 3x + q \sin 3x$ $y' = -3p \sin 3x + 3q \cos 3x$ $\Rightarrow (-3p + 4q) \sin 3x + (4p + 3q) \cos 3x = 5 \cos 3x$ $\Rightarrow \left. \begin{matrix} -3p + 4q = 0 \\ 4p + 3q = 5 \end{matrix} \right\} \Rightarrow p = \frac{4}{5}, q = \frac{3}{5}$ GS $(y =) Ae^{-4x} + \frac{4}{5} \cos 3x + \frac{3}{5} \sin 3x$	B1 M1 A1 M1 A1 A1 B1√ 7	For stating PI of correct form For substituting y and y' into DE For correct equation For equating coeffs and solving For correct value of p , and of q For GS f.t. from their CF+PI with 1 arbitrary constant in CF and none in PI
	SR Integrating factor method may be used, followed by 2-stage integration by parts or C+iS method Marks for (i) are awarded only if CF is clearly identified		
(iii)	$e^{-4x} \rightarrow 0, \frac{4}{5} \cos 3x + \frac{3}{5} \sin 3x = \frac{\sin}{\cos}(3x + \alpha)$ $\Rightarrow -1 \leq y \leq 1 \quad \text{OR} \quad -1 \lesssim y \lesssim 1$	M1 A1√ 2	For considering either term For correct range (allow <) CWO f.t. as $-\sqrt{p^2 + q^2} \leq y \leq \sqrt{p^2 + q^2}$ from (ii)
11			
4 (i)	$abc = (ab)c = (ba)c = b(ac) =$ $b(ca) = (bc)a = (cb)a = cba$ Minimum working: $abc = bac = bca = cba$ OR $abc = acb = cab = cba$ OR $abc = bac = bca = cba$	M1 A1	For using commutativity correctly 2 For correct proof (use of associativity may be implied)
(ii)	$\{e, a\}, \{e, b\}, \{e, c\}, \{e, bc\}, \{e, ca\}, \{e, ab\}, \{e, abc\}$	B1 B1	For any 5 subgroups 2 For the other 2 subgroups and none incorrect
(iii)	$\{e, a, b, ab\}, \{e, a, c, ca\}, \{e, b, c, bc\}$ $\{e, a, bc, abc\}, \{e, b, ca, abc\}, \{e, c, ab, abc\}$ $\{e, bc, ca, ab\}$	B1 B1 B1	For any 3 subgroups For 1 more subgroup 3 For 1 more subgroup (5 in total) and none incorrect
(iv)	All elements ($\neq e$) have order 2 OR all are self-inverse OR no element of G has order 4 OR no order 4 subgroup has a generator or is cyclic OR subgroups are of the form $\{e, a, b, ab\}$ (the Klein group) \Rightarrow all order 4 subgroups are isomorphic	B1* B1 (*dep)2	For appropriate reference to order of elements in G For correct conclusion
9			

5 (i)	$\frac{dy}{dx} = ku^{k-1} \frac{du}{dx}$	M1	For using chain rule
		A1	For correct $\frac{dy}{dx}$
	$\Rightarrow xku^{k-1} \frac{du}{dx} + 3u^k = x^2u^{2k}$	M1	For substituting for y and $\frac{dy}{dx}$
	$\Rightarrow \frac{du}{dx} + \frac{3}{kx}u = \frac{1}{k}xu^{k+1}$	A1	4 For correct equation AG
(ii)	$k = -1$	B1	1 For correct k
(iii)	$\frac{du}{dx} - \frac{3}{x}u = -x \Rightarrow \text{IF } e^{-\int \frac{3}{x} dx} = e^{-3 \ln x} = \frac{1}{x^3}$	B1√	For correct IF f.t. for IF = $x^{\frac{3}{k}}$ using k or their numerical value for k
	$\Rightarrow \frac{d}{dx} \left(u \cdot \frac{1}{x^3} \right) = -\frac{1}{x^2}$	M1	For $\frac{d}{dx} (u \cdot \text{their IF}) = -x \cdot \text{their IF}$
	$\Rightarrow u \cdot \frac{1}{x^3} = \frac{1}{x} (+c) \Rightarrow y = \frac{1}{cx^3 + x^2}$	A1 A1	For correct integration both sides 4 For correct solution for y
9			
6 (a)	Closure $(ax+b) + (cx+d) = (a+c)x + (b+d)$	B1	For obtaining correct sum from 2 distinct elements
	$\in P$	B1	For stating result is in P <i>OR</i> is of the correct form SR award this mark if any of the closure result, the identity or the inverse element is stated to be in P <i>OR</i> of the correct form
	Identity $0x+0$	B1	For stating identity (allow 0)
	Inverse $-ax-b$	B1	4 For stating inverse
(b) (i)	Order 9	B1*	1 For correct order
(ii)	$x+2$	B1	1 For correct inverse element
(iii)	$(ax+b) + (ax+b) + (ax+b) = 3ax+3b$	M1	For considering sums of $ax+b$ and obtaining $3ax+3b$
	$= 0x+0$		For equating to $0x+0$ <i>OR</i> 0
	$\Rightarrow ax+b$ has order $3 \forall a, b$ (except $a=b=0$)	A1	and obtaining order 3 SR For order 3 stated only <i>OR</i> found from incomplete consideration of numerical cases award B1
	Cyclic group of order 9 has element(s) of order 9	M1 (*dep)	For reference to element(s) of order 9
	$\Rightarrow (Q, +(\text{mod } 3))$ is not cyclic	A1	4 For correct conclusion
10			

7 (i)



B1 For sketch of tetrahedron labelled in some way
At least one right angle at O must be indicated or clearly implied

M1 For using $\Delta = \frac{1}{2} \text{base} \times \text{height}$

$$\Delta OPQ = \frac{1}{2} pq, \Delta OQR = \frac{1}{2} qr, \Delta ORP = \frac{1}{2} rp$$

A1 **3** For all areas correct **CAO**

(ii)

$$\frac{1}{2} \left| \vec{RP} \times \vec{RQ} \right| = \frac{1}{2} |\vec{RP}| |\vec{RQ}| \sin R = \Delta PQR$$

B1 **1** For correct justification

(iii)

$$\text{LHS} = \left(\frac{1}{2} pq\right)^2 + \left(\frac{1}{2} qr\right)^2 + \left(\frac{1}{2} rp\right)^2$$

B1 For correct expression

$$\Delta PQR = \frac{1}{2} |(p\mathbf{i} - q\mathbf{j}) \times (p\mathbf{i} - r\mathbf{k})|$$

B1 For ΔPQR in vector form

$$\text{OR } \frac{1}{2} |(p\mathbf{i} - r\mathbf{k}) \times (q\mathbf{j} - r\mathbf{k})|$$

$$\text{OR } \frac{1}{2} |(p\mathbf{i} - q\mathbf{j}) \times (q\mathbf{j} - r\mathbf{k})|$$

$$\Delta PQR = \frac{1}{2} |qr\mathbf{i} + pr\mathbf{j} + pq\mathbf{k}|$$

M1 For finding vector product of their attempt at ΔPQR

A1 For correct expression

$$\text{RHS} = \frac{1}{4} \left((pq)^2 + (qr)^2 + (rp)^2 \right)$$

M1 For using $|a\mathbf{i} + b\mathbf{j} + c\mathbf{k}| = \sqrt{a^2 + b^2 + c^2}$

A1 **6** For completing proof of **AG WWW**

10

8 (i)	$\operatorname{Re}(c+is)^4 = \cos 4\theta = c^4 - 6c^2s^2 + s^4$	M1*	For expanding $(c+is)^4$: at least 2 terms and 1 binomial coefficient needed
	$\cos 4\theta = c^4 - 6c^2(1-c^2) + (1-c^2)^2$	A1	For 3 correct terms
	$\Rightarrow \cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$	M1 (*dep)	For using $s^2 = 1-c^2$
	(ii) $\cos 4\theta \cos 2\theta = (8c^4 - 8c^2 + 1)(2c^2 - 1)$	A1	4 For correct expression for $\cos 4\theta$ CAO
	$= 16\cos^6\theta - 24\cos^4\theta + 10\cos^2\theta - 1$	B1	For multiplying by $(2c^2 - 1)$
	(iii) $16c^6 - 24c^4 + 10c^2 - 2 = 0$	M1	1 to obtain AG WWW
	$\Rightarrow (c^2 - 1)(8c^4 - 4c^2 + 1) = 0$		For factorising sextic
	For quartic, $b^2 - 4ac = 16 - 32 < 0$	A1	with $(c-1)$, $(c+1)$ or (c^2-1)
	$\Rightarrow c = \pm 1$ only $\Rightarrow \theta = n\pi$	A1	For justifying no other roots CWO
		A1	3 For obtaining $\theta = n\pi$ AG
			Note that M1 A0 A1 is possible
		SR	For verifying $\theta = n\pi$ by substituting $c = \pm 1$ into $16c^6 - 24c^4 + 10c^2 - 2 = 0$ B1
	(iv) $16c^6 - 24c^4 + 10c^2 = 0$		
	$\Rightarrow c^2(8c^4 - 12c^2 + 5) = 0$	M1	For factorising sextic with c^2
	For quartic, $b^2 - 4ac = 144 - 160 < 0$	A1	For justifying no other roots CWO
	$\Rightarrow \cos \theta = 0$ only	A1	3 For correct condition obtained AG
			Note that M1 A0 A1 is possible
		SR	For verifying $\cos \theta = 0$ by substituting $c = 0$ into $16c^6 - 24c^4 + 10c^2 = 0$ B1
		SR	For verifying $\theta = \frac{1}{2}\pi$ and $\theta = -\frac{1}{2}\pi$ satisfy $\cos 4\theta \cos 2\theta = -1$ B1

