1	$\pm(5.4\cos45^{o} - 8.7)$	M1		For attempting to find Δv in **i** dir'n
		M1		For using I = m(Δv) in **i** direction
	$I\cos\theta = \pm 0.4(5.4\cos45^{o} - 8.7)$	A1		(= $\mp$1.953)
	$I\sin\theta$ = 0.4x5.4sin45	B1		(= 1.527)
	$I = \sqrt{(1.527^2 + 1.953^2)}$ or $\theta = \tan^{-1}[1.527/(-1.953)]$	M1		For using Pythagoras or trig.
	Magnitude is 2.48 $kgms^{-1}$	A1		
	Direction is 142^{o} to original dir'n.	A1	[7]	Accept $\theta = 38.0^{o}$ with θ shown appropriately
OR		M1		For using Impulse = mass x Δv
		M1		For appropriate use of cosine rule
	$I = 0.4(5.4^2 + 8.7^2 - 2x5.4x8.7\cos45^{o})^{½}$	A1		
	Magnitude is 2.48 $kgms^{-1}$	A1		
		M1		For appropriate use of sine rule
	$\sin\theta/5.4 = \sin45^{o}/6.1976$	A1		
	$\theta = 38.0^{o}$	A1		

2	(i)	M1		For correct use of Newton's 2^{nd} law
	$0.5dv/dt = 1 + kt^2$	A1		
	$v = 2t + 2kt^3/3$	A1	[3]	SR(max 1/3) for omission of mass but otherwise correct $v = t + kt^3/3$ B1
	(ii) $x = t^2 + kt^4/6$	M1		For integration w.r.t. t
	2 = 1 + k/6	M1		For substitution and attempting to solve for k
	k = 6	A1		
		M1		For attempting to solve quadratic in t^2 for t
	t = 2	A1	[5]	With no extra solutions

3	(i)	M1		For use of EE formula
	EE = λ x $(5-3)^2$ /(2 x 3)	A1		
	$2\lambda/3$ = 1.6 x 9.8 x 5	M1		For equating EE and PE
	λ = 117.6 N	A1	[4]	AG
	(ii)	M1		For use of conservation of energy
	$0.5x1.6v^2 = 1.6x9.8x4.5 - 117.6x1.5^2/(2x3)$	A2,1,0		-1 each error
	v = 5.75 ms^{-1}	A1	[4]	

4	Perp. vel. of A after impact = 4	B1	
		M1	For using cons'n of m'm'tum // l.o.c
	[5x0] - 2x4 = 5a + 2b	A1	
		M1	Using N.E.L. // l.o.c.
	0.75 x 4 = b-a	A1	
		M1	For solving sim. equ.
	Speed of B is 1ms^{-1}; direction //l.o.c. and to the right	A1	
	$v_A = \sqrt{(4^2 + (-2)^2)}$	M1	For method of finding the speed of A
	tan(angle) = 4/2	M1	For method of finding the direction of A
	Speed of A is 4.47 ms^{-1}; direction is 63.4^o to l.o.c. and to the left	A1 [10]	

5	(i)	M1	For any moment equ. that includes F and all other relevant forces
	1.8F = 0.9x40 +1.4x9	A2,1,0	-1 each error
	Magnitude is 27 N	A1 [4]	AG
	(ii) Vertical comp. is 22 N downwards	B1	
		M1	For any moment equ. that includes X and all other relevant forces
	1.2X = (40+9-27)x(3.8-1.8) + 64 x1 (1.2X = 44 + 64)	A2,1,0 ft	-1 each error. ft wrong vert. comp.
	Horizontal comp. is 90 N to the left	A1 [5]	
	(iii) μ = 27/[90]	M1	For use of μ =F/R
	Coefficient of friction is 0.3	A1 ft [2]	ft wrong answer in (ii)

6	(i)	M1	For use of conservation of energy
	$0.5x0.3v^2 - 0.5x0.3x2^2 = 0.3x9.8x0.5\cos60 - 0.3x9.8x0.5\cos\theta$	A2,1,0	-1 each error
	$v^2 = 8.9 - 9.8\cos\theta$	A1 [4]	AG
	(ii)	M1	For using Newton's 2nd law radially
	$T + 0.3x9.8\cos\theta = 0.3v^2/0.5$	A1	
	$T + 2.94\cos\theta = 0.6(8.9 - 9.8\cos\theta)$	M1	For correct substitution for v^2
	Tension is$(5.34 - 8.82\cos\theta)$N	A1 [4]	Accept any correct form
	(iii)	M1	For using T = 0
	Basic value $\theta = 52.7^o$	A1 ft	ft any T of the form a - b$\cos\theta$
	Angle = (360-52.7) - 60	M1	
	Angle turned through is 247^o	A1 [4]	

7	(i)	M1		For using $T = \lambda e/L$ once
	For 180e/1 or 360(0.8-e)/1.2 **or** T_A = 180 x 0.5/1 or T_B = 360 x 0.3/1.2	A1		
	480e = 240 or T_A = 90, T_B = 90	M1		For using $T_A(e) = T_B(e)$ or attempting to show $T_A = T_B$ when BQ = 1.5
	BQ = 1 + 0.5 = 1.5 m or $T_A = T_B$	A1	[4]	AG
	(ii) $T_B = 360(0.3 - x)/1.2$	B1		
	$T_A = 180(0.5 + x)$	B1		
	$1.2d^2x/dt^2 = 300(0.3-x) - 180(0.5+x)$	M1		For using Newton's 2nd law
	$d^2x/dt^2 = -400x$	A1		
	Period is $2\pi/\sqrt{[400]}$ = 0.314 s	A1	[5]	AG
	(iii)	M1		For using $T_B = 0$
	Max amplitude = 1.5 - 1.2 = 0.3 m	A1		
	amplitude = $u/\sqrt{400}$ or $180 \times 0.5^2/(2 \times 1) + 360 \times 0.3^2/(2 \times 1.2) + ½\, 1.2u_{max}^2 = 180 \times 0.8^2/(2 \times 1)$	M1		For using Amp. = u/ω or 'energy at equil. pos'n = energy at max. displ.'
	Maximum value of u is 6	A1	[4]	AG
	(iv) -0.2 = 0.3sin20t	M1		For relevant trig. equation
	20t = 0.7297 + 3.142	M1		For method of obtaining relevant solution
	Time taken is 0.194s	A1	[3]	

1	**(i)**		M1		For using I = Δ(mv) in the direction of the original motion (or equivalent from use of relevant vector diagram).
		$20\cos\theta = 0.4 \times 25$	A1		
		Direction at angle 120^o to original motion	A1	3	Accept $\theta = 60^o$ with θ correctly identified.
	(ii)		M1		For using I = Δ(mv) perp. to direction of the original motion (or equivalent from use of relevant vector diagram).
		$20\sin 60^o = 0.4v$	A1ft		
		Speed is 43.3 ms^{-1}	A1	3	
2			M1		For applying Newton's 2^{nd} Law.
			M1		For using a = v(dv/dx).
		$2v(dv/dx) = -(2v + 3v^2)$	A1		
			M1		For separating variables and attempting to integrate.
		$2/3\ln(2 + 3v) = -x$ (+C)	A1ft		ft absence of minus sign,
		[$2/3\ln 14 = C$]	M1		For using v(0) = 4.
		[$2/3\ln 2 = -x + 2/3\ln 14$]	M1		For attempting to solve v(x) = 0 for x.
		Comes to rest after travelling 1.30m	A1	8	AG

3	**(i)**		M1		For taking moments about C for the whole structure.
		$1.4R = 0.35 \times 360 + 1.05 \times 200$	A1		
		Magnitude is 240N	A1		AG
			M1		For taking moments about A for the rod AB.
		$0.7 \times 240 = 0.35 \times 200 + 1.05T$	A1		
		Tension is 93.3N	A1	6	
	OR **(i)**		M1		For taking moments about A for AB and AC.
		$0.7R_B = 70 + 1.05T$ and $0.7R_C = 126 + 1.05T$	A1		
			M1		For eliminating T or for adding the equations, and then using $R_B + R_C = 560$.
		$0.7(560 - R_B) - 0.7R_B = 126 - 70$ or $0.7 \times 560 = 70 + 126 + 2.1T$	A1		For a correct equation in R_B only or T only
		Magnitude is 240N	A1		AG
		Tension is 93.3N	A1	6	
	(ii)	Horizontal component is 93.3 N to the left	B1ft		
		$Y = 240 - 200$	M1		For resolving forces vertically.
		Vertical component is 40 N downwards	A1	3	

4	**(i)**		M1		For using Newton's 2nd Law perp. to string with a = L$\ddot{\theta}$.
		L(m)$\ddot{\theta}$ = -(m)gsinθ or (m)$\ddot{s}$ = -(m)gsin(s/L)	A1		
		$\ddot{\theta} \approx$ -kθ or $\ddot{s}$ = -ks [and motion is therefore approx. simple harmonic]	B1		
			M1		For using T = 2π/n and k = w^2 or T = $2\pi\sqrt{L/g}$ for simple pendulum.
		Period is 3.14s.	A1	5	AG
	(ii)		M1		For using $\dot{\theta}^2 = n^2(\theta_0{}^2 - \theta^2)$ or the principle of conservation of energy
		$\dot{\theta}^2$ =4(0.1^2 – 0.06^2) or ½ m(2.45$\dot{\theta}$)^2 = 2.45mg(cos0.06 – cos0.1)	A1		
		Angular speed is 0.16 rad s^{-1}.	A1	3	(0.1599… from energy method)
	OR	(in the case for which (iii) is attempted before (ii))			
	(ii)	[$\dot{\theta}$= -0.2sin2t]	M1		For using $\dot{\theta}$= d(Acos nt)/dt
		$\dot{\theta}$= -0.2sin(2x0.464)	A1ft		
		Angular speed is 0.16 rad s^{-1}.	A1	3	
	(iii)		M1		For using θ= Acos nt or Asin(π/2 – nt) or for using θ= Asin nt and T =$t_{0.1}$ – $t_{0.06}$
		0.06 = 0.1cos2t or 0.1sin(π/2 – 2t) or 2T = π/2 – sin^{-1}0.6	A1ft		ft angular displacement of 0.04 instead of 0.06
		Time taken is 0.464s	A1	3	

5			M1		Σmv conserved in **i** direction.
		$2x12\cos60^o - 3x8 = 2a + 3b$	A1		
			M1		For using NEL
		For LHS of equation below	A1		
		$0.5(12\cos60^o + 8) = b - a$	A1		Complete equation with signs of a and b consistent with previous equation.
			M1		For eliminating a or b.
		Speed of B is $0.4ms^{-1}$ in **i** direction	A1		
		a = -6.6	A1		
		Component of A's velocity in **j** direction is $12\sin60^o$	B1		May be shown on diagram or implied in subsequent work.
		Speed of A is $12.3ms^{-1}$	B1ft		
			M1		For using $\theta = \tan^{-1}$(**j**comp/$\pm$ **i** comp)
		Direction is at 122.4^o to the **i** direction	A1ft	12	Accept $\theta = 57.6^o$ with θ correctly identified.
6	**(i)**	T = 1470x/30	B1		
		[49x = 70x9.8]	M1		For using T = mg
		x = 14	A1		
		Distance fallen is 44m	A1ft	4	
	(ii)	PE loss = 70g(30 + 14)	B1ft		
		EE gain = $1470x14^2/(2x30)$	B1ft		
		[½ $70v^2$ = 30184 – 4802]	M1		For a linear equation with terms representing KE, PE and EE changes.
		Speed is $26.9ms^{-1}$	A1	4	AG
	OR (ii)	[0.5 v^2 = 14g – 68.6 + 30g]	M1		For using Newton's 2^{nd} law (vdv/dx = g – 0.7x), integrating (0.5 v^2 = gx – $0.35x^2$ + k), using $v(0)^2$ = 60g➔ k = 30g, and substituting x = 14.
		For 14g + 30g	B1ft		
		For $\mp$68.6	B1ft		Accept in unsimplified form.
		Speed is $26.9ms^{-1}$	A1	4	AG
	(iii)	PE loss = 70g(30 + x)	B1ft		
		EE gain = $1470x^2/(2x30)$	B1ft		
		[x^2 – 28x – 840 = 0]	M1		For using PE loss = KE gain to obtain a 3 term quadratic equation.
		Extension is 46.2m	A1	4	
	OR (iii)		M1		For identifying SHM with n^2 = 1470/(70x30)
			M1		For using v_{max} = An
		$A = 26.9/\sqrt{0.7}$	A1		
		Extension is 46.2m	A1	4	

7	**(i)**	½ $0.3v^2$ + ½ $0.4v^2$	B1		
		$\pm 0.3g(0.6\sin\theta)$	B1		
		$\pm 0.4g(0.6\theta)$	B1		
		$[0.35v^2 = 2.352\theta - 1.764\sin\theta]$	M1		For using the principle of conservation of energy.
		$v^2 = 6.72\theta - 5.04\sin\theta$	A1	5	AG
	(ii)		M1		For applying Newton's 2nd Law radially to P and using $a = v^2/r$
		$0.3(v^2/0.6) = 0.3g\sin\theta - R$	A1		
		[½ $(6.72\theta - 5.04\sin\theta) =$ $0.3g\sin\theta - R]$	M1		For substituting for v^2.
		Magnitude is $(5.46\sin\theta - 3.36\theta)$N	A1		AG
		$[5.46\cos\theta - 3.36 = 0]$	M1		For using $dR/d\theta = 0$
		Value of θ is 0.908	A1	6	
	(iii)	$[T - 0.3g\cos\theta = 0.3a]$	M1		For applying Newton's 2nd Law tangentially to P
		$[0.4g - T = 0.4a]$	M1		For applying Newton's 2nd Law to Q [If $0.4g - 0.3g\cos\theta = 0.3a$ is seen, assume this derives from $T - 0.3g\cos\theta = 0.3a$ ……. M1 and $T = 0.4g$ ……. M0]
		Component is $5.6 - 4.2\cos\theta$	A1	3	
	OR (iii)	$0.4g - 0.3g\cos\theta = (0.3 + 0.4)a$	B2		
		Component is $5.6 - 4.2\cos\theta$	B1	3	
	OR (iii)	$[2v(dv/d\theta) = 6.72 - 5.04\cos\theta]$	M1		For differentiating v^2 (from (i)) w.r.t. θ
		$2(0.6a) = 6.72 - 5.04\cos\theta$	M1		For using $v(dv/d\theta) = ar$
		Component is $5.6 - 4.2\cos\theta$	A1	3	

1		M1		For using the principle of conservation of energy
	½ 0.6×5^2 – ½ $0.6v^2 = 0.6g(2 \times 0.4)$ [$v^2 = 9.32$]	A1		
	[T + 0.6g = 0.6a]	M1		For using Newton's second law
	[a = 9.32/0.4]	M1		For using $a = v^2/r$
	T + 0.6g = 0.6x9.32/0.4	A1ft		ft incorrect energy equation
	Tension is 8.1N	A1	6	

2	$28\cos30^o – 10\cos30^o$ [= $\lvert \Delta v_H \rvert$ = $(I/m)\cos\theta$]	B1		
	$10\sin30^o + 28\sin30^o$ [= $\lvert \Delta v_V \rvert$ = $(I/m)\sin\theta$]	B1		
	[X = - $I\cos\theta$ = -0.8885, Y = $I\sin\theta$ = 1.083]	M1		For using mv change for component or resultant
		M1		For using $I^2 = X^2 + Y^2$
	I = 1.40	A1		
	[$\tan\theta$ = 1.083/0.8885 or 19/15.588..]	M1		For using $\theta = \tan^{-1}(Y/-X)$ or $\tan^{-1}(\lvert \Delta v_V \rvert / \lvert \Delta v_H \rvert)$
	θ = 50.6	A1	7	

	ALTERNATIVELY			
2		M1		For using cosine rule in correct triangle
	$(I/m)^2 = 28^2 + 10^2 – 2 \times 28 \times 10\cos60^o$ [=604]	A1		
	[$I = 0.057\sqrt{604}$]	M1		For using I = mv change
	I = 1.40	A1		
		M1		For using sine rule in correct triangle
	$(I/m)/\sin60^o$ = $10/\sin(\theta-30^o)$ or $28/\sin(150^o-\theta)$	A1		
	θ = 50.6	A1	7	

3	(i) $160a = 2aY$	M1		For taking moments for AB about B
	Component at B is 80N	A1		
	Component at C is 240N	B1ft	3	ft 160 + Y
	(ii)	M1		For taking moments for BC about B or C (and using X = F) or for whole about A
	$160a\cos60^o + 2aF\sin60^o = 240x2a\cos 60^o$ or $80x2a\cos60^o + 160a\cos60^o = 2aX\sin60^o$ or $240(2 + 2\cos60^o)a = 160a + 160(2 + \cos60^o)a + 2aF\sin60^o$	A1ft		
	Frictional force is 92.4N	A1		
	Direction is to the left	B1	4	
	(iii) [92.4/240]	M1		For using $F = \mu R$
	Coefficient is 0.385	A1ft	2	

4	(i)	M1		For using T = mg and $T = \lambda e/L$
	$3.5e/0/7 = 0.2g$ [e = 0.392]	A1		
	Position is 1.092m below O.	A1	3	AG
	(ii)	M1		For using Newton's second law
	$0.2g – 3.5(0.392 + x)/0.7 = 0.2a$	A1ft		ft incorrect e
	$a = -25x$	A1ft		ft incorrect e
	[$25A^2 = 1.6^2$ or $\frac{1}{2}(0.2)1.6^2 + 3.5x0.392^2/(2x0.7) + 0.2gA = 3.5x(0.392 + A)^2/(2x0.7)$]	M1		For using $A^2n^2 = v_{max}^2$ or Energy at lowest point = energy at equilibrium point (4 terms needed including 2 EE terms)
	Amplitude is 0.32m	A1ft	5	
	(iii) [$x = 0.32\sin2^c$]	M1		For using x = Asin nt or Acos(π/2 - nt)
	$x = 0.291$	A1		
	[$v = 0.32x5\cos2^c$ or $v^2 = 25(0.32^2 – 0.291^2)$ or $0.256 + 0.38416 + 0.2g(0.291) = \frac{1}{2} 0.2v^2 + 2.5(0.683)^2$]	M1		For using v = Ancos nt or $v^2 = n^2(A^2 – x^2)$ or Energy at equilibrium point = energy at x = 0.291
	$v^2 = 0.443$	A1		May be implied
	v = -0.666 (or 0.666 upwards)	A1	5	