Question		Answer	Marks	Guidance
1	(i)	$(y = xu \Rightarrow) \frac{dy}{dx} = x \frac{du}{dx} + u$ $x \frac{du}{dx} + u = \frac{2 + u^2}{u}$ $\Rightarrow x \frac{du}{dx} = \frac{2}{u}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>For a correct statement</p> <p>For using the substitution to eliminate y (If B0, then y must be eliminated from LHS, but $\frac{d(uv)}{dx}$ sufficient)</p> <p>For correct equation AG</p>
1	(ii)	$\int u \, du = \int \frac{2}{x} \, dx$ $\Rightarrow \frac{1}{2}u^2 = 2\ln((k)x) \text{ OR } \frac{1}{2}u^2 = 2\ln x (+c)$ $\Rightarrow \frac{1}{2}\left(\frac{y}{x}\right)^2 = 2\ln(kx) \text{ OR } \frac{1}{2}\left(\frac{y}{x}\right)^2 = 2\ln x + c$ $\Rightarrow y^2 = 4x^2 \ln(kx) \text{ OR } y^2 = 4x^2 \ln x + Cx^2$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>For separating variables and writing/attempting integrals</p> <p>For correct integration both sides (k or c not required here)</p> <p>For substituting for u into integrated terms with constant (on either side)</p> <p>For correct solution AEF $y^2 = f(x)$ Do not penalise “c” being used for different constants e.g. $2\ln x + c = 2\ln(cx)$</p>
2	(i)	$(z^n - e^{i\theta})(z^n - e^{-i\theta}) \equiv z^{2n} - 2z^n \left(\frac{e^{i\theta} + e^{-i\theta}}{2} \right) + 1$ $\equiv z^{2n} - (2\cos\theta)z^n + 1$	<p>B1</p> <p>[1]</p>	<p>For multiplying out to AG with evidence of $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ (Can be implied by $2\cos\theta = (e^{i\theta} + e^{-i\theta})$)</p>

Question		Answer	Marks	Guidance
2	(ii)	<p>METHOD 1</p> $2\cos\theta = 1 \Rightarrow \theta = \frac{1}{3}\pi$ $\Rightarrow z^4 - z^2 + 1 \equiv \left(z^2 - e^{\frac{1}{3}\pi i}\right)\left(z^2 - e^{-\frac{1}{3}\pi i}\right)$ $\equiv \left(z + e^{\frac{1}{6}\pi i}\right)\left(z - e^{\frac{1}{6}\pi i}\right)\left(z + e^{-\frac{1}{6}\pi i}\right)\left(z - e^{-\frac{1}{6}\pi i}\right)$ $\equiv \left(z - e^{\frac{1}{6}\pi i}\right)\left(z - e^{\frac{5}{6}\pi i}\right)\left(z - e^{\frac{7}{6}\pi i}\right)\left(z - e^{\frac{11}{6}\pi i}\right)$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>For using (i) to find θ</p> <p>For correct quadratic factors (Or $\frac{5\pi}{3}i$ in place of $-\frac{\pi}{3}i$)</p> <p>For factorising $(z^2 - a^2)$</p> <p>For correct linear factors</p> <p>For adjusting arguments (must attempt correct range and “(z – root)”)</p> <p>For correct factors CAO</p> <p>Correct answer www gets 6</p>
		<p>METHOD 2</p> $z^4 - z^2 + 1 = 0 \Rightarrow z^2 = \frac{1}{2} \pm \frac{1}{2}\sqrt{3}i = e^{\frac{1}{3}\pi i}, e^{-\frac{1}{3}\pi i}$ $\Rightarrow z = \pm e^{\frac{1}{6}\pi i}, \pm e^{-\frac{1}{6}\pi i}$ $= e^{\frac{1}{6}\pi i}, e^{\frac{7}{6}\pi i}, e^{\frac{5}{6}\pi i}, e^{\frac{11}{6}\pi i}$ $\Rightarrow \left(z - e^{\frac{1}{6}\pi i}\right)\left(z - e^{\frac{5}{6}\pi i}\right)\left(z - e^{\frac{7}{6}\pi i}\right)\left(z - e^{\frac{11}{6}\pi i}\right)$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>For solving quadratic</p> <p>For correct roots in exp form</p> <p>For attempt to find 4 roots</p> <p>For correct roots $\pm e^{i\alpha}$</p> <p>For adjusting arguments</p> <p>For correct factors CAO</p>
3	(i)	<p>METHOD 1</p> $(yx)(yx)^{-1} = e \Rightarrow x(yx)^{-1} = y^{-1}$ $\Rightarrow (yx)^{-1} = x^{-1}y^{-1}$ <p>METHOD 2</p> <p>Compare $(yx)(yx)^{-1} = e$ with $yx x^{-1} y^{-1} = e$</p> $\Rightarrow (yx)^{-1} = x^{-1}y^{-1}$	<p>M1</p> <p>A1</p> <p>[2]</p> <p>M1</p> <p>A1</p>	<p>For starting point and appropriate multiplication</p> <p>For correct result AG</p> <p>For appropriate comparison</p> <p>For correct result AG</p> <p>For A1, proof cannot be written in the form ‘LHS = RHS $\rightarrow \dots \rightarrow e = e$’</p>

Question		Answer	Marks	Guidance
3	(ii)	$x^n y^n = (xy)^n = x(yx)^{n-1} y$ $\Rightarrow x^{-1} x^n y^n y^{-1} = x^{-1} x (yx)^{n-1} y y^{-1}$ $\Rightarrow x^{n-1} y^{n-1} = (yx)^{n-1}$	M1 M1 A1 [3]	For using associativity or an inverse with respect to LHS, RHS or initial equality www beforehand For using $(xy)^n = x(yx)^{n-1} y$ oe For correct result AG SR for numerical n used, allow M1 M1 only
3	(iii)	METHOD 1 All steps in (ii) are reversible \Rightarrow result follows METHOD 2 Show working for (ii) in reverse \Rightarrow result follows	B1*dep B1*dep [2] B1* B1*dep	For correct reason. Dep on correct part(ii) For correct conclusion For correct working For correct conclusion

Question	Answer	Marks	Guidance	
4	(i)	<p>METHOD 1 (M, then distance) $M = (1 + 2t, 1 + 3t, -1 + 2t)$ $\mathbf{AM} = (\pm)[2t - 6, 3t - 2, 2t - 8]$ \mathbf{AM} perp. $l \Rightarrow 2(2t - 6) + 3(3t - 2) + 2(2t - 8) = 0$ $\Rightarrow t = 2, M = (5, 7, 3)$ $AM = \sqrt{2^2 + 4^2 + 4^2} = 6$</p> <p>METHOD 2(a) (distance, then M) $(C = (1, 1, -1)) \mathbf{AC} = \pm[6, 2, 8]$ $\mathbf{n} = \mathbf{AC} \times [2, 3, 2] = k[-20, 4, 14]$ $d = \frac{ \mathbf{n} }{ [2, 3, 2] } = \frac{\sqrt{612}}{\sqrt{17}} = 6$ $CM = \sqrt{(6^2 + 2^2 + 8^2)} - 6 = 2\sqrt{17}$ $[2, 3, 2] = \sqrt{17} \Rightarrow t = 2, M = (5, 7, 3)$</p> <p>METHOD 2(b) $(C = (1, 1, -1)) \mathbf{AC} = \pm[6, 2, 8]$ $\cos \theta = \frac{\mathbf{AC} \cdot (2, 3, 2)}{ \mathbf{AC} (2, 3, 2) }, \theta = 36.0(39..)$ or $\sin \theta = \frac{153}{\sqrt{442}}$ $AM = \mathbf{AC} \sin \theta = 6$ $M = (5, 7, 3)$</p>	<p>B1 B1 FT M1 A1 A1 M1 A1 [7] B1 M1 A1 FT A1 M1 B1 A1 B1 M1,A1 M1,A1 M1,A1</p>	<p>Coordinates or vectors allowed throughout</p> <p>For correct parametric form soi</p> <p>For correct vector. FT from M</p> <p>For using perpendicular condition</p> <p>For correct equation</p> <p>For correct coordinates</p> <p>For using distance formula</p> <p>For correct distance</p> <p>For correct vector</p> <p>For finding $\mathbf{AC} \times$ direction of l</p> <p>For correct \mathbf{n}. FT from \mathbf{n}</p> <p>For correct distance</p> <p>For a correct method for finding position of M</p> <p>For $[2, 3, 2] = \sqrt{17}$ soi</p> <p>For correct vector</p> <p>As above</p>

Question		Answer	Marks	Guidance
4	(ii)	$\mathbf{AM} = [-2, 4, -4]$ or $\mathbf{MA} = [2, -4, 4]$ $\Rightarrow B = (7, 3, 7) + \frac{3}{4}(-2, 4, -4) = \left(7 - \frac{3}{2}, 3 + 3, 7 - 3\right)$ OR $B = (5, 7, 3) + \frac{1}{4}(2, -4, 4) = \left(5 + \frac{1}{2}, 7 - 1, 3 + 1\right)$ OR $B = \frac{3}{4}(5, 7, 3) + \frac{1}{4}(7, 3, 7) = \left(\frac{15}{4} + \frac{7}{4}, \frac{21}{4} + \frac{3}{4}, \frac{9}{4} + \frac{7}{4}\right)$ $B = \left(\frac{11}{2}, 6, 4\right)$	M1 M1 A1 [3]	For using $A + k_1 \vec{AM}$ or $M + k_2 \vec{MA}$ or ratio theorem or equivalent For $B = (7, 3, 7) + \frac{3}{4}x$ their $(-2, 4, -4)$ oe (or M1 for quadratic in parameter for line AM, followed by M1 for attempt to use correct value of parameter to find B) For correct coordinates
5	(i)	$(2m^2 + 3m - 2 = 0) \Rightarrow m = \frac{1}{2}, -2$ CF = $Ae^{\frac{1}{2}x} + Be^{-2x}$	M1 A1 [2]	For attempt to solve correct auxiliary equation For correct CF
5	(ii)	$\frac{dy}{dx} = pe^{-2x} - 2pxe^{-2x}$ $\frac{d^2y}{dx^2} = -4pe^{-2x} + 4pxe^{-2x}$ $\Rightarrow (-8p + 3p + 8px - 6px - 2px)e^{-2x} = 5e^{-2x}$ $\Rightarrow p = -1$	M1 A1 M1 A1 [4]	For differentiating PI twice, using product rule For correct $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ For substituting into DE For correct p

Question		Answer	Marks	Guidance
5	(iii)	<p>GS ($y =$) $Ae^{\frac{1}{2}x} + Be^{-2x} - xe^{-2x}$</p> <p>$(0, 0) \Rightarrow A + B = 0$</p> <p>$\frac{dy}{dx} = \frac{1}{2}Ae^{\frac{1}{2}x} - 2Be^{-2x} - e^{-2x} + 2xe^{-2x}$</p> <p>$\left(0, \frac{dy}{dx} = 4\right) \Rightarrow \frac{1}{2}A - 2B = 5$</p> <p>$\Rightarrow A = 2, B = -2$</p> <p>$\Rightarrow y = 2e^{\frac{1}{2}x} - 2e^{-2x} - xe^{-2x}$</p>	<p>B1 FT</p> <p>B1 FT</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>For GS soi. FT from CF (2 constants) and p</p> <p>For correct equation. FT from GS of form $Ae^{\alpha x} + Be^{\beta x} - Cxe^{-2x}$</p> <p>For differentiating GS and substituting values, using GS of form $Ae^{\alpha x} + Be^{\beta x} - Cxe^{-2x}$</p> <p>For solving for A and B(can be gained from incorrect GS)</p> <p>For correct solution, including $y =$</p>
6	(i)	<p>METHOD 1</p> <p>$\mathbf{n} = [2, -1, -1] \times [2, -3, -5] = [2, 8, -4]$</p> <p>$\mathbf{n} = k[1, 4, -2]$</p> <p>$\Pi$ is $\mathbf{r} \cdot \mathbf{n} = [1, 6, 7] \cdot \mathbf{n}$</p> <p>$\Rightarrow \mathbf{r} \cdot [1, 4, -2] = 11$</p> <p>METHOD 2</p> <p>$y - z = -1 + 2\mu$</p> <p>$\mu = \frac{y - z + 1}{2}$</p> <p>$\lambda = 7 - z - 5 \frac{y - z + 1}{2}$</p> <p>$x = 11 + 2z - 4y$</p> <p>$r \cdot (1, 4, -2) = 11$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	<p>For finding vector product of 2 vectors in Π(or 2 scalar products = 0, with attempt to solve)</p> <p>For correct \mathbf{n}</p> <p>For attempt to find equation of Π, including cartesian equation</p> <p>For correct equation (allow multiples)</p> <p>for finding λ or μ in terms of two from x, y, z.</p> <p>For both λ & μ</p> <p>AEF</p>

Question		Answer	Marks	Guidance
6	(ii)	$[7 + 3t, 4, 1 - t] \cdot \mathbf{n} = 11 \Rightarrow t = -2$ $\Rightarrow [1, 4, 3]$	M1 A1 [2]	For attempt to find t , (or to find λ and μ by equating original equations) For correct position vector <i>OR</i> point
6	(iii)	METHOD 1 $\mathbf{c} = [1, 4, -2] \times [2, -1, -1]$ $\mathbf{c} = k[2, 1, 3]$ METHOD 2 $\mathbf{c} = [2, -3, -5] + s[2, -1, -1]$ $\mathbf{c} \cdot [2, -1, -1] = 0 \Rightarrow$ $2(2 + 2s) - 1(-3 - s) - 1(-5 - s) = 0$ $\Rightarrow s = -2 \Rightarrow \mathbf{c} = k[2, 1, 3]$	M1 M1 A1 [3] M1 M1 A1	For using given vector product (or 2 correct 'scalar products = 0') For calculating given vector product (or 2 correct scalar products = 0, with attempt to solve) (or M1 for using vector product of \mathbf{c} with \mathbf{n} or $(2, -1, -1)$ in an equation, followed by M1 for calculating vector product and attempting to solve) For correct \mathbf{c} For $\mathbf{c} =$ linear combination of $[2, -3, -5]$ and $[2, -1, -1]$ For an equation in s from $\mathbf{c} \cdot [2, -1, -1] = 0$ For correct \mathbf{c}

Question		Answer	Marks	Guidance
7	(i)	$\begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ m & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ n+m & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ m+n & 1 \end{pmatrix}$ $= \begin{pmatrix} 1 & 0 \\ m & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} \Rightarrow \text{commutative}$	M1 A1 [2]	For multiplying 2 distinct matrices of the correct form both ways, or generalised form at least one way, For stating or implying that addition is commutative and correct conclusion SR Use of numerical matrices must be generalised for any credit
7	(ii)	$(I \Rightarrow) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ <p>EITHER</p> $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$ <p>OR</p> $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow 2+n=0 \Rightarrow \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$	B1 M1 A1 [3]	For correct identity For using inverse property For correct inverse
7	(iii)	$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ has order 2 4 is not a factor of 6	B1 B1 [2]	For correct order For correct reason (Award B0 for “Lagrange” only). Must be explicit about the ‘6’
7	(iv)	$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ OR $\begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$ has order 6, (or > 3) OR (M, \times) is cyclic, G is non-cyclic (having no element of order 6) OR (M, \times) is commutative G is not commutative (being the non-cyclic group) \Rightarrow groups are not isomorphic	B1* B1*dep [2]	For stating (that there is) an element of M with order 6 Award B1* for a relevant statement about M and G For correct conclusion and no false statements attached to conclusion

Question		Answer	Marks	Guidance
8	(i)	$\cos 5\theta + i \sin 5\theta =$ $c^5 + 5ic^4s - 10c^3s^2 - 10ic^2s^3 + 5cs^4 + is^5$ $\Rightarrow \tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta} = \frac{5c^4s - 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4}$ Division of numerator & denominator by c^5 . $\Rightarrow \tan 5\theta = \frac{5 \tan \theta - 10 \tan^3 \theta + \tan^5 \theta}{1 - 10 \tan^2 \theta + 5 \tan^4 \theta}$	B1 M1 M1 A1 [4]	For explicit use of de Moivre with $n = 5$ For correct expressions for $\sin 5\theta$ and $\cos 5\theta$ For $\frac{\sin 5\theta}{\cos 5\theta}$ in terms of c and s For simplifying to AG, www with explicit mention of division by c^5
8	(ii)	$5\theta = \{1, 5, 9, 13, 17\} \frac{1}{4} \pi$ $\theta = \{1, 5, 9, 13, 17\} \frac{1}{20} \pi$	M1 A1 A1 [3]	For at least 2 of given values and no extras. For at least 3 values of θ and no extras in range For all 5 values and no extras outside range
8	(iii)	$\tan 5\theta = 1 \Rightarrow t^5 - 5t^4 - 10t^3 + 10t^2 + 5t - 1 = 0$ $\Rightarrow (t-1)(t^4 - 4t^3 - 14t^2 - 4t + 1) = 0$ $\tan \alpha = 1$ OR $\alpha = \frac{1}{4} \pi$ is not included in roots of the quartic $\Rightarrow t = \tan \alpha$ for $\alpha = \{1, 9, 13, 17\} \frac{1}{20} \pi$	M1* A1 B1 M1*dep A1 [5]	For $\tan 5\theta = 1$ and equation in t For correct factors For solution rejected (may be implied by $\frac{5}{20} \pi$ not appearing in set of solutions) For 2 correct values of t For all 4 values and no more in range

Question		Answer	Marks	Guidance	
1		<p>METHOD 1 $\mathbf{b} = [1, -3, 4] \times [3, 1, 2] = [-10, 10, 10]$ $= k[-1, 1, 1]$</p> <p>$\Rightarrow \mathbf{r} = [1, 4, 2] + t[-1, 1, 1]$</p> <p>METHOD 2 $[x, y, z] \cdot [1, -3, 4] = 0 \Rightarrow x - 3y + 4z = 0$ $[x, y, z] \cdot [3, 1, 2] = 0 \Rightarrow 3x + y + 2z = 0$</p> <p>Solving $\Rightarrow [x, y, z] = \mathbf{b} = k[-1, 1, 1]$</p> <p>$\Rightarrow \mathbf{r} = [1, 4, 2] + t[-1, 1, 1]$</p>	<p>M1 M1 A1 B1 FT [4]</p> <p>M1</p> <p>M1 A1</p> <p>B1FT</p>	<p>For attempt to find vector product of directions Correct calculation of vector product For correct \mathbf{b} . For correct equation. FT from \mathbf{b}</p> <p>For an equation from l_2 perpendicular to normal of plane and an equation from l_2 perpendicular to l_1</p> <p>For correct equation. FT. from \mathbf{b}</p>	<p>Allow 1 error</p> <p>Must show “$\mathbf{r} =$”</p>
2	(i)	<p>$z^4 = 4\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 4 \operatorname{cis} \frac{1}{3}\pi$</p> <p>$z = \sqrt[4]{4} \operatorname{cis}\left(k\frac{\pi}{12}\right), k = 1, 7, 13, 19$</p>	<p>B1 M1 A1 A1 B1</p> <p>[5]</p>	<p>For $\arg(z^4) = \frac{1}{3}\pi$ soi For dividing $\arg(z^4)$ by 4 For any 2 correct values of k For all 4 values of k and no extras. Ignore values outside range For modulus of all stated roots = $\sqrt[4]{4}$</p> <p>SR For $\arg(z^4) = \frac{1}{6}\pi$ award B0 M1 A1 FT for all $\operatorname{cis}\left(k\frac{\pi}{24}\right), k = 1, 13, 25, 37, A0 B0/B1$</p>	<p>For second A1, must be in correct form. Don't accept 1.41.. or $\sqrt[4]{4}$</p>