5	(i) $[mg - mkv^2 = ma]$	M1		For using Newton's second law
	$(v\,dv/dx)/(g - kv^2) = 1$	A1	2	AG
	(ii) $[-\frac{1}{2}[\ln(g - kv^2)]/k = x \quad (+C)]$	M1		For separating variables and attempting to integrate
	$[-(\ln g)/2k = C]$	M1		For using v(0) = 0 to find C
	$x = [-\frac{1}{2}[\ln\{(g - kv^2)/g\}]/k$	A1		Any equivalent expression for x
	$[\ln\{(g - kv^2)/g\} = \ln(e^{-2kx})]$	M1		For expressing in the form $\ln f(v^2) = \ln g(x)$ or equivalent
	$v^2 = (1 - e^{-2kx})g/k$	A1		
		M1		For using $e^{-Ax} \rightarrow 0$ for +ve A
	Limiting value is $\sqrt{g/k}$	A1ft	7	AG
	(iii) $[1 - e^{-600k} = 0.81]$	M1		For using $v^2(300) = 0.9^2 g/k$
	$[-600k = \ln(0.19)]$	M1		For using logarithms to solve for k
	k = 0.00277	A1	3	

6	(i) $[u \sin 30^o = 3]$	M1		For momentum equation for B, normal to line of centres
	u = 6	A1	2	
	(ii) $[4\sin 88.1^o = v \sin 45^o]$	M1		For momentum equation for A, normal to line of centres
	v = 5.65	A1		
		M1		For momentum equation along line of centres
	$0.4(4\cos 88.1^o) - mu\cos 30^o = -0.4v\cos 45^o$	A1		
	m = 0.318	A1	5	
	(iii)	M1		For using NEL
	$0.75(4\cos\theta + u\cos 30^o) = v\cos 45^o$	A1		
	$4\sin\theta = v\sin 45^o$	B1		
	$[3\cos\theta + 4.5\cos 30^o = 4\sin\theta]$	M1		For eliminating v
	$8\sin\theta - 6\cos\theta = 9\cos 30^o$	A1	5	AG
7	(i)(a) Extension = $1.2\alpha - 0.6$	B1		
	$[T = mg\sin\alpha]$	M1		For resolving forces tangentially
	$0.5 \times 9.8\sin\alpha = 6.86(1.2\alpha - 0.6)/0./6$	A1ft		
	$\sin\alpha = 2.8\alpha - 1.4$	A1	4	AG
	(i)(b) [0.8, 0.756.., 0.745.., 0.742.., 0.741.., 0.741..,]	M1		For attempting to find α_2 and α_3
	$\alpha = 0.74$	A1	2	
	(ii) $\Delta h = 1.2(\cos 0.5 - \cos 0.8)$ [0.217…]	B1		
	[0.5x9.8x0.217.. = 1.06355..]	M1		For using $\Delta(PE) = mg\Delta h$
	$[6.86(1.2 \times 0.8 - 0.6)^2/(2 \times 0.6) = 0.74088]$	M1		For using EE = $\lambda x^2/2L$
		M1		For using the principle of conservation of energy
	$\frac{1}{2}\, 0.5v^2 = 1.06355.. - 0.74088$	A1		Any correct equation for v^2
	Speed is 1.14ms^{-1}	A1		
	Speed is decreasing	B1ft	7	

1	(i) [$\omega = 2\pi/6.1 = 1.03$]	M1		For using $T = 2\pi/\omega$
		M1		For using $v_{max} = a\omega$
	Speed is $3.09ms^{-1}$	A1	3	
	(ii)	M1		For using $v^2 = \omega^2(A^2 - x^2)$ or for using $v = A\omega\cos\omega t$ and $x = A\sin\omega t$
	$2.5^2 = 1.03^2(3^2 - x^2)$ or $x = 3\sin(1.03x0.60996..)$	A1ft		ft incorrect ω
	Distance is 1.76m	A1	3	

2	[Magnitudes 0.6, 0.057 x 7, 0.057 x 10]	M1		For triangle with magnitudes shown
	For magnitudes of 2 sides correctly marked	A1		
	For magnitudes of all 3 sides correctly marked	A1		
		M1		For attempting to find angle (α) opposite to the side of magnitude 0.057 x 7
		M1		For correct use of the cosine rule or equivalent
	$0.399^2 = 0.57^2 + 0.6^2 - 2 \times 0.57 \times 0.6\cos\alpha$	A1ft		
	Angle is 140^o	A1	7	$(180 - 39.8)^o$

2	ALTERNATIVE METHOD			
		M1		For using I= Δmv parallel to the initial direction of motion or parallel to the impulse
	$-0.6\cos\alpha = 0.057 \times 7\cos\beta - 0.057 \times 10$ or $0.6 = 0.057x10\cos\alpha + 0.057x7\cos\gamma$	A1		
		M1		For using I= Δmv perpendicular to the initial direction of motion or perpendicular to the impulse
	$0.6\sin\alpha = 0.057 \times 7\sin\beta$ or $0.057x10\sin\alpha = 0.057x\ 7\sin\gamma$	A1		
		M1		For eliminating β *or γ
	$0.399^2 = (0.57 - 0.6\cos\alpha)^2 + (0.6\sin\alpha)^2$ or $0.399^2 = (0.6 - 0.57\cos\alpha)^2 + (0.057\sin\alpha)^2$	A1ft		
	Angle is 140^o	A1	7	$(180 - 39.8)^o$

3	(i) $[0.2v\,dv/dx = -0.4v^2]$	M1		For using Newton's second law with a = v dv/dx
	$(1/v)\,dv/dx = -2$	A1	2	AG
	(ii) $[\int (1/v)dv = \int -2dx]$	M1		For separating variables and attempting to integrate
	ln v = -2x (+C)	A1		
	[ln v = -2x + ln u]	M1		For using v(0) = u
	$v = ue^{-2x}$	A1	4	AG
	(iii) $[\int e^{2x}dx = \int u\,dt]$	M1		For using v = dx/dt and separating variables
	$e^{2x}/2 = ut$ (+C)	A1		
	$[e^{2x}/2 = ut + ½]$	M1		For using x(0) = 0
	u = 6.70	A1	4	Accept $(e^4 – 1)/8$

	ALTERNATIVE METHOD FOR PART (iii)			
	$[\int \frac{1}{v^2}dv = -2\int dt$ ➔ -1/v = -2t + A, and A = -1/u]	M1		For using a = dv/dt, separating variables, attempting to integrate and using v(0) = u
		M1		For substituting $v = ue^{-2x}$
	$-e^{2x}/u = -2t – 1/u$	A1		
	u = 6.70	A1	4	Accept $(e^4 – 1)/8$

4	$y = 15\sin\alpha$ (=12)	B1		
	$[4(15\cos\alpha) – 3 \times 12 = 4a + 3b]$	M1		For using principle of conservation of momentum in the direction of l.o.c.
	Equation complete with not more than one error	A1		
	4a + 3b = 0	A1		
		M1		For using NEL in the direction of l.o.c.
	$0.5(15\cos\alpha + 12) = b - a$	A1		
	[a = -4.5, b = 6]	M1		For solving for a and b
	[Speed = $\sqrt{(-4.5)^2 + 12^2}$, Direction $\tan^{-1}(12/(-4.50))$]	M1		For correct method for speed or direction of A
	Speed of A is 12.8ms^{-1} and direction is 111^o anticlockwise from '**i**' direction	A1		Direction may be stated in any form , including $\theta = 69^o$ with θ clearly and appropriately indicated
	Speed of B is 6ms^{-1} to the right	A1	10	Depends on first three M marks

5	(i)	M1		For taking moments of forces on BC about B
	$80 \times 0.7\cos 60^o = 1.4T$	A1		
	Tension is 20N	A1		
	$[X = 20\cos 30^o]$	M1		For resolving forces horizontally
	Horizontal component is 17.3N	A1ft		ft $X = T\cos 30^o$
	$[Y = 80 - 20\sin 30^o]$	M1		For resolving forces vertically
	Vertical component is 70N	A1ft	7	ft $Y = 80 - T\sin 30^o$
	(ii)	M1		For taking moments of forces on AB, or on ABC, about A
	$17.3 \times 1.4\sin\alpha = (80 \times 0.7 + 70 \times 1.4)\cos\alpha$ or $80 \times 0.7\cos\alpha + 80(1.4\cos\alpha + 0.7\cos 60^o) = 20\cos 60^o(1.4\cos\alpha + 1.4\cos 60^o) + 20\sin 60^o(1.4\sin\alpha + 14\sin 60^o)$	A1ft		
	$[\tan\alpha = (\frac{1}{2}\,80 + 70)/17.3 = 11/\sqrt{3}]$	M1		For obtaining a numerical expression for $\tan\alpha$
	$\alpha = 81.1^o$	A1	4	

ALTERNATIVE METHOD FOR PART (i)			
	M1		For taking moments of forces on BC about B
$H \times 1.4\sin 60^o + V \times 1.4\cos 60^o = 80 \times 0.7\cos 60^o$	A1		Where H and V are components of T
	M1		For using $H = V\sqrt{3}$ and solving simultaneous equations
Tension is 20N	A1		
Horizontal component is 17.3N	B1ft		ft value of H used to find T
$[Y = 80 - V]$	M1		For resolving forces vertically
Vertical component is 70N	A1ft	7	ft value of V used to find T

6 (i)	[T = 2058x/5.25]	M1		For using T = λx/L
	2058x/5.25 = 80 x 9.8 (x = 2)	A1		
	OP = 7.25m	A1	3	AG From 5.25 + 2
(ii)	Initial PE = (80 + 80)g(5) (= 7840) or (80 + 80)gX used in energy equation	B1		
	Initial KE = ½ (80 + 80)3.5^2 (= 980)	B1		
	[Initial EE = $2058 \times 2^2/(2 \times 5.25)$ (= 784), Final EE = $2058 \times 7^2/(2 \times 5.25)$ (= 9604), or $2058(X + 2)^2/(2 \times 5.25)$]	M1		For using EE = $\lambda x^2/2L$
	[Initial energy = 7840 + 980 + 784, final energy = 9604 or 1568X + 980 + 784 = $196(X^2 + 4X + 4)$ ➔ $196X^2 - 784X - 980 = 0$]	M1		For attempting to verify compatibility with the principle of conservation of energy, or using the principle and solving for X
	Initial energy = final energy or X = 5 ➔P&Q just reach the net	A1	5	AG
(iii)	[PE gain = 80g(7.25 + 5)]	M1		For finding PE gain from net level to O
	PE gain = 9604	A1		
	PE gain = EE at net level ➔ P just reaches O	A1	3	AG
(iv)	For any one of ‘light rope’, ‘no air resistance’, ‘no energy lost in rope’	B1		
	For any other of the above	B1	2	

FIRST ALTERNATIVE METHOD FOR PART (ii)			
[160g – 2058x/5.25 = 160v dv/dx]	M1		For using Newton’s second law with a = v dv/dx, separating the variables and attempting to integrate
$v^2/2 = gx - 1.225x^2$ (+ C)	A1		Any correct form
	M1		For using v(2) = 3.5
C = -8.575	A1		
$[v(7)^2]/2$ = 68.6 – 60.025 – 8.575 = 0 ➔P&Q just reach the net	A1	5	AG

SECOND ALTERNATIVE METHOD FOR PART (ii)			
$\ddot{x} = g - 2.45x$ (= -2.45(x – 4))	B1		
	M1		For using $n^2 = 2.45$ and $v^2 = n^2(A^2 - (x-4)^2)$
$3.5^2 = 2.45(A^2 - (-2)^2)$ (A = 3)	A1		
[(4 – 2) + 3]	M1		For using ‘distance travelled downwards by P and Q = distance to new equilibrium position + A
distance travelled downwards by P and Q = 5 ➔P&Q just reach the net	A1	5	AG

7	(i) $[a = 0.7^2/0.4]$	M1		For using $a = v^2/r$
	For not more than one error in $T - 0.8g\cos60^o = 0.8 \times 0.7^2/0.4$	A1		
	Above equation complete and correct	A1		
	Tension is 4.9N	A1	4	
	(ii)	M1		For using the principle of conservation of energy
	$\frac{1}{2}0.8v^2 = \frac{1}{2}0.8(0.7)^2 + 0.8g0.4 - 0.8g0.4\cos60^o$	A1		$(v = 2.1)$
	$(2.1 - 0)/7 = 2u$	M1		For using NEL
	Q's initial speed is 0.15ms^{-1}	A1	4	AG
	(iii)	M1		For using Newton's second law transversely
	$(m)0.4\ddot{\theta} = -(m)g\sin\theta$	A1		*Allow m = 0.8 (or any other numerical value)
	$[0.4\ddot{\theta} \approx -g\theta]$	M1		For using $\sin\theta \approx \theta$
	$[\frac{1}{2}m0.15^2 = mg0.4(1 - \cos\theta_{max})$ $\rightarrow \theta_{max} = 4.34^o\ (0.0758\text{rad})]$	M1		For using the principle of conservation of energy to find θ_{max}
	θ_{max} small justifies $0.4\ddot{\theta} \approx -g\theta$, and this implies SHM	A1	5	
	(iv) $[T = 2\pi/\sqrt{24.5} = 1.269..]$ $[\sqrt{24.5}\,t = \pi]$	M1		For using $T = 2\pi/n$ or for solving either $\sin nt = 0$ (non-zero t) (considering displacement) or $\cos nt = -1$ (considering velocity)
	Time interval is 0.635s	A1ft	2	From $t = \frac{1}{2}T$

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1	(i) $[0.5(v_x - 5) = -3.5,\ 0.5(v_y - 0) = 2.4]$	M1		For using I = m(v – u) in x or y direction
	Component of velocity in x-direction is -2ms^{-1}	A1		
	Component of velocity in y-direction is 4.8ms^{-1}	A1		
	Speed is 5.2ms^{-1}	A1	4	AG
SR For candidates who obtain the speed without finding the required components of velocity (max 2/4)				
	Components of momentum after impact are -1 and 2.4 Ns	B1		
	Hence magnitude of momentum is 2.6 Ns and required speed is $2.6/0.5 = 5.2\text{ms}^{-1}$	B1		
	(ii)	M1		For using $I_y = m(0 - v_y)$ or I_y = -y-component of 1st impulse
	Component is –2.4Ns	A1	2	

2	(i)	M1		For 2 term equation, each term representing a relevant moment
	$50\text{x}1\sin\beta = 75\text{x}2\cos\beta$	A1		
	$\tan\beta = 3$	A1	3	AG
	(ii) Horizontal force is 75N	B1		
	Vertical force is 50N	B1	2	
	(iii)	M1		For taking moments about A for the whole or for AB only
	For not more than one error in $W\text{x}1\sin\alpha + 50(2\sin\alpha + 1\sin\beta) = 75(2\cos\alpha + 2\cos\beta)$ or $W\text{x}1\sin\alpha + 50\text{x}2\sin\alpha = 75\text{x}2\cos\alpha$	A1		Where $\tan\alpha = 0.75$
	0.6W + 107.4... = 167.4… or 0.6W + 60 = 120	A1		
	W = 100	A1	4	

3	(i)	M1		For using the principle of conservation of momentum in the **i** direction
	6x4 – 3x8 = 6a + 3b (0 = 2a + b)	A1		
		M1		For using NEL
	(4 + 8)e = b – a (12e = b – a)	A1		
	Component is 4e ms^{-1} to the left	A1	5	‘to the left’ may be implied by a = -4e and arrow in diagram
	(ii) b = 8e ms^{-1}	B1ft		ft b = -2a or b = a + 12e
		M1		For using ‘**j** component of A’s velocity remains unchanged’
	$(8e)^2 = (4e)^2 + v^2$	A1ft		ft $b^2 = a^2 + v^2$
	v = 4	A1	4	

4	(i) [mg – 0.49mv = ma]	M1		For using Newton’s second law
	$mv\dfrac{dv}{dx} = mg - 0.49mv$	A1		
	$\left[\dfrac{v(dv/dx)}{g - 0.49v} = 1\right]$	M1		For relevant manipulation
	$\left[\dfrac{v}{9.8 - 0.49v} \equiv \dfrac{-1}{0.49}\left(\dfrac{(9.8 - 0.49v) - 9.8}{9.8 - 0.49v}\right)\right]$	M1		For synthetic division of v by g - 0.49v, or equivalent
	$\left(\dfrac{20}{20 - v} - 1\right)\dfrac{dv}{dx} = 0.49$	A1	5	AG
	(ii)	M1		For separating the variables and integrating
	$\int \dfrac{20}{20 - v}dv = -20\ln(20 - v)$	B1		
	-20 ln(20 – v) –v = 0.49x (+C)	A1ft		
	[-20 ln20 = C]	M1		For using v = 0 when x = 0
	x = 40.8(ln20 – ln(20 – v)) – 2.04v	A1	5	Accept any correct form

5	(i)	M1		For using Newton's second law with a = 0
	$mg\sin 30^o = 0.75mgx/1.2$	A1		
	Extension is 0.8m	A1	3	AG
	(ii) PE loss = $mg(1.2 + 0.8)\sin 30^o$ (mg)	B1		
	EE gain = $0.75mg(0.8)^2/(2x1.2)$ (0.2mg)	B1		
	[$\frac{1}{2}mv^2 = mg - 0.2mg$]	M1		For an equation with terms representing PE, KE and EE in linear combination
	Maximum speed is $3.96ms^{-1}$	A1	4	
	(iii) PE loss = $mg(1.2 + x)\sin 30^o$ or $mgd\sin 30^o$	B1ft		ft with x or d – 1.2 replacing 0.8 in (ii)
	EE gain = $0.75mgx^2/(2x1.2)$ or $0.75mg(d - 1.2)^2/(2x1.2)$	B1ft		ft with x or d – 1.2 replacing 0.8 in (ii)
	[$x^2 - 1.6x - 1.92 = 0$, $d^2 - 4d + 1.44 = 0$]	M1		For using PE loss = EE gain to obtain a 3 term quadratic in x or d
	Displacement is 3.6m	A1	4	
Alternative for parts (ii) and (iii) for candidates who use Newton's second law and a = v dv/dx: In the following x, y and z represent displacement from equil. pos^n, extension, and distance OP respectively.				
	[$mv\,dv/dx = mg\sin 30^o - 0.75mg(0.8 + x)/1.2$, $mv\,dv/dy = mg\sin 30^o - 0.75mgy/1.2$, $mv\,dv/dz = mg\sin 30^o - 0.75mg(z - 1.2)/1.2$]	M1		For using N2 with a = v dv/dx
	$v^2/2 = -5gx^2/16 + C$ or $v^2/2 = gy/2 - 5gy^2/16 + C$ or $v^2/2 = 5gz/4 - 5gz^2/16 + C$	A1		
	[$C = 0.6g + 5g(-0.8)^2/16$ or $C = 0.6g$ or $C = 0.6g - 5g(1.2/4) + 5g(1.2)^2/16$	M1		For using $v^2(-0.8)$ or $v^2(0)$ or $v^2(1.2) = 2(g\sin 30^o)1.2$ as appropriate
	$v^2 = (-5x^2/8 + 1.6)g$ or $v^2 = (y - 5y^2/8 + 1.2)g$ or $v^2 = (5z/2 - 5z^2/8 - 0.9)g$	A1		
	(ii) [$v_{max}^2 = 1.6g$ or $0.8g - 0.4g + 1.2g$ or $5g - 2.5g - 0.9g$]	M1		For using $v_{max}^2 = v^2(0)$ or $v^2(0.8)$ or $v^2(2)$ as appropriate
	Maximum speed is $3.96ms^{-1}$	A1		
	(iii) [$5x^2 - 12.8 = 0 \Rightarrow x = 1.6$, $5y^2 - 8y - 9.6 = 0 \Rightarrow y = 2.4$, $5z^2 - 20z + 7.2 = 0 \Rightarrow z = 3.6$]	M1		For solving v = 0
	Displacement is 3.6m	A1	8	
Alternative for parts (ii) and (iii) for candidates who use Newton's second law and SHM analysis.				
	[$m\ddot{x} = mg\sin 30^o - 0.75mg(0.8 + x)/1.2 \Rightarrow \ddot{x} = -\omega^2 x$; $v^2 = \omega^2(a^2 - x^2)$]	M1		For using N2 with $v^2 = \omega^2(a^2 - x^2)$
	$v^2 = 5g(a^2 - x^2)/8$	A1		
		M1		For using $v^2(-0.8) = 2(g\sin 30^o)1.2$
	$v^2 = 5g(2.56 - x^2)/8$	A1		
	(ii) [$v_{max}^2 = 5g \times 2.56 \div 8$]	M1		For using $v_{max}^2 = v^2(0)$
	Maximum speed is $3.96ms^{-1}$	A1		
	(iii) [$2.56 - x^2 = 0 \Rightarrow x = 1.6$]	M1		For solving v = 0
	Displacement is 3.6m	A1		

6	(i) [$\frac{1}{2}m7^2 = \frac{1}{2}mv^2 + 2mg$]	M1		For using the principle of conservation of energy
	Speed is 3.13ms^{-1}	A1		
	[$T = mv^2/r$]	M1		For using Newton's second law horizontally and $a = v^2/r$
	Tension is 1.96N	A1ft	4	
	(ii) [$T - mg\cos\theta = mv^2/r$]	M1		For using Newton's second law radially
		M1		For using T = 0 (may be implied)
	$v^2 = -2g\cos\theta$	A1		
		M1		For using the principle of conservation of energy
	$\frac{1}{2}m7^2 = \frac{1}{2}mv^2 + mg(2 - 2\cos\theta)$	A1		
	[$-2g\cos\theta = 49 - 4g + 4g\cos\theta$]	M1		For eliminating v^2
	$6g\cos\theta = -9.8$	A1		May be implied by answer
	$\theta = 99.6$	A1	8	
Alternative for candidates who eliminate v^2 before using T = 0.				
	(ii) [$T - mg\cos\theta = mv^2/r$]	M1		For using Newton's second law radially
		M1		For using the principle of conservation of energy
	$\frac{1}{2}m7^2 = \frac{1}{2}mv^2 + mg(2 - 2\cos\theta)$	A1		
	[$T - mg\cos\theta = m(49 - 4g + 4g\cos\theta)2$]	M1		For eliminating v^2
		M1		For using T = 0 (may be implied)
	$-2g\cos\theta = 49 - 4g + 4g\cos\theta$	A1ft		ft error in energy equation
	$6g\cos\theta = -9.8$	A1		May be implied by answer
	$\theta = 99.6$	A1	8	

7	(i) $T = 4mg(4 + x - 3.2)/3.2$	B1		
	[$ma = mg - 4mg(0.8 + x)/3.2$]	M1		For using Newton's second law
	$4\ddot{x} = -49x$	A1	3	AG
	(ii) Amplitude is 0.8m	B1		(from 4 + A = 4.8)
	Period is $2\pi/\omega$ s where $\omega^2 = 49/4$	B1		
		M1		String is instantaneously slack when shortest (4 - A = 3.2 = L). Thus required interval length = period.
	Slack at intervals of 1.8s	A1	4	AG
	(iii) [$ma = -mg\sin\theta$]	M1		For using Newton's second law tangentially
	$mL\ddot{\theta} = -mg\sin\theta$	A1		
	For using $\sin\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx -(g/L)\theta$	A1	3	AG
	(iv) [$\theta = 0.08\cos(3.5\times0.25)$] (= 0.05127..)	M1		For using $= {}_o\cos\omega t$ where $\omega^2=12.25$ (may be implied by $\dot{\vartheta} = -\omega\ {}_o\sin\omega t$)
	[$\dot{\theta} = -3.5(0.08)\sin(3.5\times0.25)$, $\dot{\theta}^2 = 12.25(0.08^2 - 0.05127..^2)$]	M1		For differentiating $= {}_o\cos\omega t$ and using $\dot{\vartheta}$ or for using $\dot{\theta}^2 = \omega^2(\theta_o{}^2 - \theta^2)$ where $\omega^2=12.25$
	$\dot{\theta} = \mp 0.215$	A1		May be implied by final answer
	[$v = 0.215\times9.8/12.25$]	M1		For using $v = L\dot{\vartheta}$ and $L = g/\omega^2$
	Speed is 0.172 ms^{-1}	A1	5	

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1	(i) $T = (1.35mg)(3 - 1.8) \div 1.8$	B1		
	[$0.9mg = ma$]	M1		For using T = ma
	Acceleration is $8.82 ms^{-2}$	A1	3	
	(ii) Initial EE = $(1.35mg)(3 - 1.8)^2 \div (2x1.8)$	B1		
	[$\frac{1}{2} mv^2 = 0.54mg$]	M1		For using $\frac{1}{2} mv^2$ = Initial EE
	Speed is $3.25 ms^{-1}$	A1	3	

2	**(i)**	M1		For using NEL vertically
	Component is $8e\sin 27^o$	A1		
	Component is $2.18 ms^{-1}$	A1	3	
	(ii) Change in velocity vertically = $8\sin 27^o(1 + e)$	B1ft		ft $8\sin 27^o$ + candidate's ans. in (i)
	$\lvert I \rvert = 0.2 \times 5.81$	M1		For using $\lvert I \rvert$ = m x change in velocity
	Magnitude of Impulse is $1.16 kgms^{-1}$	A1ft	3	ft incorrect ans. in (i) providing both M marks are scored.

3		M1		For using the principle of conservation of momentum in the **i** direction
	$0.8 \times 12\cos 60^o = 0.8a + 2b$	A1		
		M1		For using NEL
	$0.75 \times 12\cos 60^o = b - a$	A1		
	[$4.8 = 0.8a + 2(a + 4.5)$]	DM1		For eliminating b; depends on at least one previous M mark
	$a = -1.5$	A1		
	Comp. of vel. perp. to l.o.c. after impact is $12\sin 60^o$	B1		
		M1		For correct method for speed or direction
	The speed of A is $10.5 ms^{-1}$	A1ft		ft $v^2 = a^2 + 108$
	Direction of A is at 98.2^o to l.o.c.	A1ft	10	Accept $\theta = 81.8^o$ if θ is clearly and appropriately indicated; ft $\tan^{-1}\theta = (12\sin 60^o)/\lvert a \rvert$)

4	**(i)** $[mg\sin\alpha - 0.2mv = ma]$	M1		For using Newton's second law
	$5\frac{dv}{dt} = 28 - v$	A1		AG
	$[\int \frac{5}{28-v}dv = \int dt]$	M1		For separating variables and integrating
	(C) - 5ln(28 – v) = t	A1		
		M1		For using v = 0 when t = 0
	ln[(28 – v)/28] = -t/5	A1ft		ft for ln[(28 – v)/28] = t/A from C + Aln(28 – v) = t previously
	$[28 - v = 28e^{-t/5}]$	M1		For expressing v in terms of t
	$v = 28(1 - e^{-t/5})$	A1ft	8	ft for $v = 28(1 - e^{t/A})$ from ln[(28 – v)/28] = t/A previously
	(ii) $[a = 28e^{-2}/5]$	M1		For using a = (28 – v(t))/5 or $a = d(28 - 28e^{-t/5})dt$ and substituting t = 10.
	Acceleration is $0.758ms^{-2}$	A1ft	2	ft from incorrect v in the form $a + be^{ct}$ $(b \neq 0)$; Accept $5.6/e^2$

5	**(i)**	M1		For taking moments about B or about A for the whole or For taking moments about X for the whole and using $R_A + R_B = 280$ and $F_A = F_B$
	$1.4R_A = 150x0.95 + 130x0.25$ or $1.4R_B = 130x1.15 + 150x0.45$ or $1.2F - 0.9(280 - R_B) + 0.45x150 - 1.2F + 0.5R_B - 0.25x130 = 0$	A1		
	$R_A = 125N$	A1		AG
	$R_B = 155N$	B1	4	
	(ii)	M1		For taking moments about X for XA or XB
	$1.2F_A = -150x0.45 + 0.9R_A$ or $1.2F_B = 0.5R_B - 130x0.25$	A1		
	F_A or $F_B = 37.5N$	A1ft		$F_B = (1.25R_B - 81.25)/3$
	F_B or $F_A = 37.5N$	B1ft	4	
	(iii) Horizontal component is 37.5N to the left	B1ft		ft H = F or H = 56.25 – 0.75V or 12H = 325 + 5V
	$[Y + R_A = 150]$	M1		For resolving forces on XA vertically
	Vertical component is 25N upwards	A1ft	3	ft 3V = 225 – 4H or V = 2.4H -65

6	**(i)**			For applying Newton's second law
	[0.36 – 0.144x = 0.1a]	M1		
	$\ddot{x} = 3.6 - 1.44x$	A1		
	$\ddot{y} = -1.44y$ ➔ SHM or $d^2(x-2.5)/dt^2 = -1.44(x-2.5)$ ➔SHM	B1		
		M1		For using $T = 2\pi/n$
	Of period 5.24s	A1	5	AG
	(ii) Amplitude is 0.5m	B1		
		M1		For using $v^2 = n^2(a^2 - y^2)$
	$0.48^2 = 1.2^2(0.5^2 - y^2)$	A1ft		
	Possible values are 2.2 and 2.8	A1	4	
	(iii) [$t_0 = (\sin^{-1}0.6)/1.2$; $t_1 = (\cos^{-1}0.6)/1.2$]	M1		For using y = 0.5sin1.2t to find t_0 or y = 0.5cos1.2t to find t_1
	$t_0 = 0.53625$ .... or $t_1 = 0.7727$…..	A1		Principal value may be implied
	(a)			For using $\Delta t = 2t_0$ or
	[$2(\sin^{-1}0.6)/1.2$ or $(\pi - 2\cos^{-1} 0.6)/1.2$]	M1		$\Delta t = T/2 - 2t_1$
	Time interval is 1.07s	A1ft		ft incorrect t_0 or t_1
	(b)			From $\Delta t = T/2 - 2t_0$ or $\Delta t = 2t_1$; ft 2.62 – ans(a) or
	Time interval is 1.55s	B1ft	5	incorrect t_0 or t_1

7	**(i)**	M1		For using KE gain = PE loss
	½ $mv^2 = mga(1 - \cos\theta)$	A1		
	$aw^2 = 2g(1 - \cos\theta)$	B1	3	AG From v = wr
	(ii)			For using Newton's second law radially (3 terms required) with accel
		M1		$= v^2/r$ or w^2r
	$mv^2/a = mg\cos\theta - R$ or $maw^2 = mg\cos\theta - R$	A1		
				For eliminating v^2 or w^2; depends on
	[$2mg(1 - \cos\theta) = mg\cos\theta - R$]	DM1		at least one previous M1
	$R = mg(3\cos\theta - 2)$	A1ft	4	ft sign error in N2 equation
	(iii)			For using Newton's second law
	[$mg\sin\theta$ = m(accel.) or			tangentially or differentiating
	$2a(\dot{\theta})\ddot{\theta} = 2g\sin\theta(\dot{\theta})$]	M1		$aw^2 = 2g(1 - \cos\theta)$ w.r.t. t
	Accel. $(=a\ddot{\theta}) = g\sin\theta$	A1		
	[$\theta = \cos^{-1}(2/3)$]	M1		For using R = 0
				ft from incorrect R of the form mg(Acos□ +B), A $\neq$ 0, B $\neq$ 0;
	Acceleration is $7.30ms^{-2}$	A1ft	4	accept $g\sqrt{5}/3$
	(iv)			For using rate of change =
		M1		$(dR/d\theta)(d\theta/dt)$
	$dR/dt = (-3mg\sin\theta)\sqrt{2g(1-\cos\theta)/a}$			ft from incorrect R of the form
		A1ft		mg(Acos□ +B), A $\neq$ 0
		M1		For using $\cos\theta = 2/3$
				Any correct form of $\dot{R}$ with
	Rate of change is $-mg\sqrt{\dfrac{10g}{3a}}$ Ns^{-1}			$\cos\theta = 2/3$ used; ft with □ from incorrect R of the form mg(Acos□
		A1ft	4	+B), A $\neq$ 0, B $\neq$ 0

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1 (i)	For triangle sketched with sides (0.5)2.5 and (0.5)6.3 and angle θ correctly marked OR Changes of velocity in i and j directions $2.5\cos\theta - 6.3$ and $2.5\sin\theta$, respectively.	B1	May be implied in subsequent working.
	For sides 0.5x2.5, 0.5x6.3 and 2.6 (or 2.5, 6.3 and 5.2) OR $-2.6\cos\alpha = 0.5(2.5\cos\theta - 6.3)$ and $2.6\sin\alpha = 0.5(2.5\sin\theta)$	B1ft	May be implied in subsequent working.
	[$5.2^2 = 2.5^2 + 6.3^2 - 2x2.5x6.3\cos\theta$ OR $2.6^2 = 0.5^2\{(2.5\cos\theta - 6.3)^2 + (2.5\sin\theta)^2\}$]	M1	For using cosine rule in triangle or eliminating α.
	$\cos\theta = 0.6$	A1 [4]	AG
(ii)		M1	For appropriate use of the sine rule or substituting for θ in one of the above equations in θ and α
	$\sin\alpha = 2.5x0.8/5.2$ OR $-2.6\cos\alpha = 0.5(2.5x0.6 - 6.3)$	A1	
		M1	For evaluating $(180 - \alpha)^o$ or $(\pi - \alpha)^c$
	Impulse makes angle of 157^o or 2.75^c with original direction of motion of P.	A1 [4]	SR (relating to previous 2 marks; max 1 mark out of 2) $\alpha = 23^o$ or 0.395^C B1

2 (i)	[70x2 = 4X – 4Y]	M1	For taking moments about A for AB (3 terms needed)
	X – Y = 35	A1 [2]	
(ii)	[110x3 = -4X + 6Y]	M1	For taking moments about C for BC (3 terms needed)
	2X – 3Y + 165 = 0	A1 [2]	AG
(iii)		M1	For attempting to solve for X and Y
	X = 270, Y = 235	A1ft	ft any (X, Y) satisfying the equation given in (ii)
		M1	For using magnitude = $\sqrt{X^2 + Y^2}$
	Magnitude is 358N	A1ft [4]	ft depends on all 4 Ms

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3 (i)	[T_A = (24x0.45)/0.6, T_B = (24x0.15)/0.6] T_A – T_B = 18 – 6 = 12 = W ➔ P in equil'm.	M1 A1 [2]	For using T = λ x/L for PA or PB
(ii)	Extensions are 0.45 + x and 0.15 – x Tensions are 18 + 40x and 6 – 40x	B1 B1 [2]	AG From T = λ x/L for PA and PB
(iii)	[12 + (6 – 40x) – (18 + 40x) = 12 $\ddot{x}$/g] $\ddot{x}$ = -80gx/12 ➔ SHM Period is 0.777s	M1 A1 A1 [3]	For using Newton's second law (4 terms required) AG From Period = 2π $\sqrt{12/(80g)}$
(iv)	[v_{max} = 0.15$\sqrt{80g/12}$ or v_{max} = 2πx0.15/0.777 or ½ (12/g)v_{max}^2 +mg(0.15) +24{0.45^2 + 0.15^2 -0.6^2}/(2x0.6) = 0] Speed is 1.21ms^{-1}	M1 A1 [2]	For using v_{max} = An or v_{max} = 2π A/T or conservation of energy (5 terms needed)