Question		Answer	Marks	Guidance
2	(ii)		<p>B1</p> <p>B1</p> <p>B1</p> <p>[3]</p>	<p>For roots forming a square, centre O, on equal-scale axes.</p> <p>For z^4 and only one root in first quadrant with arguments in ratio approximately 3:1</p> <p>For $z^4 : z \approx 4:\sqrt{2}$ (allow (2,4):1)</p> <p>Must be roots distinct from z^4</p> <p>Penalise once use of points not lines</p> <p>For all four roots</p>
3		<p>Integrating factor = $e^{\int \cot x \, dx} = e^{\ln \sin x} = \sin x$</p> <p>$\Rightarrow \frac{d}{dx}(y \sin x) = 2x \sin x$</p> <p>$\Rightarrow y \sin x = -2x \cos x + \int 2 \cos x \, dx$</p> <p>$\Rightarrow y \sin x = -2x \cos x + 2 \sin x (+c)$</p> <p>$(\frac{1}{6}\pi, 2) \Rightarrow c = \frac{1}{6}\pi\sqrt{3}$</p> <p>$\Rightarrow y = -2x \cot x + 2 + \frac{1}{6}\pi\sqrt{3} \operatorname{cosec} x$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1*</p> <p>A1</p> <p>A1</p> <p>M1dep*</p> <p>A1 FT</p> <p>A1</p> <p>[9]</p>	<p>For IF = $e^{\pm \ln \sin x}$ OR $e^{\pm \ln \cos x}$</p> <p>For simplified IF</p> <p>For $\frac{d}{dx}(y \cdot \text{their IF}) = 2x \cdot \text{their IF}$</p> <p>For attempt to integrate RHS using parts for $\int x \begin{cases} \sin x \\ \cos x \end{cases} dx$</p> <p>For correct RHS 1st stage</p> <p>oe</p> <p>For substituting $(\frac{1}{6}\pi, 2)$ into their GS (with c)</p> <p>For correctly finding c (FT from GS)</p> <p>For correct solution AEF of standard notation $y = f(x)$</p> <p>(Must use $u = (2)x$)</p> <p>$c = 0.907$</p>

Question		Answer	Marks	Guidance
4	(i)	$\begin{array}{c ccccc} H & e & r & r^2 & r^3 \\ \hline e & e & r & r^2 & r^3 \\ r & r & r^2 & r^3 & e \\ r^2 & r^2 & r^3 & e & r \\ r^3 & r^3 & e & r & r^2 \end{array}$	B2	For correct table for H For correct table for K SR In both tables allow B1 for 1 or 2 errors
		$\begin{array}{c cccc} K & e & p & q & pq \\ \hline e & e & p & q & pq \\ p & p & e & pq & q \\ q & q & pq & e & p \\ pq & pq & q & p & e \end{array}$	B2	
		[4]		
4	(ii)	Identity = b	B1 [1]	For correct identity
4	(iii)	G is isomorphic to H	B1	For H identified as isomorphic to G (may be implied by table)
		$\begin{array}{c c c} G & H & H \\ \hline a & r^2 & r^2 \\ b & e & e \\ c & r & r^3 \\ d & r^3 & r \end{array}$	B1	For $a \leftrightarrow r^2$ at least once
			B1	For $c, d \leftrightarrow r, r^3$ either way
			B1	For $c, d \leftrightarrow r, r^3$ both ways and b corresponds to e explicit. Award fourth B1 only for completely correct answer. If none of last 3 marks gained, then SC1 for order of all elements of G and H
		[4]		
5	(i)	METHOD 1		z may be used for $e^{i\theta}$ throughout
		$\sin^3 \theta \cos^2 \theta = \left(\frac{e^{i\theta} - e^{-i\theta}}{2i} \right)^3 \left(\frac{e^{i\theta} + e^{-i\theta}}{2} \right)^2$	B1	For $\left(\frac{e^{i\theta} - e^{-i\theta}}{2i} \right)$ OR $\left(\frac{e^{i\theta} + e^{-i\theta}}{2} \right)$ soi
		$= -\frac{1}{32i} (z^3 - 3z + 3z^{-1} - z^{-3})(z^2 + 2 + z^{-2})$	M1	For expanding brackets (binomial theorem or otherwise)
			M1	For full expansion with 12 terms.
			B1	For $-\frac{1}{32i}$
			M1	For grouping terms
		$= -\frac{1}{32i} \left((z^5 - z^{-5}) - (z^3 - z^{-3}) - 2(z - z^{-1}) \right)$		This step, oe, is needed for the final mark
		$= -\frac{1}{16} \left(\frac{z^5 - z^{-5}}{2i} - \frac{z^3 - z^{-3}}{2i} - 2 \frac{z - z^{-1}}{2i} \right)$		
		$= -\frac{1}{16} (\sin 5\theta - \sin 3\theta - 2 \sin \theta)$	A1	For simplification to AG www
		[6]		two brackets expanded soi by alternate method Can be seen at any stage oe includes replacing $z^5 - z^{-5}$ with $2i \sin 5\theta$ etc

Question		Answer	Marks	Guidance	
		<p>METHOD 2</p> $\sin^3 \theta \cos^2 \theta = \sin^3 \theta - \sin^5 \theta$ $2i \sin \theta = z - \frac{1}{z}$ $-8i \sin^3 \theta = z^3 - 3z + \frac{3}{z} - \frac{1}{z^3}$ $= (z^3 - \frac{1}{z^3}) - (3z - \frac{3}{z})$ $= 2i \sin 3\theta - 6i \sin \theta$ $32i \sin^5 \theta = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$ $= (z^5 - \frac{1}{z^5}) - (5z^3 - \frac{5}{z^3}) + (10z - \frac{10}{z})$ $= 2i \sin 5\theta - 10i \sin 3\theta + 20i \sin \theta$ $\sin^3 \theta \cos^2 \theta$ $= -\frac{1}{32i} (4(2i \sin 3\theta - 6i \sin \theta) + (2i \sin 5\theta - 10i \sin 3\theta + 20i \sin \theta))$ $= -\frac{1}{16} (\sin 5\theta - 5 \sin 3\theta + 4 \sin \theta - 12 \sin \theta)$ $= -\frac{1}{16} (\sin 5\theta - \sin 3\theta - 2 \sin \theta)$	<p>B1</p> <p>M1</p> <p>M1</p> <p>B1</p> <p>B1</p> <p>A1</p>	<p>For RHS</p> <p>*</p> <p>For grouping terms</p> <p>For RHS of this line and line * above</p> <p>For $-\frac{1}{32i}$</p> <p>For ag www</p>	
5	(ii)	$\sin^3 \theta \cos^2 \theta = 0 \Rightarrow \sin \theta = 0 \text{ OR } \cos \theta = 0$ $\Rightarrow \theta = r\pi \text{ OR } \theta = (2r+1)\frac{1}{2}\pi$ $\Rightarrow \theta = \frac{n\pi}{2}$	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	<p>For either equation Accept also $\sin \theta = \pm 1$</p> <p>For either solution, A EF including a list of the first few</p> <p>For both of above solutions leading to general solution in form of AG where $k = 2$</p>	<p>Can be implied by the A mark plus at least $\sin^3 \theta = 0$ or similar. At least 2 in list (and no wrong solution)</p>

Question		Answer	Marks	Guidance	
6	(i)	METHOD 1 $m^2 + 4m = 0 \Rightarrow m = 0, -4$ CF = $A + Be^{-4x}$ PI $y = pe^{2x} \Rightarrow 4p + 8p = 12$ $\Rightarrow p = 1$ GS $y = A + Be^{-4x} + e^{2x}$ METHOD 2 Integrating $\Rightarrow \frac{dy}{dx} + 4y = 6e^{2x} + c$ IF $e^{4x} \Rightarrow \frac{d}{dx}(ye^{4x}) = 6e^{6x} + ce^{4x}$ $\Rightarrow ye^{4x} = e^{6x} + \frac{1}{4}ce^{4x} + B$ $\Rightarrow y = e^{2x} + A + Be^{-4x}$	M1 A1 B1 M1 A1 B1 FT [6] M1 B1 B1√ M1 A1 A1	For attempt to solve correct auxiliary equation For correct CF For PI of correct form seen For differentiating PI and substituting For correct p For using GS = CF + PI with 2 arbitrary constants in GS and none in PI For attempt to integrate equation For $+c$ included For correct IF. f.t. from their DE For multiplying through by their IF and attempting to integrate For correct integration both sides, including $+B$ For correct solution	Beware poor use of pxe^{2x} Scores maximum of M1 A1 B0 M1 A0 B0 Must include “y =”
		6	(ii)	$\frac{dy}{dx} = -4Be^{-4x} + 2e^{2x}$ $\left(0, \frac{dy}{dx} = 6\right) \Rightarrow -4B + 2 = 6 \Rightarrow B = -1$ $(y \approx e^{2x} \Rightarrow) A = 0$ $\Rightarrow y = -e^{-4x} + e^{2x}$	M1 A1 B1 A1 [4]
7	(i)	$\mathbf{m} = \mathbf{v} + \frac{1}{2}(\mathbf{w} - \mathbf{v}) \Rightarrow$ $\vec{UM} = \mathbf{v} + \frac{1}{2}(\mathbf{w} - \mathbf{v}) - \mathbf{u} = \frac{1}{2}(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$	M1 A1 [2]	For using vector triangle, or equivalent, for M For correct expression AG SR Allow use of ratio theorem	$\vec{UM} = \vec{UV} + \vec{VM}$ $= (\mathbf{v} - \mathbf{u}) + \frac{1}{2}(\mathbf{w} - \mathbf{v})$ Minimum $-\mathbf{u} + \frac{1}{2}(\mathbf{v} + \mathbf{w})$

Question		Answer	Marks	Guidance	
7	(ii)	<p>METHOD 1 (first 3 marks)</p> <p>\vec{UM} is $\mathbf{r} = \mathbf{u} + \frac{1}{2}t(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$</p> <p>$t = \frac{2}{3} \Rightarrow \mathbf{u} + \frac{1}{3}(\mathbf{v} + \mathbf{w} - 2\mathbf{u}) = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w})$</p> <p>METHOD 2 (first 3 marks)</p> <p>$\vec{UG} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w}) - \mathbf{u} = \frac{1}{3}(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$</p> <p>OR</p> <p>$\vec{MG} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w}) - \frac{1}{2}(\mathbf{v} + \mathbf{w}) = -\frac{1}{6}(\mathbf{v} + \mathbf{w} - 2\mathbf{u})$</p> <p>$\Rightarrow U, G, M$ collinear</p> <p>By symmetry of \vec{OG} in $\mathbf{u}, \mathbf{v}, \mathbf{w}$</p> <p>$G$ also lies on VN, WP</p> <p>$\Rightarrow UM, VN, WP$ intersect at G</p>	<p>M1*</p> <p>M1*</p> <p>A1</p> <p>M1*</p> <p>M1*</p> <p>A1</p> <p>B1</p> <p>B1dep *</p> <p>[5]</p>	<p>For equation of UM</p> <p>For attempt to find a suitable value of t</p> <p>For $t = \frac{2}{3}$ and G obtained AG</p> <p>For finding directions of UG or MG</p> <p>For comparing with UM</p> <p>For showing G lies on UM AG</p> <p>For use of symmetry, or by repeating method for UM twice more.</p> <p>For complete reasoning to AG</p>	
7	(iii)	Line is $\mathbf{r} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w}) + t(\mathbf{u} - \mathbf{v}) \times (\mathbf{u} - \mathbf{w})$ (etc)	<p>B1</p> <p>B1</p> <p>[2]</p>	<p>For $r = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w}) + t \times$ "any vector"</p> <p>For a correct \mathbf{n}, using any 2 of $\pm(\mathbf{u} - \mathbf{v}), \pm(\mathbf{v} - \mathbf{w}), \pm(\mathbf{w} - \mathbf{u})$</p>	<p>Allow $\vec{UV} \times \vec{VW}$ or similar</p>

Question		Answer	Marks	Guidance	
7	(iv)	<p>METHOD 1 $\mathbf{n} = [1, 0, -1] \times [0, 1, -1]$ (etc) = $k[1, 1, 1]$</p> <p>UVW is $\mathbf{r} \cdot \mathbf{n} = [1, 0, 0] \cdot [1, 1, 1] = 1$</p> <p>$\Rightarrow d = \frac{1}{\sqrt{3}}$</p> <p>METHOD 2 UVW is $x + y + z = 1$ (from given $\mathbf{u}, \mathbf{v}, \mathbf{w}$)</p> <p>$\Rightarrow d = \frac{1}{\sqrt{3}}$</p> <p>METHOD 3 $\vec{OG} = \frac{1}{3}(\mathbf{u} + \mathbf{v} + \mathbf{w})$</p> <p>$\Rightarrow OG = \sqrt{\frac{1}{9} + \frac{1}{9} + \frac{1}{9}}$</p> <p>$\Rightarrow d = \frac{1}{\sqrt{3}}$</p>	<p>M1*</p> <p>M1dep *</p> <p>A1</p> <p>[3]</p> <p>M2</p> <p>A1</p> <p>M1*</p> <p>M1dep *</p> <p>A1</p>	<p>For attempt to find \mathbf{n}</p> <p>For substituting a point</p> <p>For correct d</p> <p>For attempt to find cartesian equation</p> <p>For correct d</p> <p>For stating or implying \vec{OG} is d</p> <p>For finding magnitude</p> <p>For correct d</p>	<p>May see use of $\frac{ p \cdot \mathbf{n} - d }{ \mathbf{n} }$</p>

Question		Answer	Marks	Guidance	
8	(i)	For R , $\cos^2 \theta + \sin^2 \theta = 1 \Rightarrow \text{ad-bc} = 1 (\Rightarrow R \subset M)$	B1	For showing $R \subset M$	Must demonstrate use of compound angles or explain rotations.
		$R(\theta)R(\phi) = R(\theta + \phi)$ and hence closed, since $\cos \theta \cos \phi - \sin \theta \sin \phi = \cos(\theta + \phi)$ and $\pm (\cos \theta \sin \phi + \sin \theta \cos \phi) = \pm \sin(\theta + \phi)$	M1	For multiplying 2 distinct elements	
			A1	For obtaining $R(\theta)R(\phi) \in R$	
		Identity $\theta = 0 \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \in R$	B1	For identity element related to $\theta = 0$	
		Inverse $R(-\theta) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$	B1	For inverse element ...	
		$= \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix}$	B1	...converted to form of elements of R	
		SR For use of $(a, b \in R \Rightarrow ab^{-1} \in R) \Leftrightarrow R$ is a subgroup of M	[6]		
		For R , $\cos^2 \theta + \sin^2 \theta = 1 \Rightarrow R \subset M$	B1	For showing $R \subset M$	
		$R(\theta)R(\phi)^{-1} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{pmatrix}$	B1	For considering $R(\theta)R(\phi)^{-1}$	
		$= \begin{pmatrix} \cos(\theta - \phi) & -\sin(\theta - \phi) \\ \sin(\theta - \phi) & \cos(\theta - \phi) \end{pmatrix} \in R$	B1	For correct inverse	
Set is non-empty	M1	For multiplying elements			
	A1	For correct product			
	B1	Can be implied by identity element related to $\theta = 0$			

Question		Answer	Marks	Guidance	
8	(ii)	<p>For $\theta = \frac{1}{3}k\pi$ elements are</p> $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix},$ $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & -\frac{1}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[5]</p>	<p>For $\theta = \frac{1}{3}\pi$ soi</p> <p>For using “their θ” in $\begin{pmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{pmatrix}$ for at least 2 values of k, or lists all 6 values of θ</p> <p>For identity and one other element other than (-I)</p> <p>For 2 more elements</p> <p>For all 6 elements correct</p>	<p>Allow degrees instead of radians.</p>

Question	Answer	Marks	Guidance
1 (i)	$\cos \theta = \frac{\begin{vmatrix} 1 & 2 \\ 2 & -1 \\ 5 & 3 \end{vmatrix}}{\sqrt{1^2 + 2^2 + 5^2} \sqrt{2^2 + (-1)^2 + 3^2}} = \frac{15}{\sqrt{30}\sqrt{14}}$ $\theta = 0.750 \text{ or } 43.0^\circ$	M1 A1 A1 [3]	Accept unsimplified If zero, then sc1 for $n_1 \cdot n_2 = 15$ seen
1 (ii)	$\begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \times \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 11 \\ 7 \\ -5 \end{pmatrix}$ <p>(eg) $x = 0 \Rightarrow 2y + 5z = 12, -y + 3z = 5 \Rightarrow y = 1, z = 2$</p> $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 11 \\ 7 \\ -5 \end{pmatrix}$ <p>Alternative: Find one point Find a second point and vector between points</p> <p>multiple of $\begin{pmatrix} 11 \\ 7 \\ -5 \end{pmatrix}$</p> $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 11 \\ 7 \\ -5 \end{pmatrix}$	M1 A1 M1 A1 [4] M1 M1 A1 A1	M1 requires evidence of method for cross product or at least 2 correct values calculated or any valid point e.g. $(-11/7, 0, 19/7)$ $(22/5, 19/5, 0)$ Must have full equation including ' $\mathbf{r} =$ '

Question		Answer	Marks	Guidance	
		Alternative: Solve simultaneously Point found Direction found $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 11 \\ 7 \\ -5 \end{pmatrix}$	M1 A1 A1	to at least expressions for x,y,z parametrically, or two relationship between 2 variables.	
2	(i)	identity $0 + 0i$ order 25	B1 B1 [2]	Or '0'	
2	(ii)	$3 + i$	B1 [1]		
2	(iii)	$5(a + bi) = 5a + 5bi = 0 + 0i$ every non-zero element has order 5 or 25 So order is 5	M1 M1 A1 [3]	Shows 5 times any element equals e Attempt to show that order $\neq 2,3,4$ Argument is convincing, exhaustive and conclusive.	Must consider all(non-zero) elements
3		$\frac{dy}{dx} - 3\frac{y}{x} = x^3 e^{2x}$ $I = \exp\left(\int -\frac{3}{x} dx\right) = e^{-3\ln x}$ $= x^{-3}$ $x^{-3} \frac{dy}{dx} - 3x^{-4} y = e^{2x}$ $\frac{d}{dx}(x^{-3} y) = e^{2x}$ $x^{-3} y = \frac{1}{2} e^{2x} + A$ $x = 1, y = 0 \Rightarrow A = -\frac{1}{2} e^2$ $y = \frac{1}{2} x^3 (e^{2x} - e^2)$	M1 M1 A1 M1 M1 A1 M1 A1 [8]	Divide by x Multiply and recognise derivative Integrate Use condition	

Question	Answer	Marks	Guidance
4 (i)	$\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \times \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ -14 \end{pmatrix} = -2 \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix}$ $\begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}$ $\text{shortest distance} = \frac{\left \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 7 \end{pmatrix} \right }{\sqrt{2^2 + 1^2 + 7^2}} = \frac{2}{\sqrt{54}} \text{ oe}$	M1 A1 B1 M1 A1 [5]	Or any multiple Or negative Component of their vector in their direction Or use of $n.(a_1 + pb_1 + kn) = n.(a_2 + qb_2)$ B1 followed by attempt to calculate magnitude of kn M1
4 (ii)	$2x + y + 7z = \dots$ $\dots 11$	B1ft B1 dep [2]	ft from 4(i) only if 1 st M1 mark gained If zero, then sc 1 for any correct vector equation.
5 (i)	$1, e^{\frac{2}{5}\pi i}, e^{\frac{4}{5}\pi i}, e^{\frac{6}{5}\pi i}, e^{\frac{8}{5}\pi i}$ oe polar form	M1 A1 [2]	Attempt roots e.g. gives roots in an incorrect form.