4 (i)	Loss in PE = mg(0.5sinθ) [½ mv^2 – ½ $m3^2$ = mg(0.5sinθ)] v^2 = 9 + 9.8sinθ	B1 M1 A1 [3]	For using KE gain = PE loss (3 terms required) AG
(ii)	a_r = 18 + 19.6sinθ [ma_t = mg cosθ] a_t = 9.8cosθ	B1 M1 A1 [3]	Using a_r = v^2/0.5 For using Newton's second law tangentially
(iii)	[T - mg sinθ = ma_r] T – 1.96sinθ = 0.2(18 + 19.6sinθ) T = 3.6 + 5.88sinθ θ = 3.8	M1 A1 A1 B1 [4]	For using Newton's second law radially (3 terms required) AG

5	Initial **i** components of velocity for A and B are $4ms^{-1}$ and $3ms^{-1}$ respectively. $3x4 + 4x3 = 3a + 4b$ $0.75(4 – 3) = b – a$ a = 3 Final **j** component of velocity for A is $3ms^{-1}$ Angle with l.o.c. is 45^o or 135^O	 B1 M1 A1 M1 A1 M1 A1 B1 M1 A1ft [10]	 May be implied. For using p.c.mmtm. parallel to l.o.c. For using NEL For attempting to find a Depends on all three M marks May be implied For using $\tan^{-1}(v_j/v_i)$ for A ft incorrect value of a ($\neq 0$) only
			SR for consistent sin/cos mix (max 8/10) 3x3 + 4x4 = 3a + 4b and b – a = 0.75(3 – 4) M1 M1 as scheme and A1 for *both* equ's a = 4 M1 as scheme A1 j component for A is $4ms^{-1}$ B1 Angle $\tan^{-1}(4/4) = 45^o$ M1 as scheme A1

6(i)	Initial speed in medium is $\sqrt{2g \times 10}$ (= 14) [0.125dv/dt = 0.125g – 0.025v] $\int \frac{5dv}{5g - v} = \int dt$ -5 ln(5g – v) = t (+A) [-5 ln35 = A] t = 5 ln{35/(49 – v)} $v = 49 – 35e^{-0.2t}$	B1 M1 M1 A1 M1 A1 M1 A1 [8]	 For using Newton's second law with a = dv/dt (3 terms required) For separating variables and attempt to integrate For using v(0) = 14 For method of transposition AG
(ii)	 $x = 49t + 175e^{-0.2t}$ (+B) $[x(3) = (49x3 + 175e^{-0.6}) – (0 + 175)]$ Distance is 68.0m	M1 A1 M1 A1 [4]	For integrating to find x(t) For using limits 0 to 3 or for using x(0) = 0 and evaluating x(3)

7(i)	Gain in EE = $20x^2/(2\times2)$ Loss in GPE = $0.8g(2 + x)$ [$\frac{1}{2}0.8v^2 = (15.68 + 7.84x) - 5x^2$] $v^2 = 39.2 + 19.6x - 12.5x^2$	B1 B1 M1 A1 [4]	 Accept $0.8gx$ if gain in KE is $\frac{1}{2}0.8(v^2 - 19.6)$ For using the p.c.energy AG
(ii)	**(a)** Maximum extension is 2.72m	M1 A1 [2]	For attempting to solve $v^2 = 0$
	(b) [$19.6 - 25x = 0$, $v^2 = 46.8832 - 12.5(x - 0.784)^2$] $x = 0.784$ or $c = 46.9$ [$v_{max}^2 = 39.2 + 15.3664 - 7.6832$] Maximum speed is $6.85ms^{-1}$	 M1 A1 M1 A1 [4]	For solving $20x/2 = 0.8g$ or for differentiating and attempting to solve $d(v^2)/dx = 0$ or $dv/dx = 0$ or for expressing v^2 in the form $c - a(x - b)^2$. For substituting $x = 0.784$ in the expression for v^2 or for evaluating $\sqrt{c}$
	(c) $\pm(0.8g - 20x/2) = 0.8a$ or $2v\,dv/dx = 19.6 - 25x$ $a = \pm(9.8 - 12.5x)$ or $\ddot{y} = -12.5y$ where $y = x - 0.784$ [$\lvert a\rvert_{max} = \lvert 9.8 - 12.5\times2.72\rvert$ or $\lvert\ddot{y}_{max}\rvert = \lvert -12.5(2.72 - 0.784)\rvert$] Maximum magnitude is $24.2ms^{-2}$	 M1 A1 A1 M1 A1 [5]	For using Newton's second law (3 terms required) or $a = v\,dv/dx$ For substituting x = ans(ii)(a) into $a(x)$ or y = ans(ii)(a) – 0.784 into $\ddot{y}(y)$

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1 i	Horiz. comp. of vel. after impact is 4ms^{-1} Vert. comp. of vel. after impact is $\sqrt{5^2 - 4^2} = 3\text{ms}^{-1}$ Coefficient of restitution is 0.5	B1 B1 B1 [3]	May be implied AG From e = 3/6
ii	Direction is vertically upwards Change of velocity is 3 – (-6) Impulse has magnitude 2.7Ns	B1 M1 A1 [3]	 From $m(\Delta v) = 0.3\times 9$
2 i	Horizontal component is 14N $80\times1.5 = 14\times1.5 + 3Y$ or $3(80 - Y) = 80\times1.5 + 14\times1.5$ or $1.5(80 - Y) = 14\times0.75 + 14\times0.75 + 1.5Y$ Vertical component is 33N upwards	B1 M1 A1 A1 [4]	 For taking moments for *AB* about *A* or *B* or the midpoint of *AB* AG
ii	Horizontal component at *C* is 14N [Vertical component at *C* is $(\pm)\sqrt{50^2 - 14^2}$] [$W = (\pm)48 - 33$] Weight is 15N	B1 M1 DM1 A1 [4]	May be implied for using $R^2 = H^2 + V^2$ For resolving forces at *C* vertically
3 i	 $4\times3\cos60^\circ - 2\times3\cos60^\circ = 2b$ $b = 1.5$ **j** component of vel. of *B* = $(-)3\sin60^\circ$ [$v^2 = b^2 + (-3\sin60^\circ)^2$] Speed ($3\text{ms}^{-1}$) is unchanged [Angle with l.o.c. = $\tan^{-1}(3\sin60^\circ/1.5)$] Angle is 60°.	M1 A1 A1 B1ft M1 A1ft M1 A1ft [8]	For using the p.c.mmtm parallel to l.o.c. ft consistent sin/cos mix For using $v^2 = b^2 + v_y^2$ AG ft - allow same answer following consistent sin/cos mix. For using angle = $\tan^{-1}(\pm v_y/v_x)$ ft consistent sin/cos mix
ii	[$e(3\cos60^\circ + 3\cos60^\circ) = 1.5$] Coefficient is 0.5	M1 A1ft [2]	For using NEL ft - allow same answer following consistent sin/cos mix throughout.

4 i	$F - 0.25v^2 = 120v(\mathrm{d}v/\mathrm{d}x)$ $F = 8000/v$ $[32000 - v^3 = 480v^2(\mathrm{d}v/\mathrm{d}x)]$ $\dfrac{480v^2}{v^3 - 32000}\dfrac{\mathrm{d}v}{\mathrm{d}x} = -1$	M1 A1 B1 M1 A1 [5]	For using Newton's second law with $a = v(\mathrm{d}v/\mathrm{d}x)$ For substituting for F and multiplying throughout by $4v$ (or equivalent) AG
ii	$\int \dfrac{480v^2}{v^3 - 32000}\mathrm{d}v = -\int \mathrm{d}x$ $160 \ln(v^3 - 32000) = -x \quad (+A)$ $160 \ln(v^3 - 32000) = -x + 160 \ln 32000$ or $160 \ln(v^3 - 32000) - 160 \ln 32000 = -500$ $(v^3 - 32000)/32000 = e^{-x/160}$ Speed of m/c is $32.2\mathrm{ms}^{-1}$	M1 A1 M1 A1ft B1ft B1 [6]	For separating variables and integrating For using $v(0) = 40$ or $[160 \ln(v^3 - 32000)]_{40}^{v} = [-x]_0^{500}$ ft where factor 160 is incorrect but +ve, Implied by $(v^3 - 32000)/32000 = e^{-3.125}$ (or = 0.0439 ..). ft where factor 160 is incorrect but +ve, or for an incorrect non-zero value of A
5 i	$x_{\max} = \sqrt{1.5^2 + 2^2} - 1.5 \; (= 1)$ $[T_{\max} = 18\times1/1.5]$ Maximum tension is 12N	B1 M1 A1 [3]	For using $T = \lambda x/L$
      ii	**(a)** Gain in EE = $2[18(1^2 - 0.2^2)]/(2\times1.5)$ (11.52) Loss in GPE = 2.8mg (27.44m) $[2.8m \times 9.8 = 11.52]$ $m = 0.42$ **(b)** $\frac{1}{2} mv^2 = mg(0.8) + 2\times18\times0.2^2/(2\times1.5)$ or $\frac{1}{2} mv^2 = 2\times18\times1^2/(2\times1.5) - mg(2)$ Speed at M is $4.24\mathrm{ms}^{-1}$	M1 A1 B1 M1 A1 [5] M1 A1ft A1ft [3]	For using EE = $\lambda x^2/2L$ May be scored with correct EE terms in expressions for total energy on release and total energy at lowest point May be scored with correct GPE terms in expressions for total energy on release and total energy at lowest point For using the p.c.energy AG For using the p.c.energy KE, PE & EE must all be represented ft only when just one string is considered throughout in evaluating EE ft only for answer 4.10 following consideration of only one string

6 **i**	[$-mg\sin\theta = mL(d^2\theta/dt^2)$] $d^2\theta/dt^2 = -(g/L)\sin\theta$	M1 A1 [2]	For using Newton's second law tangentially with $a = Ld^2\theta/dt^2$ AG
ii	[$d^2\theta/dt^2 = -(g/L)\ \theta$] $d^2\theta/dt^2 = -(g/L)\ \theta$ ➔ motion is SH	M1 A1 [2]	For using $\sin\theta \approx \theta$ because θ is small ($\theta_{max} = 0.05$) AG
iii	[$4\pi/7 = 2\pi/\sqrt{9.8/L}$] $L = 0.8$	M1 A1 [2]	For using $T = 2\pi/n$ where $-n^2$ is coefficient of θ
iv	[$\theta = 0.05\cos 3.5\times 0.7$] $\theta = -0.0385$ $t = 1.10$ (accept 1.1 or 1.09)	M1 A1ft M1 A1ft [4]	For using $\theta = \theta_o\cos nt$ {$\theta = \theta_o\sin nt$ not accepted unless the t is reconciled with the t as defined in the question} ft incorrect L {$\theta = 0.05\cos[4.9/(5L)^{½}]$} For attempting to find 3.5t ($\pi < 3.5t < 1.5\pi$) for which $0.05\cos 3.5t$ = answer found for θ or for using $3.5(t_1 + t_2) = 2\pi$ ft incorrect L {$t = [2\pi(5L)^{½}]/7 - 0.7$}
v	$\dot{\theta}^2 = 3.5^2(0.05^2 - (-0.0385)^2)$ or $\dot{\theta} = -3.5\times 0.05\sin(3.5\times 0.7)$ ($\dot{\theta} = -0.1116..$) Speed is 0.0893ms^{-1} (Accept answers correct to 2 s.f.)	M1 A1ft A1ft [3]	For using $\dot{\theta}^2 = n^2(\theta_o^2 - \theta^2)$ or $\dot{\theta} = -n\ \theta_o\sin nt$ {also allow $\dot{\theta} = n\theta_o\cos nt$ if $\theta = \theta_o\sin nt$ has been used previously} ft incorrect θ with or without 3.5 represented by $(g/L)^{½}$ using incorrect L in (iii) or for $\dot{\theta} = 3.5\times 0.05\cos(3.5\times 0.7)$ following previous use of $\theta = \theta_o\sin nt$ ft incorrect L ($L\times 0.089287/0.8$ with $n = 3.5$ used or from $\lvert 0.35\sin\{4.9/[5L]^{½}\}/[5L]^{½}\rvert$ **SR** for candidates who use $\dot{\theta}$ as v. (Max 1/3) For v = ± 0.112 B1

7 i	Gain in PE = $mga(1 - \cos\theta)$ [$\frac{1}{2} mu^2 - \frac{1}{2} mv^2 = mga(1 - \cos\theta)$]	B1 M1	 For using KE loss = PE gain
	$v^2 = u^2 - 2ga(1 - \cos\theta)$ [$R - mg\cos\theta = m$(accel.)] $R = mv^2/a + mg\cos\theta$ [$R = m\{u^2 - 2ga(1 - \cos\theta)\}/a + mg\cos\theta$] $R = mu^2/a + mg(3\cos\theta - 2)$	A1 M1 A1 M1 A1 [7]	 For using Newton's second law radially For substituting for v^2 AG
ii	[$0 = mu^2/a - 5mg$] $u^2 = 5ag$ [$v^2 = 5ag - 4ag$] Least value of v^2 is ag	M1 A1 M1 A1 [4]	For substituting $R = 0$ and $\theta = 180^o$ For substituting for u^2 (= $5ag$) and θ = 180^o in v^2 (expression found in (i)) { but M0 if $v = 0$ has been used to find u^2} AG
iii	[$0 = u^2 - 2ga(1 - \sqrt{3}/2)$] $u^2 = ag(2 - \sqrt{3})$	M1 A1 [2]	For substituting $v^2 = 0$ and $\theta = \pi/6$ in v^2 (expression found in (i)) Accept $u^2 = 2ag(1 - \cos\pi/6)$

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1	$0.4(3\cos60^o - 4) = -I\cos\theta$ (= -1) $0.4(3\sin60^o) = I\sin\theta$ (= 1.03920) [$\tan\theta = -1.5\sqrt{3}/(1.5 - 4)$; $I^2 = 0.4^2[(1.5-4)^2 + (1.5\sqrt{3})^2]$] $\theta = 46.1$ or I = 1.44 I = 1.44 or $\theta = 46.1$	M1 A1 A1 M1 A1 M1 A1ft [7]	For using I = Δmv in one direction SR: Allow B1 (max 1/3) for $3\cos60^o - 4 = -I\cos\theta$ and $3\sin60^o = I\sin\theta$ For eliminating I or θ (allow following SR case) Allow for θ (only) following SR case. For substituting for θ or for I (allow following SR case) ft incorrect θ or I; allow for θ (only) following SR case.
	Alternatively $I^2 = 1.2^2 + 1.6^2 - 2\times1.2\times1.6\cos60^o$ or $\text{'V'}^2 = 3^2 + 4^2 - 2\times3\times4\cos60^o$ I = 1.44 $\frac{\sin\theta}{3(or 1.2)} = \frac{\sin 60}{\sqrt{13(or 2.08)}}$ or $\frac{\sin\alpha}{4(or 1.6)} = \frac{\sin 60}{\sqrt{13(or 2.08)}} and \theta = 120 - \alpha$ $\theta = 46.1$	M1 A1 M1 A1 M1 A1ft A1 [7]	For use of cosine rule For correct use of factor 0.4 (= m) For use of sine rule α must be angle opposite 1.6; (α = 73.9) ft value of I or 'V'
2	2a + 3b = 2×4 b – a = 0.6×4 [2(b - 2.4) + 3b = 8] b = 2.56 v = 2.56	M1 A1 M1 A1 M1 A1 B1ft [7]	For using the principle of conservation of momentum For using NEL For eliminating a ft v = b
3(i)	$2W(a\cos45^o) = T(2a)$ $W = \sqrt{2}\,T$	M1 A1 A1 [3]	For using 'mmt of 2W = mmt of T' AG
(ii)	Components (H, V) of force on BC at B are $H = -T/\sqrt{2}$ and $V = T/\sqrt{2} - 2W$ W(a cosα) + H(2a sinα) = V(2a cosα) [W cosα - T$\sqrt{2}$ sinα = T$\sqrt{2}$ cosα -4Wcosα] T$\sqrt{2}$ sin α = (5W - T$\sqrt{2}$) cos α tan α = 4	B1 M1 A1 M1 A1ft A1 [6]	For taking moments about C for BC For substituting for H and V and reducing equation to the form X sinα = Y cosα

	Alternatively for part **(ii)** anticlockwise mmt = W(a cosα) +2W(2a cosα + a cos45°) = T[2a cos(α – 45°) + 2a] [5W cosα + $\sqrt{2}$ W = T($\sqrt{2}$ cosα + $\sqrt{2}$ sinα) + 2] T$\sqrt{2}$ sin α = (5W - T$\sqrt{2}$) cos α tan α = 4	M1 A1 A1 M1 A1ft A1 [6]	For taking moments about C for the whole For reducing equation to the form X sinα = Y cosα
4(i)	[-0.2(v + v^2) = 0.2a] [v dv/dx = -(v + v^2) [1/(1 + v)] dv/dx = -1	M1 M1 A1 [3]	For using Newton's second law For using a = v dv/dx AG
(ii)	 ln (1 + v) = -x (+ C) ln(1+ v) = -x + ln3 [(1 + dx/dt)/3 = e^{-x} ➔ dx/dt = 3e^{-x} -1 ➔e^x dx/dt = 3 – e^x] [-e^x/(3 – e^x)] dx/dt = -1	M1 A1 A1 M1 A1 [5]	For integrating For transposing for v and using v = dx/dt AG
(iii)	[ln(3 – e^x) = -t + ln2] ln(3 – e^x) = -t + ln2 Value of t is 1.96 (or ln{2 ÷ (3 – e)}	M1 A1 A1 [3]	For integrating and using x(0) = 0
5(i)	 Loss of EE = 120(0.5^2 – 0.3^2)/(2×1.6) and gain in PE = 1.5×4 v = 0 at B and loss of EE = gain in PE (= 6) ➔distance AB is 4m	M1 A1 M1 A1 [4]	For using EE = λx^2/2L and PE = Wh For comparing EE loss and PE gain AG
(ii)	[120e/1.6 = 1.5] e = 0.02 Loss of EE = 120(0.5^2 – 0.02^2)/(2×1.6) (or 120(0.3^2 – 0.02^2)/(2×1.6)) Gain in PE = 1.5(2.1- 1.6 – 0.02) (or 1.5(1.9 + 1.6 + 0.02) loss) [KE at max speed = 9.36 – 0.72 (or 3.36 + 5.28)] ½ (1.5/9.8)v^2 = 9.36 – 0.72 Maximum speed is 10.6 ms^{-1}	M1 A1 B1ft B1ft M1 A1 A1 [7]	For using T = mg and T = λx/L ft incorrect e only ft incorrect e only For using KE at max speed = Loss of EE – Gain (or + loss) in PE
	First alternative for **(ii)** x is distance AP [½ (1.5/9.8)v^2 + 1.5x + 120(0.5 – x)^2/3.2 = 120 x0.5^2/3.2] KE and PE terms correct EE terms correct v^2 = 470.4x – 490x^2 [470.4 – 980x = 0] x = 0.48 Maximum speed is 10.6 ms^{-1}	 M1 A1 A1 A1 M1 A1 A1	 For using energy at P = energy at A For attempting to solve dv^2/dx = 0

	Second alternative for **(ii)** [120e/1.6 = 1.5] e = 0.02 [1.5 – 120(0.02 + x)/1.6 = 1.5 $\ddot{x}$/g] n = $\sqrt{490}$ a = 0.48 Maximum speed is 10.6 ms^{-1}	M1 A1 M1 M1 A1 A1 A1	For using T = mg and T = λx/L For using Newton's second law For obtaining the equation in the form $\ddot{x}$ = $-n^2x$, using (AB – L – e_{equil}) for amplitude and using v_{max} = na.
6(i)	PE gain by P = 0.4g × 0.8 sin θ PE loss by Q = 0.58g × 0.8 θ ½ (0.4 + 0.58)v^2 = g × 0.8(0.58 θ –0.4sin θ) v^2 = 9.28 θ - 6.4sin θ	B1 B1 M1 A1ft A1 [5]	 For using KE gain = PE loss AEF
(ii)	 0.4g sin θ – R = 0.4v^2/0.8 [0.4g sin θ – R = 4.64 θ – 3.2 sin θ] R = 7.12 sin θ – 4.64 θ	M1 A1 M1 A1 [4]	For applying Newton's second law to P and using a = v^2/r For substituting for v^2 AG
(iii)	 R(1.53) = 0.01(48...), R(1.54) = -0.02(9...) or simply R(1.53) > 0 and R(1.54) < 0 R(1.53) × R(1.54) < 0 ➔1.53 < α < 1.54	M1 A1 M1 A1 [4]	For substituting 1.53 and 1.54 into R(θ) For using the idea that if R(1.53) and R(1.54) are of opposite signs then R is zero (and thus P leaves the surface) for some value of θ between 1.53 and 1.54. AG
7(i)	 T_{AP} = 19.6e/1.6 and T_{BP} = 19.6(1.6-e)/1.6 0.5g sin30° + 12.25(1.6 – e) = 12.25e Distance AP is 2.5m	M1 A1 M1 A1ft A1 [5]	For using T = λe/L For resolving forces parallel to the plane
(ii)	Extensions of AP and BP are 0.9 + x and 0.7 – x respectively 0.5g sin 30° + 19.6(0.7 – x)/1.6 – 19.6(0.9 + x)/1.6 = 0.5 $\ddot{x}$ $\ddot{x}$ = -49x Period is 0.898 s	B1 B1ft B1 M1 A1 [5]	 AG For stating k < 0 and using T = 2π/$\sqrt{-k}$
(iii)	 $2.8^2 = 49(0.5^2 – x^2)$ x^2 = 0.09 x = 0.3 and -0.3	M1 A1ft A1 A1ft [4]	For using $v^2 = \omega^2(A^2 – x^2)$ where ω^2 = -k ft incorrect value of k May be implied by a value of x ft incorrect value of k or incorrect value of x^2 (stated)

1	For included angle marked α or for $0.8(10.5 – 8.5\cos\alpha) = 4\cos\beta$ For opposite side marked 4/0.8 (or 4) or for -- $0.8 \times 8.5\sin\alpha = 4\sin\beta$ $8.4^2 + 6.8^2 – 2 \times 8.4 \times 6.8 \cos\alpha = 4^2$ $\alpha = 28.1^o$	M1 A1 A1 M1 A1ft A1 [6]	For triangle with two of its sides marked 0.8 x 10.5 and 0.8 x 8.5 (or 10.5 and 8.5) or for using I = Δmv in one direction. Allow B1 for omission of 0.8 Allow B1 for omission of 0.8 For using the cosine rule or for eliminating β ft 0.8 mis-used or not used
2(i)	[$100a = 2aV_B$] Vertical component at B is 50 N Vertical component at C is 150 N	M1 A1 A1 [3]	For taking moments about A for AB
(ii)	$100(0.5a) + (\sqrt{3}\ a)F = 150a$ or $100a + 100(1.5a) = 150a + (\sqrt{3}\ a)F$ Frictional force is 57.7 N Direction is to the right	M1 A1ft A1 B1 [4]	For taking moments about B for BC (3 terms needed) or about A for the whole (4 terms needed)
3(i)	u = 4 v = 2	B1 B1 [2]	
(ii)	mu = ma + mb (or u = b – a) u = b – a (or mu = ma + mb) a = 0 and b = 4ms^{-1} Speed of A is 2ms^{-1} and direction at 90^o to the wall Speed of B is 4ms^{-1} and direction parallel to the wall	M1 A1 B1 A1ft A1ft A1ft [6]	For using the principle of conservation of momentum or for using NEL with e = 1 ft incorrect u ft incorrect v ft incorrect u
4(i)	[$0.25\ dv/dt = 3/50 – t^2/2400$] $v = 12t/50 – t^3/1800$ [v(12) = 1.92] [$0.25\ dv/dt = t^2/2400 - 3/50$ ➔ $v = t^3/1800 - 12t/50 + C_2$] [$1.92 = 0.96 – 2.88 + C_2$] $v = t^3/1800 - 12t/50 + 3.84$ $v(24) = 5.76 = 3 \times v(12)$	M1 M1 A1 M1 M1 M1 A1 A1 [8]	For using Newton's second law (1st or 2nd stage) For attempting to integrate (1st stage) and using v(0) = 0 (may be implied by the absence of + C_1) For evaluating v when force is zero For using Newton's second law (2nd stage) and integrating For using v(12) = 1.92 AG

(ii)	Sketch has v(0) = 0 and slope decreasing (convex upwards) for 0 < t < 12 Sketch has slope increasing (concave upwards) for 12 < t < 24 Sketch has v(t) continuous, single valued and increasing (except possibly at t = 12) with v(24) seen to be > 2v(12)	B1 B1 B1 [3]	
5(i)	For using amplitude as a coefficient of a relevant trigonometric function. For using the value of ω as a coefficient of t in a relevant trigonometric function. $x_1 = 3\cos t$ and $x_2 = 4\cos 1.5t$	B1 B1 B1 [3]	
(ii)	Part distance is 20m [20 – (-3.62)] Distance travelled by P_2 is 23.6 m	M1 A1 M1 A1 [4]	For using distance travelled by P_2 for $0 < t < 5\pi/3$ is $5A_2$ For subtracting displacement of P_2 when t = 5.99 from part distance.
(iii)	$\dot{x}_1 = -3\sin t$; $\dot{x}_2 = -6\sin 1.5t$ $v_1 = 0.867$, $v_2 = -2.55$; opposite directions	M1 A1 M1 A1 [4]	For differentiating x_1 and x_2 For evaluating when t = 5.99 (must use radians)
	Alternative for (iii): $v_1^2 = 3^2 - 2.87^2$, $v_2^2 = 2.25[4^2 - (-3.62)^2]$ [$\pi < 5.99 < 2\pi \Rightarrow v_1 > 0$, $4\pi/3 < 5.99 < 2\pi \Rightarrow v_2 < 0$] $v_1 = 0.867$, $v_2 = -2.55$; opposite directions	M1 A1 M1 A1	For using $v^2 = n^2(a^2 - x^2)$ (must use radians to find values of x) For using the idea that v starts –ve and changes sign at intervals of T/2 s
6(i)	PE loss at lowest allowable point = 25W EE gain = $32000\text{x}5^2/(2\text{x}20)$ [25W = 20000] Value of W is 800	B1 M1 A1 M1 A1 [5]	For using EE = $\lambda x^2/(2L)$; may be scored in (i) or in (ii) For equating PE loss and EE gain and attempting to solve for W
(ii)	[800 = 32000x/20] ½ $(800/9.8)v^2$ = 800 x 20.5 - $32000\text{x}0.5^2/(2\text{x}20)$ Maximum speed is 19.9ms^{-1}	M1 M1 A1 A1 [4]	For using W = $\lambda x/L$ at max speed For using the principle of conservation of energy (3 terms required)
(iii)	$(800)\ddot{x}/g = 800 - 32000 \times 5/20$ Max. deceleration is 88.2 ms^{-2}	M1 A1 A1 [3]	For applying Newton's second law to jumper at lowest point (3 terms needed)

7(i)	[½ mv^2 – ½ m 6^2 = mg(0.7)] Speed of P before collision is $7.05ms^{-1}$ Coefficient of restitution is 0.695	M1 A1 B1ft [3]	For using the principle of conservation of energy for P (3 terms needed) ft 4.9 ÷ speed of P before collision
(ii)	[½ mv^2 = ½ m 4.9^2 - mg0.7(1 - cos θ)] v^2 = 3.43(3 + 4 cos θ) T – mgcos θ = mv^2/0.7 [T – m9.8cos θ = m3.43(3 + 4 cos θ)/0.7] Tension is 14.7m(1 + 2cos θ)N	M1 A1 M1 A1 M1 A1 [6]	For using the principle of conservation of energy for Q Accept any correct form For using Newton's second law radially with $a_r = v^2/r$ For substituting for v^2 AG
(iii)	T = 0 → θ = 120° Radial acceleration is (±)4.9 ms^{-1} **or** transverse acceleration is (±)8.49 ms^{-1} Radial acceleration is (±)4.9 ms^{-1} **and** transverse acceleration is (±)8.49 ms^{-1}	B1 M1 A1 B1 [4]	For using a_r = -gcos θ {or 3.43(3 + 4cos θ)/0.7} or a_t = –gsin θ
			SR for candidates with a sin/cos mix in the work for M1 A1 B1 immediately above. (max. 1/3) Radial acceleration is (±)8.49 ms^{-1} **and** transverse acceleration is (±)4.9 ms^{-1} B1
(iv)	[V^2 = 3.43{3 + 4(-0.5)}x0.5^2 or V^2 = (-gcos120° x 0.7) x $\cos^2 60°$] V^2 = 0.8575 [mgH = ½ m(4.9^2 – 0.8575) or mg(H – 1.05) = ½ m(3.43 – 0.8575)] Greatest height is 1.18 m	M1 A1 M1 A1 [4]	For using V = v(120°) x cos60° AG For using the principle of conservation of energy

1 i	$(-)15\cos\alpha = (0 -)\ 0.5\text{x}22$ or $15\sin\beta = 0.5\text{x}22$ Impulse makes angle 42.8^o (0.748 rads) with negative x-axis	M1 A1 A1 [3]	M1 for using I = Δ(mv) in 'x' direction or for sketching Δ reflecting **I** = m(**v** – **u**) AEF, but angle must be clear
ii	$15\sin\alpha = 0.5\text{v}$ or $15\cos\beta = 0.5\text{v}$ or $(0.5\text{v})^2 = 15^2 - 11^2$ Correct explicit expression for v Speed is 20.4 ms^{-1}	M1 A1 A1 [3]	For using I = Δ(mv) in 'y' direction or using sketched Δ

2	½ $(\text{m})(\text{v}^2 - 6^2) = -(\text{m})\text{g} \times 0.5$ in (i) or ½ $(\text{m})(\text{v}^2 – 6^2) = -(\text{m})\text{g} \times 1$ in (ii) $\text{v}^2 = 26.2$ in (i) and 16.4 in (ii) $\text{T} = 0.4\text{v}^2/0.5$ in (i) or $\text{T} + 0.4\text{g} = 0.4\text{v}^2/0.5$ Tension is 21.0N in (i) (20.96) 9.2N in (ii)	M1 A1 M1 A1 A1 A1 [6]	For using the principle of conservation of energy in (i) or (ii) soi For using Newton's second law with $a = \text{v}^2/\text{L}$. M1 for either attempt, A1 for both right

3 i	$2.8V = 1.4\text{x}72$ Vertical component at P is 36 N	M1 A1 [2]	For taking moments about Q for PQ or for using symmetry
ii	$36 + N = 72 + 54$ Normal component at R is 90 N	M1 A1 [2]	For resolving forces vertically on both rods AG
iii	$1.44\text{F} = 1.2\text{x}90 – 0.8\text{x}54$ or $72\text{x}1.4 + 54\text{x}3.6 + 1.44\text{F} = 90\text{x}4$ with not more than 1 error in either case Equation correct and leading to F = 45 For using $\text{F} = \mu\text{R}$ Coefficient is 0.5	M1 A1 A1 M1 A1 [5]	For taking moments about Q for QR or about P for the whole structure (all terms needed)

4 i	$0.4(7\times0.6) - 0.3\times2.8 = 0.4a + 0.3b$ $0.7(7\times0.6 + 2.8) = b - a$ Speed of B is $4ms^{-1}$	M1 A1 M1 A1 M1 A1 [6]	For using the principle of conservation of momentum For using $e(\Delta u) = \Delta v$ For eliminating a from equations
ii	$a = (-)0.9$ Component perp. to l.o.c. is 5.6 $\tan \alpha = 5.6/0.9$ $\alpha = 80.9°$ Angle turned through is $46.0°$ (0.803^c)	B1 B1 M1 A1 A1ft [5]	For attempting to find α - the angle between the direction of motion of A after collision and the l.o.c. to the left, or $90° - \alpha$ $126.9° - \alpha$

5 i	$2.45e/0.5 = 0.05g$ $(e = 0.1)$ Distance from O is $0.5 + 0.1 = 0.6$m	M1 A1 A1 [3]	For using $T = \lambda e/L$ and resolving forces vertically accept use of 0.1 to show both sides equal to 0.49 AG
ii	$mg - T = m\ddot{x}$ $0.05g - 2.45(0.1 + x)/0.5 = 0.05\ddot{x}$ $\ddot{x} = -98x$	M1 A1 A1 [3]	For using Newton's second law with 3 terms AG
iii	$a = 0.075$ $n = 7\sqrt{2}$ oe $x = 0.075\cos(7\sqrt{2}\,t)$ $x(0.2) = -0.0298$ $v = -0.075(7\sqrt{2})\sin(7\sqrt{2}\,t)$ $v(0.2) = -0.681 \Rightarrow$ velocity is $0.681ms^{-1}$ upwards	B1 B1 M1 A1 M1 A1ft A1 [7]	 accept 9.90 For using $x = a\cos nt$ oe For differentiating $x = a\cos nt$ and using it ft incorrect a and/or n If from $v^2 = n^2(a^2 - x^2)$ the direction must be clearly established

6 i	$112e/4 = 3.5 \times 9.8 \times \frac{40}{49}$	M1	For using $mg\sin\theta$ and $\lambda e/L$
		A1	
	$V^2 = 2\times8\times(4 + 1)$	M1	For using $s = 4 + e$ and a = 8 in $v^2 = 2as$, or by energy
	$V^2 = 80$	A1	
	$0.5\sqrt{80} = (0.5 + 3.5)u$	M1	For using the principle of conservation of momentum
	Initial speed of combined particles is $\frac{1}{2}\sqrt{5}$ ms^{-1}	A1	
		[6]	AG
ii		M1	For using EE = $\lambda x^2/2L$
	Gain in EE = $(112/(2\times4))\{(X + 1)^2 - 1^2\}$	A1	
	Loss of KE = ½ (0.5 + 3.5) x 5/4	B1	
	Loss of PE = $(0.5 + 3.5) \times 9.8 \times \frac{40}{49}X$	B1	
			For using the principle of conservation of energy
	$14(X^2 + 2X) = 2.5 + 32X$	M1	
	$28X^2 - 8X - 5 = 0$	A1	AG
		[6]	
OR	$T - mg\sin\theta = -ma$	M1	For use of $F = ma$
	$\frac{112(x+1)}{4} - 4g\frac{40}{49} = -4a$	A1	allow one sign slip for A1
	$\int (7x-1)dx = -\int v\,dv \ (+c)$	M1	Using a = $v\frac{dv}{dx}$ and integrating
	$\frac{7x^2}{2} - x = -\frac{v^2}{2} + c$	A1	
	$c = \frac{5}{8}$	A1	
		A1	AG Convincingly
	$28X^2 - 8X - 5 = 0$	[6]	

7 i	$0.2g - v^2/2000 = 0.2v(dv/dx)$ $(\frac{400v}{3920 - v^2})\frac{dv}{dx} = 1.$	M1 A1 [2]	For using Newton's second law with $a = v(dv/dx)$ AG Convincing, with no slips.
ii	$-200 \ln(3920 - v^2) = x + (A)$ $-200 \ln(3920) = A$ $x = 200 \ln(\frac{3920}{3920 - v^2})$ $e^{x/200} = 3920/(3920 - v^2)$ $v^2 = 3920(1 - e^{-x/200})$ $0 < e^{-x/200} \Rightarrow v^2 < 3920$	M1 A1 M1 A1 M1 A1 B1 [7]	For separating variables and integrating For using v(0) = 0 For using inverse ln process AG Convincingly – dep on correct answer
iii	Using $0.2g - v^2/2000 = 0.2a$ $v = 40$ Gain in KE = ½ 0.2x1600 (=160J) $x = 200 \ln(\frac{3920}{3920 - 1600})$ (= 104.90) 0.2g x (104.9) – 160 Work done is 45.6 J	M1 A1 B1ft B1ft M1 A1 [6]	 For using WD = loss of PE – gain in KE
OR	Using $0.2g - v^2/2000 = 0.2a$ $v = 40$ $x = 200 \ln(\frac{3920}{3920 - 1600})$ (= 104.90...) WD = $\int \frac{v^2}{2000} dx + c$ $= \int \frac{3920}{2000}(1 - e^{-x/200})\, dx$ $= 3920/2000(x + 200e^{(-x/200)} - 392$ Work done is 45.6 J	M1 A1 B1ft M1 A1 A1 [6]	 Use of WD = $\int F dx$ and subst for v^2

1	$[5\cos\theta - 4 = 0]$ $\cos\ \theta = 0.8$ $[I = 0.3(5\sin\theta - 0)$ or $\sin\theta = I \div (0.3 \times 5)]$ $I = 0.9$	M1 A1 M1 A1 [4]	For using $v_x - u_x = 0$ **or** for a triangle sketched with sides $I/0.3$, 4 and 5 with angles θ and 90° opposite I/m and 5 respectively. AG For using I = $m(\Delta v)$ in 'y' direction or $I = \sqrt{\left((0.3\times 5)^2 - (0.3\times 4)^2\right)}$ M1

2 i	$(1.8 + 3.2)R_B = (3.2 + 0.9)$x300 + 1.6x400 Force exerted on *AB* is 374 N Force exerted on *AC* is 326 N	M1 A1 A1 B1 [4]	For taking moments about *C* for the whole for M1 need 3 terms; allow 1 sign error and/or 1 length error and/or still including sin/cos or for taking moments about *B* for whole $(1.8 + 3.2)R_C = (1.8 + 1.6)$x400 + 0.9x300 giving force on *AC* first: M1A1A1A1
ii	0.9x300 + 1.2T = 1.8x374 Tension is 336 N	M1 A1 A1 [3]	For taking moments about *A* for *AB* for M1 need 3 terms, allow 1 sign error and/or 1 length error and/or still including sin/cos or moments about *A* for *AC* 1.6x400 + 1.2T = 3.2x326
iii	Horizontal component is 336 N to the left [Y = 374 – 300] Vertical component is 74 N downwards	B1ft M1 A1ft [3]	For resolving forces on *AB* vertically

Give credit for part (ii) done on the way to part (i) if not contradicted in (ii).