Question	Answer	Marks	Guidance
5 (ii)	$z^5 = (z+1)^5 = z^5 + 5z^4 + 10z^3 + 10z^2 + 5z + 1$ $\Leftrightarrow 5z^4 + 10z^3 + 10z^2 + 5z + 1 = 0$ so $z+1 = ze^{\frac{2k\pi i}{5}}$, $k=0,1,2,3,4$ $k=0$ no solution $1 = z\left(e^{\frac{2k\pi i}{5}} - 1\right)$ $z = \frac{1}{e^{\frac{2k\pi i}{5}} - 1}, k=1,2,3,4$	M1 A1 M1 B1 A1 [5]	soi If B0, then give A1 ft for correct solution plus $k=0$
6 (i)	PI: $y = ax \cos 2x + bx \sin 2x$ $\frac{dy}{dx} = a \cos 2x - 2ax \sin 2x + b \sin 2x + 2bx \cos 2x$ $\frac{d^2y}{dx^2} = -4a \sin 2x - 4ax \cos 2x + 4b \cos 2x - 4bx \sin 2x$ in DE: $-4a \sin 2x - 4ax \cos 2x + 4b \cos 2x - 4bx \sin 2x$ $+4(ax \cos 2x + bx \sin 2x)$ compare coefficients: $-4a = 1, 4b = 0$ $\Rightarrow a = -\frac{1}{4}, b = 0$ AE: $\lambda^2 + 4 = 0$ $\lambda = \pm 2i$ CF: $A \cos 2x + B \sin 2x$ GS: $y = \left(A - \frac{1}{4}x\right) \cos 2x + B \sin 2x$	B1 M1 M1 A1 M1 A1 A1ft [7]	For correct $\frac{dy}{dx}$ or better Differentiate twice and substitute For correct auxiliary equation and attempt to solve oe form Must be real and contain 2 unknowns

Question			Answer	Marks	Guidance
6	(ii)		oscillations unbounded	B1 B1 [2]	oe (accept sketch) dep consistent with 6(i) oe (accept sketch) dep consistent with 6(i) If zero, then sc1 for recognition that $x\cos 2x$ term becomes dominant
6	(iii)		If $k \neq 2$ then PI $y = \alpha \cos kx + \beta \sin kx$ So bounded oscillations	B1 B1 [2]	oe (accept sketch)
7	(i)	(a)	$e^{i\theta} + e^{2i\theta} + \dots + e^{10i\theta} = \frac{e^{i\theta} \left((e^{i\theta})^{10} - 1 \right)}{e^{i\theta} - 1}$ $= \frac{e^{\frac{1}{2}i\theta} \left(e^{10i\theta} - 1 \right)}{e^{\frac{1}{2}i\theta} - e^{-\frac{1}{2}i\theta}}$ $= \frac{e^{\frac{1}{2}i\theta} \left(e^{10i\theta} - 1 \right)}{2i \sin \left(\frac{1}{2}\theta \right)}$	M1 A1 M1 A1 [4]	Sum of a GP AG
7	(i)	(b)	$\theta = 2n\pi \Rightarrow \text{sum} = 10$	B1 [1]	

Question		Answer	Marks	Guidance
7	(ii)	$\cos \theta + \cos 2\theta + \dots + \cos 10\theta = \operatorname{Re} \left(\frac{e^{\frac{1}{2}i\theta} (e^{10i\theta} - 1)}{2i \sin(\frac{1}{2}\theta)} \right)$ $= \frac{\operatorname{Re}(-ie^{\frac{1}{2}i\theta} (e^{10i\theta} - 1))}{2 \sin(\frac{1}{2}\theta)} = \frac{\operatorname{Re}(-ie^{\frac{21}{2}i\theta} + ie^{\frac{1}{2}i\theta})}{2 \sin(\frac{1}{2}\theta)}$	M1 M1	Take real parts Manipulate expression Must at least make genuine progress in sorting real part of numerator, or in converting numerator to trig terms.
		$= \frac{\sin(\frac{21}{2}\theta) - \sin(\frac{1}{2}\theta)}{2 \sin(\frac{1}{2}\theta)}$ $= \frac{\sin(\frac{21}{2}\theta)}{2 \sin(\frac{1}{2}\theta)} - \frac{1}{2}$	A1 [3]	AG
7	(iii)	$\cos \frac{1}{11}\pi + \cos \frac{2}{11}\pi + \dots + \cos \frac{10}{11}\pi = \frac{\sin(\frac{21}{22}\pi)}{2 \sin(\frac{1}{22}\pi)} - \frac{1}{2}$ <p>But $\sin \frac{21}{22}\pi = \sin(\pi - \frac{21}{22}\pi) = \sin \frac{1}{22}\pi$</p> <p>So RHS = $\frac{1}{2} - \frac{1}{2} = 0$, so $\frac{1}{11}\pi$ is a root</p> <p>Using $\sin(2\pi + x) = \sin x$ gives</p> $2\pi + \frac{1}{2}\theta = \frac{21}{2}\theta \Rightarrow \theta = \frac{1}{5}\pi$	M1 M1 A1 A1 [4]	AG For second M1, must convince that solution is exact and not simply from calculator.

8	(i)	$wa^2 = waa = a^3 wa = a^3 a^3 w$ $= a^4 a^2 w = ea^2 w$ $= a^2 w$ <p>Either result $\Rightarrow wa^3 = a^3 wa^2$</p> $= a^3 a^2 w$ $= eaw = aw$	M1 B1 A1 M1 M1 A1 [6]	Use $wa = a^3 w$ to simplify Use $a^4 = e$ (oe) in either proof Complete argument AG AG	
8	(ii)	$(aw)^2 = (aw)(aw)$ $= awwa^3 = eea^3 = a^4 = e \text{ so order 2}$ $(a^2 w)(a^2 w) = a^2 wwa^2 = a^2 ea^2 = a^4 = e \text{ so order 2}$ $(a^3 w)(a^3 w) = a^3 wwa = a^3 ea = a^4 = e \text{ so order 2}$	M1 M1 A1 A1 [4]	for squaring any of elements for attempt to simplify to e for at least two squared elements shown equal to e for complete argument	
8	(iii)	$\{e, a^2, w, a^2 w\}$ $\{e, a^2, aw, a^3 w\}$ $a^2, w, aw, a^2 w, a^3 w \text{ all of order 2}$ so not cyclic as no element of order 4 in either	B1 B1 M1 A1 [4]	Consider orders Or considers form $\{e, x, y, xy\}$ where x, y order 2 Dep on both groups correct	Condone equivalents Condone 'no generator' or 'Klein (V) group' in place of 'no element of order 4'

Question		Answer	Marks	Guidance
1	(i)	<p>vectors in plane: two of $\begin{pmatrix} -4 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 6 \\ 4 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$</p> $\mathbf{r} = \begin{pmatrix} 1 \\ 6 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>Differences between two pairs</p> <p>Aef of parametric equation</p> <p>Must have “$\mathbf{r} = \dots$”</p>
1	(ii)	$\begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} \times \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ -12 \end{pmatrix}$ $\left(\mathbf{r} - \begin{pmatrix} 1 \\ 6 \\ 2 \end{pmatrix} \right) \cdot \begin{pmatrix} 5 \\ 8 \\ -12 \end{pmatrix} = 0$ $5x + 8y - 12z = 29$ <p>Alternate method</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p> <p>M1</p> <p>A1</p> <p>M1A1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p>	<p>Calculate vector product or multiple</p> <p>Aef of cartesian equation, isw.</p> <p>EITHER</p> <p>x, y, z in parametric form both parameters in terms of e.g. x, y substitute into parametric form of z</p> <p>OR</p> <p>x, y, z in parametric form 2 equations in x, y, z and one parameter eliminate parameter</p> <p>M1 can be awarded where vector product has method shown or only one term wrong</p> <p>Or Cartesian form = d with attempt to compute d</p>

Question		Answer	Marks	Guidance																										
2	(i)	<table style="border-collapse: collapse; margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;"></td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">3</td> <td style="padding: 2px 10px;">5</td> <td style="padding: 2px 10px;">7</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">3</td> <td style="padding: 2px 10px;">5</td> <td style="padding: 2px 10px;">7</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;">3</td> <td style="padding: 2px 10px;">3</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">7</td> <td style="padding: 2px 10px;">5</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;">5</td> <td style="padding: 2px 10px;">5</td> <td style="padding: 2px 10px;">7</td> <td style="padding: 2px 10px;">1</td> <td style="padding: 2px 10px;">3</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px 10px;">7</td> <td style="padding: 2px 10px;">7</td> <td style="padding: 2px 10px;">5</td> <td style="padding: 2px 10px;">3</td> <td style="padding: 2px 10px;">1</td> </tr> </table> <p>From table clearly closed</p> <p>1 is identity</p> <p>$3^{-1} \equiv 3, 5^{-1} \equiv 5, 7^{-1} \equiv 7 \pmod{8}$</p>		1	3	5	7	1	1	3	5	7	3	3	1	7	5	5	5	7	1	3	7	7	5	3	1	<p>B2</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>[5]</p>	<p>-1 each error</p> <p>Superfluous fact/s gets -1</p>	<p>Must be clear they are referring to tabulated results</p> <p>Or "1 appears in every row"</p>
	1	3	5	7																										
1	1	3	5	7																										
3	3	1	7	5																										
5	5	7	1	3																										
7	7	5	3	1																										
2	(ii)	1 has order 1 and 3, 5, 7 all have order 2	<p>B1</p> <p>[1]</p>																											
2	(iii)	{1, 3}, {1, 5}, {1, 7} (and {1})	<p>B1</p> <p>[1]</p>	<p>All correct, no extras</p> <p>Allow {1} included or omitted</p>																										
2	(iv)	<p>in H $3^2 \equiv 9 \pmod{10}$ so 3 not order 2</p> <p>no element of order > 2 in G so not isomorphic</p>	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>Shows and states that 3 or that 7 is not order 2 (or is order 4)</p> <p>Completely correct reasoning</p> <p>Or, if zero, then SC1 for merely stating comparable orders and then saying that "orders don't correspond, so not isomorphic"</p> <p>Or</p> <p>table for H with saying "not all elements self inverse, so not isomorphic"</p>																										