3 i	$0.25(dv/dt) = -0.2v^2$ $0.25\int v^{-2}dv = -0.2t(+C)$ $-v^{-1}/4 = -t/5 + C$ $[1/4v = t/5 + 1/20]$ $v = \dfrac{5}{4t+1}$ oe	M1 dep M1 A1 M1 A1 [5]	For using Newton's second law with $a = dv/dt$. Allow sign error and/or omitting mass For separating variables and attempting to integrate (ie get v^{-1} and t). For using $v(0) = 5$ to obtain C
ii	$x = (5/4)\ln(4t + 1)$ (+ B) Subst $v = 0.2$ in (i) to find t Obtain $x(6)$ (= 1.25 ln25 oe (4.02359...)) Average speed is 0.671 ms^{-1}	M1 A1 M1 M1 A1 [5]	For using $v = dx/dt$ and integrating Implied by $t = 6$ May be written as $\frac{5}{12}\ln 5$
	Alternatively $\ln v = -0.8x + B$ Subst $v = 0.2$ in (i) to find t Obtain $x(0.2)$ (= 1.25 ln(5/0.2) oe (4.0239...)) Average speed is 0.671 ms^{-1}	M1 A1 M1 M1 A1	For using $mv(dv/dx) = -0.2v^2$, separating variables and integrating. Allow sign error and/or omitting mass. Implied by $t = 6$ May be written as $\frac{5}{12}\ln 5$

4 i	$[-0.2\text{x}2\ddot{\theta} = 0.2g\sin\theta]$ $\dfrac{d^2\theta}{dt^2} = -4.9\sin\theta$ For small θ, $\sin\theta \approx \theta$ **and** $\ddot{\theta} = -4.9\theta$ represents SHM	M1 A1 B1 [3]	For using Newton's second law transversely. Allow sign error and/or sin/cos error and/or missing 0.2, g or l. AG
ii	$\theta = 0.15\cos(\sqrt{4.9}\,t)$ oe $t = 1.04$ at first occasion $t = 1.80$ at second occasion	M1 A1 A1 M1 A1 [5]	For using $\theta = A\cos(nt)$ or $A\sin(nt + \varepsilon)$. Allow sin/cos confusion for using $t_1 + t_2 = 2\pi / n$
iii	Angular speed is (-) 0.297 rads s^{-1} Linear speed is (-) 0.594ms^{-1}	M1 A1 A1ft [3]	For using $\dot{\theta} = -An\sin(nt)$ oe. Allow sign error and/or ft from θ in (ii).

In (ii) & (iii) allow M marks if angular displacement/speed has been confused with linear.

5 i	[sin γ = 0.96 ÷ 1.2] sin γ = 0.8	M1 A1 [2]	For using $v_B \sin\gamma = u_B \sin\beta$
ii	$(m)2 - (m)u_B\cos\beta = (m)v_B\cos\gamma$ $2 = v_B(0.6 + 0.28 \div 1.2)$ $v_B = 2.4$, $u_B = 2$	M1 A1 M1 A1 A1 [5]	For using the principle of conservation of momentum. Allow sign error and/or $u_A\cos\alpha$ (instead of 2) for M1. allow $u_A\cos\alpha$ (instead of 2) for A1 For eliminating u_B or v_B. Allow with cos Or $2 = 0.28u_B + 0.72u_B$
iii	$[(2 + u_B\cos\beta)e = v_B\cos\gamma]$ $(2 + 2 \times 0.28)e = 2.4 \times 0.6$ $e = \frac{9}{16}$ or 0.5625	M1 A1ft A1 [3]	For applying Newton's exp'tal law. Allow sign error and/or $u_A\cos\alpha$ (instead of 2) for M1. ft u_B and v_B only
iv	$[(y\text{-component})^2 = 13 - 4]$ $v_A = (y\text{-component})_{\text{before}} = 3$	M1 A1 [2]	For using ½ $(m)v^2 = 6.5(m)$ **and** $(y\text{-component})^2 = v^2 - 2^2$. Allow 1 slip.

6 i	PE gain = $6 \times 0.8(\sqrt{3}/2 - 1/\sqrt{2})$ $= 2.4(\sqrt{3} - \sqrt{2})$ EE loss = $\frac{9}{2(\pi/10)}[(0.8\pi/4 - \pi/10)^2 - (0.8\pi/6 - \pi/10)^2]$ EE loss = $45\pi[(0.2 - 0.1)^2 - (0.4 - 0.3)^2 \div 9]$ $= 5\pi(9 \times 0.01 - 0.01) = 40\pi/100 = 0.4\pi$ J	M1 A1 M1 A1 A1 [5]	For using PE gain = $W(h_Y - h_X)$ Shown fully, with no slips AG For using EE loss = $\lambda(e_X^2 - e_Y^2)/2l$. Allow slips for M1. Fully correct No slips in simplification AG
ii	$T = 9(0.8\pi/6 - \pi/10) \div (\pi/10)$ $W\sin\theta - T = 6 \times \sin(\pi/6) - 90 \times (0.2 \div 6) = 0$ ➔ transverse acceleration is zero ½ $(6/9.8)v^2 = 0.4\pi - 2.4(\sqrt{3} - \sqrt{2})$ Maximum speed is 1.27 ms^{-1}	B1 M1 A1 M1 A1 A1 [6]	For attempting to show that $W\sin\theta - T = 0$ at Y by subst $\theta = \pi/6$ AG No slips For using KE gain = EE loss – PE gain at Y. Need 3 terms, allow sign errors and/or g omitted.

7 i		M1	For using the principle of conservation of energy. Allow sign error, sin/cos; need 3 terms.
	½ mv^2 = ½ $m5.6^2 - mg0.8(1 - \cos\theta)$	A1	
	$v^2 = 15.68(1 + \cos\theta)$	A1	AG No slips
	$T - mg\cos\theta = mv^2/r$	M1 A1	For using Newton's second law. Allow sign error and/or sin/cos and/or m omitted
	[$T - 0.3g\cos\theta = 0.3\text{x}15.68(1 + \cos\theta)/0.8$] Tension is $2.94(3\cos\theta + 2)$ N oe	M1 A1 [7]	For substituting for v^2
ii	θ is 131.8° (or 2.3 rads) Accept 132° (exact) v is 2.29	M1 A1 B1 [3]	For putting $T = 0$ and attempting to solve accept $\theta = \cos^{-1}(-2/3)$ $\sqrt{15.68/3}$ exact
iii	[speed = $\lvert v\cos(180 - \theta)\rvert$ = $\sqrt{15.68/3} \times (2/3)$]	M1	For using 'speed at max. height = horiz. comp. of vel. when string becomes slack'
	Speed at greatest height is 1.52 ms^{-1}	A1	
	$0.3gH$ = ½ $0.3(5.6^2 - 1.52...^2)$ Greatest height is 1.48 m	M1 A1 [4]	For using the principle of conservation of energy 40/27 exact
	ALTERNATIVE for (iii) [$0 = 2.286..^2 \times (1\text{-}4/9)\ \ -19.6y$, $H = 0.8(1 + 2/3) + y$] $H = 1.3333.. + 0.1481...$ (4/3 + 4/27) Greatest height is 1.48 m (40/27)	 M1 A1	For using $0^2 = \dot{y}^2 - 2gy$ and $H = 0.8\{1 + \cos(180 - \theta)\} + y$
	[½ $m(2.286...^2 - \text{speed}^2) = mg \times 0.1481....$ $\text{speed}^2 = 2.286..^2 - 19.6 \times 0.1481....$] or [½ $m(5.6^2 - \text{speed}^2) = mg \times 1.481....$ $\text{speed}^2 = 5.6^2 - 19.6 \times 1.481....$] Speed at greatest height is 1.52 ms^{-1}	 M1 A1	For using the principle of conservation of energy

Question			Answer	Marks	Guidance	
1	(i)		Triangle of velocities/momentum All correct Use of Pythagoras' theorem to find I $I = 0.075$	 M1 A1 M1 A1 **[4]**	For right angled triangle with at least one side correctly shown (2.5, 2, 20I or 0.125, 0.1, I) or vector equation $(v_1, v_2) =$ $(0, 20I) + (2, 0)$ with at least 3 of the 4 components on the RHS correct $400I^2 + 2^2 = 2.5^2$ or $I^2 = 0.125^2 - 0.1^2$	 may be implied by $v_1^2 + v_2^2 =$ 2.5^2 or $\sin\alpha = 0.6$
1	(ii)		Components of velocity parallel to the wall before and after are 2 and 2 Components of velocity perpendicular to the wall before and after are (–) 1.5 and 1.5e $[2^2 + (1.5e)^2 = 5]$ Coefficient is $\frac{2}{3}$ or 0.667	 B1 B1 M1 A1 **[4]**	 For using $v_1^2 + v_2^2 = 5$ Must be perp to wall	 may be implied
2	(i)		$2mu\cos\alpha - mu\cos\alpha = 2ma + mb$ $0.5(u\cos\alpha + u\cos\alpha) = b - a$ Comp of B's velocity along l.o.c. is $u\cos\alpha$ Establishing B's speed unchanged	M1 M1 A1 A1ft A1 **[5]**	For using the p.c.m. parallel to l.o.c. For using NEL parallel to l.o.c. for both p.c.m and NEL correct & consistent dep on M1M1 gained by stating vel perp l.o.c. still $u\sin\alpha$, hence result, dep on all previous marks	allow sign errors, $m/2m$, sin/cos allow sign errors, e left in or by showing speed is still u condone 'vertical' in this part
2	(ii)		$a = 0$ correct interpretation of direction of A Direction of B is at angle α to l.o.c.., with an indication that removes ambiguity (eg in sketch)	B1 B1 B1 **[3]**	may be shown in (i) perp to l.o.c.	 condone 'vertical' for perpendicular, accept sketch, and refs to sketch in (i)

Question			Answer	Marks	Guidance	
3	(i)		 $0.3v(\mathrm{d}v/\mathrm{d}x) = -1.2v^3$ $[-v^{-1} = -4x + A]$ $[-u^{-1} = 0 + A]$ $v = \dfrac{u}{4ux+1}$	M1 A1 M1* *M1 A1 **[5]**	For using Newton's second law and $a = v(\mathrm{d}v/\mathrm{d}x)$ For finding $\mathrm{d}v/\mathrm{d}x$ in terms of v and attempting to integrate For using $v(0) = u$ AG	allow missed – sign / stray g / missed 0.3 allow $A/v = Bx + C$ oe
3	(ii)		$\int(4ux+1)dx = \int udt$ $2ux^2 + x = ut + B$ $[(2 \times 4 - 9)u = -2]$ $u = 2$	M1* A1 *M1 A1 **[4]**	For using $v = \mathrm{d}x/\mathrm{d}t$, separating the variables and attempting to integrate one side For using $x(0) = 0$ (may be implied by absence of B) and $x(9) = 2$ – dep on int being done	$-1.2v^3=0.3\,\mathrm{d}v/\mathrm{d}t$ and attempt to int one side M1* $8t = 1/v^2 - 1/u^2$ and subst for v A1 then as main scheme
4	(i)		EE gain = $44.1x^2 \div (2\mathrm{x}0.75)$ PE loss = $1.8g(0.75 + x)$ $[x^2 - 0.6x - 0.45 = 0]$ Extension is 1.03 m	B1 B1 M1 A1 **[4]**	 ignore signs For using EE gain = PE loss	allow use of $(e + x)$ for x $44.1x^2-26.46x-19.845=0$ allow sign errors 1.0348469...
4	(ii)		 $\dfrac{44.1\times1.03}{0.75} - 1.8 \times 9.8 = -1.8\,\ddot{x}$ Acceleration is -24.0 ms^{-2}	M1 M1 A1ft A1 **[4]**	For using $T = \lambda x/L$ For using Newton's 2nd law ft their '1.03' from (i) direction must be clear	 allow missed g, m, sign error allow sign error 1.03 → − 23.84666 1.035 → − 24.01

Question			Answer	Marks	Guidance	
5	(i)		 84.5 × 12L/13 = T(2L) Tension is 39 N	M1 A1 A1 **[3]**	For taking moments about B for BC must use 12/13 for cos β	must be 2 terms involving T, L, 84.5 and sin/cos β
5	(ii)		 X = 39 × 5/13 Y = 84.5 – 39 ×12/13 X is to the left and Y is upwards	 M1 A1 FT A1 FT A1cao **[4]**	For resolving forces on BC horiz or vert explicit expression for X explicit expression for Y AG (numerical values – must be correct) dep M1A1A1	must involve their T and sin/cos β accept on diagram
5	(iii)		 84.5 × $L\cos\alpha$ +48.5 ×2$L\cos\alpha$ = 15 × 2$L\sin\alpha$ [$\tan\alpha = \frac{84.5+97}{30}$] α = 1.41^c or 80.6^o	M1* A1 *M1 A1 **[4]**	For taking moments about A for AB For obtaining a numerical expression for tan α	must involve 3 terms, 84.5, 48.5, 15,sinα and cos α; allow sign errors, L/2L similar scheme for those who take moments about A for whole system
6	(i)		[0.4π = 2π/n] n = 5 Distance OA is 0.8 m	M1 A1 M1 A1 **[4]**	For using $T = 2\pi/n$ For using $v_{max} = n(OA)$	
6	(ii)		[x = 0.8cos(5×1)] x = 0.227 [$\dot{x}$ = –0.8 × 5sin(5×1)] Velocity is 3.84 ms^{-1}	M1 A1 M1 A1 **[4]**	For using $x = a\cos nt$ For using $\dot{x} = -an\sin nt$	 Use of $v^2 = n^2(a^2 - x^2)$ M1 Direc needs to be shown for A1

Question			Answer	Marks	Guidance	
6	(iii)		t and x for one point t and x for second point t and x for third point correctly stating precisely 3 points	B2 B1 B1 B1	Values of t are = 0.257, 0.372, 0.885 Values of x are 0.227, –0.227 , –0.227 sc all 3 x values B2 all 3 t values B2 one t value B1 one x value B1	0.4π – 1, 1 – 0.2π, 0.6π – 1 ignore ref to point when t = 1 can show on graph
			If B1 or B0 scored (out of first 4) on above scheme, allow, subject to max mark 2, Number of occasions is 3	(M1) (A1) **[5]**	For $t = 1 \approx 0.8T \Rightarrow 3/4T < 1 < 4/4T$ or equiv	
7	(i)		Tension in string $T = mg\sin\alpha$ For using $e = R\alpha - 2R/3$	M1 B1 B1	For using $T = \lambda x/L$ $mg \sin\alpha = 1.2mg\left(Ra - \frac{2R}{3}\right) \div \frac{2R}{3}$	
			$1.8\alpha - \sin\alpha - 1.2 = 0$ Finding f(1.175) and f(1.185) correctly correct conclusion	A1 M1 A1 A1 **[7]**	AG establish result ≈ -0.008, and $\approx +0.0065$ AG $\alpha = 1.18$ correct to 3 significant figures	By iteration $\alpha = (1.2 + \sin\alpha)/1.8$ M1 start [1, 2], and 1 iteration A1 at least 1 more iteration, and conclusion 1.18(0427) A1
7	(ii)		Direction is towards O	B1 **[1]**		
7	(iii)		 Gain in EE = $1.2mg(1.18R - 2R/3)^2 \div (2\text{x}2R/3)$ PE loss = $mgR(\cos 2/3 - \cos 1.18)$ $v^2 =$ $2gR[\cos 2/3 - \cos 1.18 - 0.9(1.18 - 2/3)^2]$ Acceleration is 3.29 ms^{-2}.	M1* A1 A1 M1 A1 *M1 A1 **[7]**	For using EE = $\lambda e^2 \div (2L)$ and PE = mgh ignore signs For using ½ mv^2 = PE loss – EE gain For using acceleration = v^2/R	 allow α for 1.18 for A1A1 allow sign errors need 1.18 here If candidates use $mR\ddot{\theta}$ use equivalent scheme

Question			Answer	Marks	Guidance
1		(i)	[40*d* = 30 × 2] Distance is 1.5 m	M1 A1 **[2]**	For taking moments about *B* for *BC*
		(ii)	30 = 0.75 *R* Horizontal component on *AB* at *B* is 40 N to the left For resolving forces on *BC* vertically, or taking moments about *C* Vertical component on *AB* at *B* is 10 N down	B1 B1 M1 A1 **[4]**	$Y + 30 = 40$, or 40 x ½ = Y x 2 Accept directions on diagram, if not contradicted in text SR A1 if both magnitudes correct but directions wrong/not stated
		(iii)	(+/-)10 × 2 + 60 × 0.8*d* = (+/-)40 × 1.5 Distance is 0.833 m	M1 A1 FT A1 **[3]**	For taking moments about *A* for *AB* FT magnitudes of components at *B*; need to use '$x = d\cos\theta$' May see moments about *A* for *ABC* (60 x 0.8*d* + 40 x 3.5 = 30 x 4 + '40' x 1.5) or moments about *B* for *AB* – need to get equation with only '*d*' unknown for M1
2		(i)	Since plane is smooth impulse is perpendicular to plane(so $\theta = 15$)	B1 **[1]**	
		(ii)	Use of $v^2 = (u^2) + 2 \times g \times 2.5$ $v = 7$ ms^{-1} after impact: Speed parallel to plane is 7sin15° u = 7sin15° / cos 60° u = 3.62 I = 0.45(7 cos15° + $u$sin60°) I = 4.45 Or For using a triangle with sides 3.15 (0.45 x 7), I and 0.45 x u (or 7, I/0.45 and u) and correct angles 135°, 15° and 30° Use of sin rule or cos rule (correct) u = 3.62 I = 4.45	M1 A1 B1 M1 A1 M1 A1 **[7]** M1 A1 M1 A1 A1	 1.81(173…) Allow sin/cos errors Allow sin/cos errors or I = 0.45(7 cos15° + 7sin15°tan60°) 4.45477…. May see e = 0.464 Need 2 correct sides and 1 correct angle All correct OR Icos15° = 3.15 + 0.45 $u$cos45° M1 Isin15° = mucos45° B1 Solve sim equations M1, dep attempt at two comps of I Answers A1A1

Question			Answer	Marks	Guidance
3		(i)		M1	For using N's 2nd law with $a = v\,dv/dx$; 3 terms
			$v\,dv/dx = g - 0.0025v^2$	A1	
			$\int \frac{vdv}{g - 0.0025v^2} = \int dx$	M1	For correctly separating variable and attempting to integrate
			$-200\ln(g - 0.0025v^2) = x\ (+ A)$	A1	
			$A = -200\ln g$	M1*	Attempt to find A from $B\ln(C - Dv^2)$
			$[g - 0.0025v^2 = ge^{-0.005x}]$	*M1	For transposing equation to remove ln
			$v^2 = 400g(1 - e^{-0.005x})$	A1	
			$0 < e^{-0.005x} \leq 1 \Rightarrow v^2$ cannot reach $400g$ ie cannot reach 3920	B1	dependent on getting other 7 marks. Need '0 <' oe
				[8]	
		(ii)	$v^2 = 400g(1 - e^{-0.5})$	M1	For substituting for x and evaluating v must have $v^2 = A + Be^{Cx}$ for (i), but not neces in this form
			Speed of P is 39.3 ms^{-1}	A1	
				[2]	
4		(i)		M1	For using the pce condone sin/cos and sign errors; need KE before and after and difference in PE
			$\frac{1}{2}mv^2 + mg(0.6)(1 - \cos\theta) = \frac{1}{2}m4^2$	A1	
			$v^2 = 4.24 + 11.76\cos\theta$	A1	AG
				M1	For using Newton's 2nd law, condone sin/cos and sign erorrs; 3 terms needed
			$R - 0.45g\cos\theta = 0.45v^2/0.6$	A1	
			$R = 3.18 + 13.23\cos\theta$	A1	
				[6]	
		(ii)		M1	For using $R = 0$
			$\cos\theta = -3.18/13.23$	A1 FT	$-0.24036\ldots$ or $-106/441$ or $\theta = 103.9°$ ft from $R = A + B\cos\theta$, where $A, B \neq 0$
			$[v^2 = 4.24 - 11.76 \times 3.18/13.23]$	M1	For substituting for $\cos\theta$
			Speed is 1.19 ms^{-1}	A1	CAO without wrong working
				[4]	

Question			Answer	Marks	Guidance
5		(i)	[0.8mgx/0.78 = mg(5/13)]	M1	For resolving forces and using $T = \lambda x / L$ at equilibrium position
			x = 0.375	A1	Accept 1.155 for $e + l$
			PE = mg(0.78 + 0.375) × 5/13	B1 FT	FT value of x
			EE = 0.8mg × 0.375^2 ÷ (2 × 0.78)	B1 FT	FT value of x
			[½ mv^2 = m(4.353... – 0.7067...)]	M1	For using ½ mv^2 = PE loss – EE gain
			Maximum speed is 2.70 ms^{-1}	A1	
				[6]	
			OR at extension x		
			PE = $mg(x+0.78)\times\frac{5}{13}$	B1	
			EE = $\dfrac{0.8mgx^2}{2\times 0.78}$	B1	
			$mg(x+0.78)\times\dfrac{5}{13}=\dfrac{1}{2}mv^2+\dfrac{0.8mgx^2}{2\times 0.78}$	M1	For using ½ mv^2 = PE loss – EE gain
			$v^2 = -10.05x^2 + 7.53x + 5.88$		$v^2 = -\dfrac{40\times 9.8}{39}x^2+\dfrac{98}{13}x+\dfrac{9.8\times 3.9\times 2}{13}$
			$v^2 = -10.05(x^2 - 0.749x - 0.585)$		$v^2 = -\dfrac{392}{39}(x^2-\dfrac{3}{4}x-\dfrac{3\times 3.9\times 2}{40})$
			for attempting to complete square	M1	
			$v^2 = -10.05((x-0.375)^2 - 0.726)$	A1	$v^2 = -\dfrac{392}{39}((x-\dfrac{3}{8})^2 - 0.725625)$
			Max speed is 2.70 ms^{-1}	A1	Note, after getting equation for v^2, can instead Differentiate v^2 wrt x M1 Establish max at x = 0.375 A1 Max speed 2.70 ms^{-1} A1

Question			Answer	Marks	Guidance
		(ii)		M1*	For using PE loss = EE gain
			$mg(0.78 + x) \times 5/13 = 0.8mgx^2 \div (2 \times 0.78)$	A1	or $mg(x) \times 5/13 = 0.8mg(x - 0.78)^2 \div (2 \times 0.78)$ if PO = x or $mg(x+0.78+0.375) \times 5/13 = 0.8mg(x + 0.375)^2 \div (2 \times 0.78)$ if $PO = x + 0.78 + 0.375$
			[$x^2 - 0.75x - 0.585 = 0$ if x is extension]	*M1	For arranging in quadratic form and attempting to solve All nec terms required
			$x = 1.2268$ so Distance is 2.01 m	A1	[$x^2 - 2.31x + 0.6084 = 0$ if $PO = x$] [$20x^2 = 14.5125$, if $PO = x + 0.78 + 0.375$]
				[4]	[$x = 2.0068$] [$x = 0.8518….$]
			OR put v = 0 in v^2 equation from above	M1A1ft	
			Solve to get $x = 1.23$ (+0.78) = 2.01 m	M1A1	

Question			Answer	Marks	Guidance
6		(i)		M1	For using ½ $m(u^2 - v^2) = 7.56$ and solving for v; *must use '5', allow sign error/ missing ½, missing m.*
			½ × $2(5^2 - v^2) = 7.56$ ($v^2 = 17.44$)	A1	
			Speed is 4.18 ms^{-1}	A1	Do not award if this is not candidate's final answer.
				[3]	
		(ii)	$v_{Ay} = u_{Ay} = 5\sin\alpha = 4$	B1	
			$[v_{Ax}^2 + 4^2 = 17.44 \rightarrow v_{Ax}^2 = 1.44]$	M1	For using $v_{Ax}^2 + v_{Ay}^2 = 17.44$
			$v_{Ax} = \pm 1.2$ and v_{Ax} must be less than 0.8		
			➔Component has magnitude 1.2 ms^{-1} and direction to the left	A1	
				[3]	
		(iii)		M1	For using the pcm parallel to loc must use 5cosα, 2, 0.8 and '1.2', 4 terms or equivalent, allow sign errors, condone one mass missing
			$2 \times 3 - m \times 2 = 2 \times (-1.2) + m \times 0.8$	A1 FT	FT incorrect v_{AX}
			$m = 3$	A1	CAO
				[3]	
		(iv)	$[e(3 + 2) = (1.2 + 0.8)]$	M1	For using NEL with their '1.2' and 5cosα, 2 and 0.8; allow sign errors. Must be right way up
			$e = 0.4$	A1	
				[2]	

Question			Answer	Marks	Guidance
7		(i)	$E_{(AP = 2.9)} = 120 \times 0.9^2/4 + 180 \times 0.1^2/6$ = (24.3 + 0.3) and $E_{(AP = 2.1)} = 120 \times 0.1^2/4 + 180 \times 0.9^2/6$ = (0.3 + 24.3) ➔same for each position Conservation of energy ➔ v = 0 when AP = 2.1, string taut here so taut throughout motion – oe,	M1 A1 B1 **[3]**	For using EPE = $\lambda x^2/2L$ for both strings for one position 24.6 seen twice Need to point out that v = 0 when AP = 2.1 or KE = 0 Dep on M1A1
		(ii)	$T_A = 120(0.5 + x)/2$, $T_B = 180(0.5 - x)/3$ [(30 – 60x) – (30 + 60x) = (+/−)0.8a] $a = -150x$	B1 M1 A1 **[3]**	soi For using Newton's 2nd law; allow omission of 0.8 With no wrong working
		(iii)	SHM because $a = -k$ (where $k > 0$) [$T = 2\pi/\sqrt{150}$] Time interval is 0.257 s	M1 M1 A1 FT **[3]**	SHM because $a = -\omega^2 x$ or in words For using $T = 2\pi/n$; must follow from (ii) FT $\pi \div$ candidate's n 0.256509…
		(iv)	[$x = 0.4\cos(\sqrt{150} \times 0.6) = 0.194$] [distance = 4a + (a – 0.194)] Distance travelled is 1.81 m	M1 M1 A1 **[3]**	For using $x = a\cos(0.6n)$, where n follows from (ii) and a is numerical. For using $T < 0.6 < 1.25\,T$ ➔ distance = 4 a + ($a - x$); may be implied by 1.6 < distance < 2.0 CAO, no wrong working
		(v)	Speed is 4.29 ms^{-1}.	M1 A1 **[2]**	For using $\dot{x} = -an\sin(0.6n)$, where n follows from (ii) Or using $v^2 = n^2(a^2 - x^2)$, where n follows from (ii) and x follows from (iv) or using $\dot{x} = an\cos(0.6n)$ if $x = a\sin(0.6n)$ used in (iv), where n follows from (ii) Condone −4.29

		Answer	Marks	Guidance	
1			M1		Use of cos rule; condone + for − / missing 2/ missing ‘0.6’; angle as ‘θ’ for M1
		$I^2 = 2.04^2 + 0.9^2 - 2\text{x}2.04\text{x}0.9\text{x}\frac{15}{17}$	A1	And attempt to square root	Condone + for −
		1.32 (N)	A1	CAO	(1.3159)
			M1	Correct use of sin rule from their diagram oe	Can be in terms of I α and θ
		46.8(°) with initial direction of ball	A1	CAO OR $0.9 + I\cos\theta = 0.6\text{x}3.4\text{x}15/17$ M1 $I\sin\theta = 0.6\text{x}3.4\text{x}8/17$ M1 square and add to find I^2; or divide to find θ M1 I, θ A1 A1 CAO	(46.8476) (0.8176 rads) Accept 46.7 from using I = 1.32 Allow missing 0.6 and/or sign or trig error for these 2 marks, then M0A0A0
			[5]		
2	**(i)**	Vel unchanged perp to L o C	M1		Stated or used
		$0.6\sin30° = v\cos30°$	M1		Allow 1 sign or trig error
		$0.2\sqrt{3}$ (ms^{-1})	A1		(0.34641)
			[3]		
2	**(ii)**	Use momentum equation	M1		Allow their v; allow sign errors / omission of m
		$0.3m - 0.6m\cos 30° = am + 0.2\sqrt{3}m\cos 60°$	A1ft	Follow through on v	m’s not necessary;
		(a =) 0.393 to left	A1	Direction must be clearly stated or implied from working. WWW	(0.39282) Away from B/opp direction to before
			[3]		
2	**(iii)**	Use of NLR	M1		Allow sign error and/or trig error
		$(0.2\sqrt{3})\cos 60° - (-0.393) = e(0.6\cos 30° + 0.3)$	A1ft	Ft on a and v	
		0.691	A1	CAO	(0.69082 or 0.6905679)
			[3]		

		Answer	Marks	Guidance	
3	(i)	Use of $F = ma$, using $v\frac{dv}{dx}$	M1*		Allow sign error / 0.3 omitted
		$0.3v\frac{dv}{dx} = 1.5x$	A1		
		Attempt to rearrange and integrate	*M1	$0.3v^2 = 1.5x^2 (+c)$	No need for c. At least one side integrated correctly
		$v = \sqrt{5}x$ **AG**	A1 **[4]**	correct derivation WWW	
3	(ii)	Integrate to find x in terms of t $\ln x = \sqrt{5}t + c$ $x = e^{\sqrt{5}t}$ $v = \sqrt{5}\,e^{\sqrt{5}t}$	M1 A1 A1 A1 **[4]**	$dx/x = \sqrt{5}dt$ and int 1 side correctly CAO	Need to separate variables No need for c for first 2 marks Must include showing c = 0.
		OR Integrate to find v in terms of t	M1	Use jn $0.3\frac{dv}{dt} = 1.5x$ and int 1 side correctly	No need for c for first 2 marks
		$\frac{dv}{v} = \sqrt{5dt}$			
		$\ln v = \sqrt{5}t + c$ $\ln v = \sqrt{5}t + \ln(\sqrt{5})$ $v = \sqrt{5}\,e^{\sqrt{5}t}$	A1 A1 A1	 CAO	 Must include showing c = ln(√5)

Answer			Marks	Guidance	
4	(i)	Conservation of energy	M1		Need 4 terms; allow sign & trig errors
			M1		Both KE or both PE correct
		$\frac{1}{2}0.4v^2+\frac{1}{2}0.6v^2+0.4ga\sin\theta-0.6ga\theta=0$	A1		completely correct
			M1	Attempt to find v^2 dep both earlier M1s	Allow with sign and trig errors
		$v^2=3.92a(3\theta-2\sin\theta)$	A1	**AG**	No errors
		F = ma radially for P	M1*		Allow sign and trig errors
		$0.4g\sin\theta-R=\frac{0.4v^2}{a}$	A1		
			*M1	Manipulation attempted, leading to $a\theta+b\sin\theta$	Allow sign and trig errors
		$R=-4.704\theta+7.056\sin\theta$	A1		$2.352(-2\theta+3\sin\theta)$
			[9]		
4	(ii)	Using $R = 0$	M1	$0=-4.704\theta+7.056\sin\theta$	
		(k =) $\frac{2}{3}$	A1		Must be from correct expression in (i)
			[2]		
5	(i)	2.5g = 36.75 e/3	M1	P in equilibrium	Allow missing g
		$e = 2$	A1		
		$v^2 = 0^2 + 2g(3 + e)$	M1		
		$v = 7\sqrt{2}$	A1		May be implied by $v^2 = 98$
		1 x v = 3.5 V	M1		
		Combined speed = $2\sqrt{2}$ (ms-1)	A1	**AG**	Convincing derivation, no errors
			[6]		

Answer			Marks	Guidance	
5	(ii)	change in PE is $3.5gX$ change in KE is $0.5 \text{x} 3.5\ (2\sqrt{2})^2$ change in EE is $36.75(X+2)^2/(2\times3)-36.75\times2^2/(2\times3)$ Use conservation of energy $35X^2 - 56X - 80 = 0$	B1 B1 M1 A1 M1 A1 **[6]**	$34.3X$ 14 $\frac{36.75(X+2)^2}{2\times3} = \frac{36.75\times2^2}{2\times3} + 3.5gX + \frac{3.5}{2}V^2$ **AG**	 Allow sign errors / omission of 2; Allow '*x*' or '*x* + 5' for '*x* + 2'; 2 terms or difference Allow sign errors; at least PE, KE, EE term Convincing derivation, no errors may see $36.75X^2 - 58.8X - 84 = 0$
6	(i)	Moments about *C* for *CD* $Wl\sqrt{3}/2(\cos30°) = Ql\sqrt{3}(\cos30°)$ $(Q =)\ W/2$ Resolve vert $(R =)\ \frac{3}{2}W$	M1 A1 A1 M1 A1 **[5]**	 **AG** CAO	allow M if sin/cos wrong
6	(ii)	$X = 0$ Resolve vert for *CD* or *AB* $Y = W/2$ Vertically downwards	B1 B1* *B1 **[3]**	 $Y + Q = W$ or $Y + W = R$	

Answer			Marks	Guidance	
6	(iii)	Moments about C for AB	M1		Allow M if sin/cos wrong or sign errors; need all terms
		$Pl\cos30° + Fl\cos30° = Rl\sin30°$	A1	Correct	
		Use P in terms of F	M1	$F = P$ or other correct 2nd step	Allow if missing term above
		Find F in terms of W, or in terms of R	M1	$F = \frac{\sqrt{3}}{4}W$	Or getting 'their' F oe, ie putting $F = \mu R$ in moment equation.
		$\mu = (F/R) = \sqrt{3}/6$	A1 **[5]**	Accept decimal answers from 0.288675	
		OR Moments about A for AB	M1		Allow M if sin/cos wrong or sign errors; need all terms
		$Wl\sin30° + (Y)l\sin30° + F2l\cos30° = R2l\sin30°$	A1		May have X term if not 0 in (ii)
		Write Y (and X) in terms of W	M1		
		Find F in terms of W, or in terms of R, oe	M1	$F = \frac{\sqrt{3}}{4}W$	
		$\mu = (F/R) = \sqrt{3}/6$	A1	Accept decimal answers from 0.288675	
7	(i)	Use of energy equation	M1		Allow M1 if sign error and/or 9.8 missing and/or missing m or l
		$0.5\ m\ (0.3)^2 = m\text{x}9.8\text{x}0.8\text{x}(1 - \cos\theta)$	A1		
		$\theta = 0.107$	A1 **[3]**	No errors **AG**	0.107194171
7	(ii)	Use $F = ma$	M1	$m \text{ x } 9.8 \sin\theta = -\ m \text{ x } 0.8\ \ddot{\theta}$	allow M1 if sign error, or 9.8 missing
		$\ddot{\theta} = -\ 12.25\ \theta$	A1		Allow fraction
		small θ	B1	Dep on having seen acc = $k\sin\theta$	Rigorous
		Use of $T = \frac{2\pi}{\omega}$	M1	or sight of $\omega = 3.5$	
		$T = 1.80$	A1 **[5]**		accept $\frac{4\pi}{7}$ (1.795195)