Question	Answer	Marks	Guidance
3	$u = y^3 \Rightarrow \frac{du}{dx} = 3y^2 \frac{dy}{dx}$ <p>in DE gives $x \frac{du}{dx} + 2u = \frac{\cos x}{x}$</p> $\frac{du}{dx} + \frac{2}{x}u = \frac{\cos x}{x^2}$ $I = \exp\left(\int \frac{2}{x} dx\right) = e^{2 \ln x}$ $= x^2$ $x^2 \frac{du}{dx} + 2xu = \cos x$ $\frac{d}{dx}(x^2 u) = \cos x$ $x^2 u = \sin x \quad (+A)$ $u = \frac{\sin x + A}{x^2}$ $y = \left(\frac{\sin x + A}{x^2}\right)^{\frac{1}{3}}$	<p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[8]</p>	<p>Or $\frac{dy}{dx} = \frac{1}{3}u^{-\frac{2}{3}} \frac{du}{dx}$</p> <p>Divide</p> <p>Correctly integrates</p> <p>Integrate</p> <p>Or gives GS in implicit form</p> <p>Both sides</p> <p>Must have form $\frac{du}{dx} + f(x)u = g(x)$</p> <p>Can be implied by subsequent work</p> <p>Must include constant at this stage</p>

Question		Answer	Marks	Guidance
4	(i)	Sketch $OA = 3 = 3, OB = \left 3e^{\frac{1}{3}\pi i} \right = 3$ and $\angle BOA = \frac{1}{3}\pi$ hence $\triangle OAB$ equilateral	B1 M1 A1 [3]	Must have axes, A shown 3 across and either scale (or co-ordinates) with B in rough position, or angle and distance on argand diagram. No inconsistencies Can be seen on diagram Alt. Attempts AB or second angle
4	(ii)	$3e^{-\frac{1}{3}\pi i}$	M1A1 [2]	Or $3e^{\frac{5}{3}\pi i}$. Isw M1 for evidence they are considering BA, or for $\frac{3}{2} - \frac{3}{2}\sqrt{3}i$ For full marks can use CiS form, or clear polar co-ordinates, in radians. Not C-iS
4	(iii)	$\left(3 - 3e^{\frac{1}{3}\pi i} \right)^5 = 3^5 e^{-\frac{5}{3}\pi i}$ $= 243 \left(\cos \frac{5}{3}\pi - i \sin \frac{5}{3}\pi \right)$ $= \frac{243}{2} + \frac{243}{2}\sqrt{3}i$	M1 A1ft B1 [3]	For mod^5 and $\text{arg} \times 5$ aef “Hence” so must use ‘their $3e^{-\frac{1}{3}\pi i}$, Condone use of “121.5”.

Question		Answer	Marks	Guidance	
5		AE: $\lambda^2 + 2\lambda + 5 = 0$ $\lambda = -1 \pm 2i$ CF: $e^{-x}(A \cos 2x + B \sin 2x)$ PI: $y = ae^{-x}$ $ae^{-x} - 2ae^{-x} + 5ae^{-x} = e^{-x}$ $4a = 1$ $a = \frac{1}{4}$ GS: $y = e^{-x}\left(\frac{1}{4} + A \cos 2x + B \sin 2x\right)$ $\frac{dy}{dx} = -e^{-x}\left(\frac{1}{4} + A \cos 2x + B \sin 2x\right)$ $+ e^{-x}(-2A \sin 2x + 2B \cos 2x)$ $x = 0, \frac{dy}{dx} = 0 \Rightarrow -\left(\frac{1}{4} + A\right) + 2B = 0$ $x = 0, y = 0 \Rightarrow \frac{1}{4} + A = 0$ $A = -\frac{1}{4}, B = 0$ $y = \frac{1}{4}e^{-x}(1 - \cos 2x)$	M1 A1 A1ft B1 M1 A1 A1ft M1* *M1 A1ft A1 [11]	Differentiate & substitute Differentiate their GS of form $y = e^{-x}(P + A \cos nx + B \sin nx)$ where P is constant or linear term, n not 0 or 1 Use conditions From their GS	Or $Ae^{-x} \cos(2x + \alpha)$ Must be in real form If PI $y = axe^{-x}$, then max of M1, A1, A1, B0, M1, A0, A0 (since cannot be consistent) M1, M1, A1. Must have a constant in "their PI" Must have "y =" Allow one error But M0 if leads to solution of $y = 0$ Must have 'y =' and be in real form
6	(i)	$x = 2t + 1, y = 5t - 1, z = t + 2$ $(2t + 1) + 2(5t - 1) - 2(t + 2) = 5$ $\Rightarrow 10t = 10 \Rightarrow t = 1$ Intersect at (3, 4, 3)	B1 M1 A1 [3]	Parameterise Substitute into plane Solve cao	or B1 for y and z correctly in terms of x e.g. $2y = 5x - 7, 2z = x + 3$ Then M1 for full simultaneous equations method. Accept vector form

Question		Answer	Marks	Guidance	
7	(i)	$(ab)^6 = abab\dots ab = a^6b^6$ as commutative $= (a^2)^3 (b^3)^2 = e^3e^2 = e$ So ab has order 1, 2, 3, or 6 $(b \neq a \Rightarrow ab \neq a^2 \Rightarrow ab \neq e$ so ab not order 1) $(ab)^2 = a^2b^2 = eb^2 = b^2$ and b not order 2, so ab not order 2 $(ab)^3 = a^3b^3 = aa^2e = aee = a \neq e$, so ab not order 3 (So must be order 6)	M1 A1 M1 A1 [4]	Must give reason Using orders of a and b Consider other cases AG Complete argument	Some demonstration that they understand commutativity Condone absence of this line Insufficient to merely have simplified all $(ab)^n$
7	(ii)	ac has order 18 18 is LCM of 2 and 9, (so we can use a similar argument to part (i)) So as G has an element of order 18 it must be cyclic.	B1 M1 A1 [3]	or explicit consideration of other cases AG Complete argument	Or abc or generator
8	(i)	$\cos 5\theta + i \sin 5\theta = (\cos \theta + i \sin \theta)^5$ $= c^5 + 5ic^4s - 10c^3s^2 - 10ic^2s^3 + 5cs^4 + is^5$ $\cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$ $= c^5 - 10c^3(1-c^2) + 5c(1-c^2)^2$ $= c^5 - 10c^3 + 10c^5 + 5c - 10c^3 + 5c^5$ $\cos 5\theta = 16c^5 - 20c^3 + 5c$	B1 M1 M1 M1 A1 [5]	Or $\cos 5\theta = \operatorname{re}\{(\cos \theta + i \sin \theta)^5\}$ Take real parts AG	No more than 1 error, can be unsimplified

Question		Answer	Marks	Guidance
8	(ii)	<p>Multiplying by x gives $16x^5 - 20x^3 + 5x = 0$</p> <p>letting $x = \cos \alpha$ gives $\cos 5\alpha = 0$</p> <p>hence $5\alpha = \frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi, \frac{7}{2}\pi, \frac{9}{2}\pi$</p> <p>$\alpha = \frac{1}{10}\pi, \frac{3}{10}\pi, \frac{5}{10}\pi, \frac{7}{10}\pi, \frac{9}{10}\pi$</p> <p>$\cos \frac{5}{10}\pi = 0$ which is not a root</p> <p>so roots $x = \cos \frac{1}{10}\pi, \cos \frac{3}{10}\pi, \cos \frac{7}{10}\pi, \cos \frac{9}{10}\pi$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>[4]</p>	Hence, so no marks for using quadratic at this stage.
8	(iii)	<p>$16x^4 - 20x^2 + 5 = 0 \Leftrightarrow x^2 = \frac{20 \pm \sqrt{80}}{32}$</p> <p>$\cos$ decreases between 0 and π so $\cos \frac{1}{10}\pi$ is greatest root</p> <p>so $\cos \frac{1}{10}\pi = \sqrt{\frac{20 + \sqrt{80}}{32}} = \sqrt{\frac{5 + \sqrt{5}}{8}}$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>Can be gained if seen in (ii)</p> <p>Dep on full marks in (ii)</p>

Question	Answer	Marks	Guidance
1 (i)	$\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ -7 \\ 7 \end{pmatrix} = 7 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ <p>(eg) $z = 0 \Rightarrow 2x + y = 4, 3x + 5y = 13 \Rightarrow x = 1, y = 2$</p> $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	<p>M1 A1</p> <p>M1</p> <p>A1</p>	<p>M1 requires evidence of method for cross product or at least 2 correct values calculated</p> <p>or any valid point e.g.(0, 3, -1), (3, 0, 2)</p> <p>Must have full equation including 'r ='</p> <p>oe vector form</p>
	<p>Alternative: Find one point Find a second point and vector between points</p> <p>multiple of $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$</p> $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ <p>Alternative: Solve simultaneously</p> <p>Point and direction found</p> $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	<p>M1 M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>M1 A1</p> <p>A1</p> <p>[4]</p>	<p>to at least expressions for x,y,z parametrically, or two relationship between 2 variables.</p>

Question	Answer	Marks	Guidance	
1 (ii)	$\frac{ 2 \times 2 + 5 - -2 - 4 }{\sqrt{2^2 + 1^2 + 1^2}} = \frac{7}{\sqrt{6}}$	M1 A1	Condone lack of absolute signs for M1 oe surd form. isw	2.86 with no workings scores M1
	Alternative: find parameter for perpendicular meets plane and use to find distance	M1 [2]	For complete method with calculation errors	look for $\lambda = -7/6$
2	$u = y^2 \Rightarrow \frac{du}{dx} = 2y \frac{dy}{dx}$ <p>so DE $\Rightarrow 2y \frac{dy}{dx} - 4y^2 = 2e^x$</p> $\Rightarrow \frac{du}{dx} - 4u = 2e^x$ $I = \exp \int -4 dx = e^{-4x}$ $e^{-4x} \frac{du}{dx} - 4e^{-4x} u = 2e^{-3x}$ $u e^{-4x} = -\frac{2}{3} e^{-3x} (+A)$ $u = -\frac{2}{3} e^x + A e^{4x}$ $y = \sqrt{-\frac{2}{3} e^x + A e^{4x}}$ <p>Alternative from 4th mark to 6th mark</p> <p>CF: $(u = \dots) A e^{4x}$</p> <p>PI: $u = k e^x, \frac{du}{dx} = k e^x$</p> $k e^x - 4k e^x = 2e^x, \quad k = -\frac{2}{3}$	M1 M1 A1 A1ft M1* *M1dep* M1dep* A1 A1 M1* M1 dep* [8]	Correctly finds or for complete unsimplified substitution Multiples through by IF of form e^{kx} , simplifying RHS Integrates Rearranges to make u or y^2 the subject Cao PI chosen & differentiated correctly Substitutes and solves	Or $\frac{dy}{dx} = \frac{1}{2} u^{-\frac{1}{2}} \frac{du}{dx}$ Can be implied by next A1 Must have form $\frac{du}{dx} + f(x)u = g(x)$ for this mark and any further marks Can be implied by subsequent work No more than 1 numerical error at this step ignore use of ' \pm '

Question	Answer	Marks	Guidance
3 (i)	$z^6 = 1 \Rightarrow z = e^{2k\pi i/6}$ $k = 0, 1, 2, 3, 4, 5$ Diagram	M1 A1 B1 B1 [4]	Oe exactly 6 roots 6 roots in right quadrant, correct angles and moduli accept roots 1, -1 given as integers. as evidenced by labels, circles, or accurate diagram, or by co-ordinates
3 (ii)	$(1+i)^6 = \left(\sqrt{2} e^{\frac{1}{4}\pi i}\right)^6$ $8e^{\frac{6}{4}\pi i}$ $= -8i$ <p>Alternative:</p> $(1+i)^6 = 1 + 6i + 15i^2 + 20i^3 + 15i^4 + 6i^5 + i^6$ $= 1 + 6i - 15 - 20i + 15 + 6i - 1$ $= -8i$ <p>Alternative: $(1+i)^2 = 2i$</p> $(1+i)^6 = (2i)^3$ $= -8i$	M1 M1 A1 M1 M1 A1 M1 M1 A1 [3]	Attempts modulus-argument form, getting at least 1 correct for $(\text{mod})^6$ and $\arg x$ ag complete argument including start line no more than 1 term wrong ag ag Sc 2 for only lines 2 & 3 correct

Question		Answer	Marks	Guidance																		
4	(i)	<table border="1"> <tr> <td>element</td> <td>(1)</td> <td>3</td> <td>7</td> <td>9</td> <td>11</td> <td>13</td> <td>17</td> <td>19</td> </tr> <tr> <td>inverse</td> <td>(1)</td> <td>7</td> <td>3</td> <td>9</td> <td>11</td> <td>17</td> <td>13</td> <td>19</td> </tr> </table>	element	(1)	3	7	9	11	13	17	19	inverse	(1)	7	3	9	11	17	13	19	B1 B1 B1 [3]	2 or more 4 or more all 7 correct Ignore 1
element	(1)	3	7	9	11	13	17	19														
inverse	(1)	7	3	9	11	17	13	19														
4	(ii)	<p>(1 has order 1) 9,11,19 have order 2</p> <p>$3^2 = 9 \Rightarrow 3^4 = 1$ so order 4 similarly 7,13,17 order 4</p> <p>no element of order 8 so not cyclic</p>	M1 B1 A1 [3]	Correctly identifies order of all elements justifies order for at least 1 element of order 4 www Allow one error must show workings towards a^4 for demonstration that these elements are order 4` condone “no generator” in place of “no element of order 8”																		
4	(iii)	<p>{1,13, 9, 17} and {1, 3, 9, 7}</p> <p>$1 \leftrightarrow 1, 9 \leftrightarrow 9, 3 \leftrightarrow 13, 7 \leftrightarrow 17$</p>	M1 B1 A1 M1 A1 [5]	For two sets which both contain “1” and all (4) elements’ inverses One subgroup of order 4 for correspondence of “their” elements of same order or $3 \leftrightarrow 17, 7 \leftrightarrow 13$																		

Question	Answer	Marks	Guidance
5	<p>AE: $\lambda^2 + 5\lambda + 6 = 0$ $\lambda = -2, -3$ CF: $Ae^{-2x} + Be^{-3x}$ PI: $y = ae^{-x}$ $ae^{-x} - 5ae^{-x} + 6ae^{-x} = e^{-x}$ $2a = 1$ $a = \frac{1}{2}$ GS: $(y =) \frac{1}{2}e^{-x} + Ae^{-2x} + Be^{-3x}$ $x = 0, y = 0 \Rightarrow \frac{1}{2} + A + B = 0$ $y' = -\frac{1}{2}e^{-x} - 2Ae^{-2x} - 3Be^{-3x}$ $x = 0, y' = 0 \Rightarrow -\frac{1}{2} - 2A - 3B = 0$ $A = -1, B = \frac{1}{2}$ $y = \frac{1}{2}e^{-x} - e^{-2x} + \frac{1}{2}e^{-3x}$</p>	<p>B1 B1ft B1ft M1 A1 A1ft M1 M1* M1dep* A1 [10]</p>	<p>Differentiate and substitute Use condition on GS Differentiate their GS of form $y = ke^{-x} + Ae^{mx} + Be^{nx}$ where k, m, n are non-zero constants and m, n not 1 Use condition and attempt to find A, B www Must have 'y ='</p>

Question	Answer	Marks	Guidance
6 (i)	$l \parallel \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} \quad \Pi \perp \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix} \quad \text{so} \quad \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix} = 0 \Rightarrow l \parallel \Pi$ <p>$(1, -2, 7)$ on l but $4 \times 1 - 2 - 7 = -1 \neq 8$ so not in Π</p> <p>hence l not in Π</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>dot product of correct vectors = 0</p> <p>substitute point on line into Π and calculate d</p> <p>Full argument includes key components</p> <p>Argument can be about a general point on line</p>
6 (ii)	$(\mathbf{r} =) \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix}$ <p>closest point where meets Π</p> $4(1 + 4\lambda) - (-2 - \lambda) - (7 - \lambda) = 8$ $\Rightarrow \lambda = \frac{1}{2}$ $\Rightarrow \mathbf{r} = \begin{pmatrix} 3 \\ -\frac{5}{2} \\ \frac{13}{2} \end{pmatrix}$	<p>B1</p> <p>M1</p> <p>A1ft</p> <p>A1</p> <p>[4]</p>	<p>parametric form of (x, y, z) substituted into plane</p>
6 (iii)	$\mathbf{r} = \begin{pmatrix} 3 \\ -\frac{5}{2} \\ \frac{13}{2} \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$	<p>B1ft</p> <p>[1]</p>	<p>oe</p> <p>must have “$\mathbf{r} =$”</p>

Question	Answer	Marks	Guidance
7 (i)	$2i \sin \theta = e^{i\theta} - e^{-i\theta}$ $2i \sin n\theta = e^{in\theta} - e^{-in\theta}$ $(2i \sin \theta)^5 = (e^{i\theta} - e^{-i\theta})^5$ $= e^{i5\theta} - 5e^{i3\theta} + 10e^{i\theta} - 10e^{-i\theta} + 5e^{-i3\theta} - e^{-i5\theta}$ $32i \sin^5 \theta = (e^{5i\theta} - e^{-5i\theta}) - 5(e^{3i\theta} - e^{-3i\theta}) + 10(e^{i\theta} - e^{-i\theta})$ $= 2i \sin 5\theta - 5(2i \sin 3\theta) + 10(2i \sin \theta)$ $\sin^5 \theta = \frac{1}{16}(\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta)$	B1 M1* M1dep* A1 [4]	any equivalent form binomial expansion grouping terms AG If use z , must define it can be unsimplified Award B1 then sc M1A1 for candidates who omit this stage from otherwise complete argument must convince on the $\frac{1}{16}$ and on the elimination of i
7 (ii)	$16 \sin^5 \theta - 10 \sin \theta = \sin 5\theta - 5 \sin 3\theta$ $16 \sin^5 \theta - 6 \sin \theta = 0$ $\sin \theta = 0, \pm \sqrt[4]{\frac{3}{8}}$ $\theta = 0, \pm 0.899$	M1* A1 M1dep* A1 [4]	Attempts to eliminate $\sin 5\theta$ and $\sin 3\theta$ must have 3 values for $\sin \theta$ Or $16 \sin^5 \theta = 6 \sin \theta$

Question	Answer	Marks	Guidance
8 (i)	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ is identity}$ $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}^{-1} = \frac{1}{a^2 + b^2} \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \in G$ $\begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} c & -d \\ d & c \end{pmatrix} = \begin{pmatrix} ac - bd & -bc - ad \\ bc + ad & ac - bd \end{pmatrix}$ <p>and</p> $(ac - bd)^2 + (bc + ad)^2 = a^2c^2 + b^2d^2 + b^2c^2 + a^2d^2$ $= (a^2 + b^2)(c^2 + d^2) \neq 0$	<p>B1</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1</p> <p>[6]</p>	<p>for M1, must at least get matrix in form $\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$, or state existence of inverse due to non-singularity</p> <p>Must not attempt to prove commutativity in (i)</p>
8 (ii)	$\begin{pmatrix} c & -d \\ d & c \end{pmatrix} \begin{pmatrix} a & -b \\ b & a \end{pmatrix} = \begin{pmatrix} ac - bd & -bc - ad \\ bc + ad & ac - bd \end{pmatrix}$ $= \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} c & -d \\ d & c \end{pmatrix} \text{ so commutative}$	<p>M1</p> <p>A1</p> <p>[2]</p>	<p>must also consider matrices reversed, but may be seen in (i)</p>
8 (iii)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ <p>order 4</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>g^2 must be correct</p> <p>allow 1 error in getting g^4</p>