Answer			Marks	Guidance	
7	(iii)	identifying amplitude as 0.107	B1		ft from (i)
		Use of ($\dot{\theta}$) = 0.107x3.5xcos(3.5t)	M1	or sin(3.5t+ε), ε not 0	ft for a and ω; allow sign error
		Use of $\dot{\theta}$ = −0.25	A1	Consistent angle or length	
		t = 0.658	A1		(0.6576339)
		Use of θ = 0.107 sin(3.5t)	M1	ft from velocity equation (matches, ignore sign)	
		(θ =) 0.0797rads	A1 **[6]**	accept 5.20°	(0.0796678 or 0.079576)

Question			Answer	Marks	Guidance	
1			Use of $T=\frac{\lambda e}{l}$	M1	Attempt at one tension; allow use of x	allow $2l$ for M1
				A1	$\frac{20(d-0.4)}{0.4}$ or $\frac{30(d-0.6)}{0.6}$	either term seen, accept in terms of x
			Weight = tension 1 + tension 2	M1		condone Wg and W/g
				A1	$100 = 50d - 20 + 50d - 30$	fractions and brackets removed
			$(AW =)$ 1.5 (m)	A1 **[5]**		
2	**(i)**		Use of correct formula Vert speed imm before bounce = 2.8 ($\mathrm{m\,s^{-1}}$) Time between bounces = 0.286 (s) (2/7)	M1 A1 B1 **[3]**	$v^2 = 0^2 + 2 \times 9.8 \times 0.4$	or by energy
2	**(ii)**		Use of their t in a correct formula Vert speed imm after bounce = 1.4 ($\mathrm{m\,s^{-1}}$) Coeff of rest = 0.5	M1 A1 B1ft **[3]**	$0 = u + 9.8 \times 0.5(t)$ Allow their value of t Their values for v after/v before	or $-u = u - 9.8t$ must be worked out to fraction or decimal; $0 \le e \le 1$
2	**(iii)**		Imp = change of mom $I = 1.26$ (N s)	M1 A1 **[2]**	$I = 0.3 \times (v) + 0.3 \times (u)$ Allow their u, v CAO	allow sign errors for M1, allow if answer implies use of their values
3	**(i)**		Use of $F = ma$	M1	$\frac{3}{2}t - 1 = 0.2\frac{dv}{dt}$	allow sign errors or m omitted
			Integrate correctly	A1	$v = \frac{15}{4}t^2 - 5t(+c)$	allow if c missing or wrong
			$v = \frac{15}{4}t^2 - 5t + 0.8$	A1 **[3]**		oe

Question			Answer	Marks	Guidance	
3	(ii)		Use vel = 0.8	M1	$\frac{15}{4}t^2 - 5t + 0.8 = 0.8$	ft their **(i)**
			t = 1.33 (s) or 1 1/3 (s)	A1 **[2]**	must come from correct equation for v	Accept 4/3
3	(iii)		Integrate to find x	M1*	At least 2 terms with powers increased by 1	
			$x = \frac{15}{12}t^3 - \frac{5}{2}t^2 + 0.8t$	A1	Need to state c = 0, or use limits	
			Solve for x = 0	*M1		
			t = 1.6 (s) or 0.4 (s)	A1 **[4]**	Both answers needed; must be from correct work to find equation	Ignore t = 0
3	(iv)		$x(3) - x(2)$	M1	Allow for $x(2)$ or $x(3)$ worked out from (iii)	13.65 or 1.6
			Distance is 12.05 (m)	A1 **[2]**		Accept 12 or 12.1
4	(i)		Conservation of momentum	*M1	Must have 4 terms	allow sign errors, cosθ omitted
				A1	$0.1 \times 3 + 0.2 \times 1 \times \cos\theta = 0.1 \times a + 0.2 \times b$	a and b are vel components of A and B to right, respectively, after collision
			Newton's experimental law	*M1	Must have 4 terms and 0.8	allow sign errors, cosθ omitted
				A1	$b - a = -0.8(1 \times \cos\theta - 3)$	
			Attempt to solve their 2 sim eqns	M1*	Dep both previous M marks	allow 1 slip
			0.12 in same direction as before	A1 **[6]**	Direction may be implied by working	withhold if direction stated to left
4	(ii)		b = 2.04	B1	Must be seen/used in (ii)	
			vel of B perp to line of centres = 0.8	B1	$(1 \times \sin\theta)$	
			Direction of B after collision makes angle 21.4° with line of centres	M1 A1	$\tan\varphi = 0.8/2.04$; or 0.374 rads	Allow with their 0.8 and 2.04 (b from (i)); allow $\tan\varphi = 2.04/0.8$, if angle clear, leading to 68.4° for A1
			Angle turned through by B is 31.7°	A1ft **[5]**	or 0.554 rads; allow +/−	53.1(3) – φ, 0.927 – 0.374 rads

Question			Answer	Marks	Guidance	
5	(i)		Use of energy equation at A and B	M1	3 terms needed $mg0.6\cos\frac{\pi}{6} = mg0.6\cos\theta + \frac{1}{2}mv^2$	allow sign error, missing m / g / r
				A1		allow if θ replaced by $\varphi + \pi/6$
			$F = ma$ radially	M1 A1	$mg\cos\theta - R = \frac{mv^2}{0.6}$	allow sign error, missing m / g
			Use of $R = 0$	M1	May be incorporated in previous step	
			$\cos TOB = \frac{\sqrt{3}}{3}$ AG	A1	Completely correct	not given if decimals used for angle.
				[6]		
5	(ii)		Use of $\sqrt{3}/3$ in 'correct' equation in (i)	M1	$mg0.6\cos\frac{\pi}{6} = mg0.6\times\frac{\sqrt{3}}{3} + \frac{1}{2}mv^2$ or $mg\frac{\sqrt{3}}{3} = \frac{mv^2}{0.6}$	equation must have gained M1 in (i) but allow restart here
			1.84 (m s^{-1})	A1		
				[2]		
5	(iii)		Use of $F = ma$ tangentially	M1	$mg\sin\theta = ma$ seen	allow missing m/g, − sign; allow M1 if angular accel found
			8.00 (m s^{-2})	A1		
				[2]		
6	(i)		Moments about B for equilibrium of BC	M1	$2Wl\cos 60° + F2l\sin 60° = R2l\cos 60°$	3 moment terms, condone sin/cos errors and missing l. Need trig terms for M1
			$W + \sqrt{3}F = R$ AG	A1	Must be formula for R	correct, with sin/cos evaluated
				[2]		

Question			Answer	Marks	Guidance	
6	(ii)		Moments about A for equilibrium of whole system	M1	At least one of F and R terms must involve lengths of both rods	At least 3 moment terms, condone sin/cos errors, sign errors and $l/2l$ confusion/missing. Wrong use of forces at B gets M0
				A1	$Wl\cos 30 + 2W(2l\cos 30 + l\cos 60) + F(2l\sin 60 + 2l\sin 30) = R(2l\cos 30 + 2l\cos 60)$	4 terms, accept sin/cos errors and $l/2l$ confusion/missing and sign errors for A1
				A1	sin/cos left in, but correct	
			$W\left(\frac{5\sqrt{3}}{2}+1\right)+F\left(\sqrt{3}+1\right)=R\left(\sqrt{3}+1\right)$	A1	fully correct, oe. Mark final answer	accept $5.33W + 2.73F = 2.73R$, $W\left(\frac{13}{4}-\frac{3\sqrt{3}}{4}\right)+F=R$
					Allow full credit for candidates who work out internal forces at B and work correctly from there.	Eg $3R = \sqrt{3}F + 7.5W$
				[4]		
6	(iii)		Solving 2 sim equations to eliminate F or R	M1	Both equations must involve W, F and R	allow slips in working
				A1	$F=\frac{3\sqrt{3}}{4}W$	$F = 1.299\ W$
				A1	$R=\frac{13}{4}W$	$R = 3.25\ W$
			Use $F = \mu R$ to find μ	M1	At any point	
			$(\mu =)\frac{3\sqrt{3}}{13}$ (0.39970)	A1		Accept 0.4 if with correct working $5.33(R - 1.73F) + 2.73F = 2.73R$ $2.6R = 6.52F$
					Or eliminate W M1A1A1 Use $F = \mu R$ M1 cao A1	
				[5]		

Question			Answer	Marks	Guidance	
7	(i)		Use of $F = ma$ when string stretched	M1	Must have mg – tension term (involving $39.2m$, 0.8 and x) $= ma$	allow if sign errors; x could be length or ext of string, or from eq^m pos.
					$mg - \frac{39.2m(x-0.8)}{0.8} = m\ddot{x}$	$mg - \frac{39.2mx}{0.8} = m\ddot{x}$ leads to
				A1	$\ddot{x} = -49(x-1)$	$\ddot{x} = -49(x-0.2)$
						$mg - \frac{39.2(x+0.2)}{0.8} = m\ddot{x}$ leads to
						$\ddot{x} = -49x$
			Show $x = 1$ is centre of SHM or that $x = 1$ is equilibrium position.	B1	and state about $x = 1$	Convincingly
				[3]		
7	(ii)		By energy	M1	Must be PE term and EE term	Allow for missing '2', wrong 'g' or inconsistent lengths
				A1	$mg(0.8+e) = \frac{39.2me^2}{2\times 0.8}$	Or $mgh = \frac{39.2m(h-0.8)^2}{2\times 0.8}$ and
						$h = 0.8 + e$
						$2.5e^2 - e - 0.8 = 0$
			$e = 0.8$ satisfies this equation **AG**	A1	Or by solving quadratic in e	Convincingly
					Allow full credit if done correctly from $v^2 = \omega^2(a^2 - x^2)$	Allow integration of $v\frac{dv}{dx} = g - 49x$
				[3]		

Question			Answer	Marks	Guidance	
7	**(iii)**		For SHM, $\omega = 7$	B1		To be awarded if seen in **(i) or (iv)**
			$a = 0.6$	B1		or seen or used here
			Correct use of appropriate SHM distance equation	M1	$-0.2 = 0.6 \cos (7t)$ or $-0.2 = 0.6 \sin (7t)$	Allow +0.2, allow their a and ω
			$t = 0.272(9476)$ from bottom ($x = 1.6$) to $x = 0.8$	A1	Could be 0.0485 + 0.224	
			$t = 0.404(061)$ from O to $x = 0.8$	B1	Or $\frac{2\sqrt{2}}{7}$	May be seen first
			Time to reach lowest point = 0.677 s	A1ft	(‘0.273’ + ‘0.404’)	
				[6]		
7	**(iv)**		Use of $v = -a\omega\sin\omega t$ or $a\omega\cos\omega t$	M1	Must ft from their ‘x’ equation in (iii), or shown here	Allow use of their a and ω, sign error
			$v = -0.6 \times 7\sin7t$	A1	or $0.6 \times 7\cos7t$	
			Use of $t = 0.8 - 0.677 = 0.123$ after bottom point	B1ft	Or use of $t = 0.3475$ in ‘cos’ version	Must be between 0 and 0.8
			$v = 3.19$ (3.185677...)	A1	(−)3.187	Do not allow if direction stated to be down.
				[4]		

Answer			Marks	Guidance	
1	**(i)**	realising impulse must be in same direction as velocity, or opposite max speed 2.8 (m s^{-1}) min speed 1.2 (m s^{-1})	M1 A1 A1 **[3]**	 0.8 +/− 0.6/0.3 − 1.2 is wrong	 various methods
	(ii)	Impulse momentum diagram $\cos\theta = \dfrac{0.6^2 + 0.24^2 - 0.75^2}{2 \times 0.6 \times 0.24}$ $\theta = 120°$ (2.098 rad) angle shown correctly	M1 A1 M1 A1 **[4]**	Triangle with sides labelled 0.24, 0.6 and 0.75 or 0.8, 2 and 2.5 accept 59.8° (1.04 rad) consistent with their θ; dep M1A1M1	Allow M1 if positions wrong. Diagram must be correct. $v_x = 0.8 + 2\cos\theta$ M1 either $v_y = 2\sin\theta$ and correct diag A1 both Square, add, giving 1.61 = 3.2cosθ M1 120.(21)...A1
2	**(i)**	By energy $\dfrac{30(d-0.6)^2}{2 \times 0.6} = 48 \times d$ $25d^2 - 78d + 9 = 0$ or $30d^2 - 93.6d + 10.8 = 0$ (d =) 3 (m)	M1* A1 *M1 A1 **[4]**	Attempt at elastic energy get 3 term quadratic and attempt to solve ignore d = 0.12, unless given as answer	Allow M1 for $\dfrac{30y^2}{(2)\times 0.6} = kd$ $\dfrac{30x^2}{2 \times 0.6} = 48(x + 0.6)$ allow 1 slip or $25x^2 - 48x - 28.8 = 0$ (x =) 2.4 leading to (d =) 3
	(ii)	Use $F = ma$ $48 - \dfrac{30 \times (3 - 0.6 - 1.3)}{0.6} = (\pm)\dfrac{48}{g}a$ (a =) (+/−) 1.43 upwards	M1 A1ft A1 A1 **[4]**	 ft their ‘3’ 1.4291666 depends on a being right	allow missing g, allow 1.3 or 0.6 to be omitted Using energy: $a = v\frac{dv}{dx} = \frac{g}{48}(50x - 72)$ M1A1

		Answer	Marks	Guidance	
3	(i)	Using conservation of momentum along loc 0.1 x 2.8 + 0.4 x 1x0.8 = 0.4 x b Using NEL $b - 0 = -e(1\text{x}0.8 - 2.8)$ $e = 0.75$	M1 A1 M1 A1 A1 **[5]**	3 (or 4) terms, correct dimensions Vel diff after = e x vel diff before	Allow sign errors, (sin/cos) may see $b = 1.5$ Allow $\pm e$
	(ii)	$b(perp) = 0.6$ $\tan\beta = \frac{b(perp)}{\text{their } 1.5}$, angle turned through is $36.9° - \beta$ $= 15.1°$ (0.262 rad)	B1 M1* *M1 A1 **[4]**	 $\beta = 21.8°$; ft 1.5 from (i) Must be $36.9° -$ their β (soi)	May be on diagram 21.8014...(0.381 rad) 36.86989 15.068 scB1 for 165° after B1M1
4	(i)	Use $F = mv\frac{dv}{dx}$ $-4v = \frac{dv}{dx}$ $-4x = \ln v + c$ $0 = \ln 2 + c$ $\ln\frac{v}{2} = -4x$ $v = 2e^{-4x}$	M1 A1 M1 M1 A1 **[5]**	 expression for $\frac{dv}{dx}$ required get (+/–) $Ax = \ln v + c$ valid attempt to find c need a step leading to given answer AG	Allow sign error, missing m or g inc
	(ii)	$e^{4x}\,dx = 2\,dt$ $\frac{1}{4}e^{4x} = 2t + c$ $\frac{1}{4} = 0 + c$ $e^{4x} = 4(1 + \frac{1}{4})$ $x = \frac{1}{4}\ln 5$	M1* A1 *M1 *M1 A1 **[5]**	Write v as $\frac{dx}{dt}$ and separate variables must have c or use limits valid attempt to find c or subst limits find x when $t = 0.5$ - need to remove exp; allow even if no c Accept 0.402(359…)	$dv/4v^2 = -dt$ $\frac{1}{v} = 4t + \frac{1}{2}$ $\frac{dx}{dt} = \frac{2}{8t+1}$ OR t = 0.5 gives v = 0.4 $x = \frac{1}{4}\ln(8t+1) + c$ OR $-4x = \ln 0.2$ $x = \frac{1}{4}\ln 5$
5	(i)	Take moments about A for whole body $W\text{x}2L\cos 60° + 2W\text{x}6L\cos 60° = R\text{x}8L\cos 60°$ $R = 1.75W$ $S = 1.25W$	M1 A1 A1 B1 **[4]**	Correct 3 terms needed; dim correct cos60° may be omitted at least 1 correct step to show given answer	Allow sign errors, $W/2W$, cos/sin, R is reaction at C S is reaction at A For less efficient methods, M1 can only be earned when equation with one unknown, R, is reached.

		Answer	Marks	Guidance	
	(ii)	Take moments about B for equil of BC $T\text{x}L\sin 60° + 2W\text{x}2L\cos 60° =$ $1.75W\text{x}4L\cos 60°$ solve to get $T = \sqrt{3}W$	M1* A1 *M1 A1 **[4]**	Correct 3 resolved terms needed; dim correct; or for BA $T\text{x}L\sin 60° + W\text{x}2L\cos 60° =$ $1.25W\text{x}4L\cos 60°$ accept $T = 1.73W$	allow sign errors, $W/2W$, cos/sin,
	(iii)	Resolve vertically for AB $Y + 1.25W - W = 0$ $Y = 0.25W$, downwards $X = \sqrt{3}W$ to left	 M1 A1CAO B1ft **[3]**	 direction must be clear direction must be clear	 Weight and normal term must be for same rod
6	**(i)**	$\frac{1}{2}mv^2 = mg \times 0.8(1 - \sin 30°)$ $v = 2.8$ m s^{-1} Speed of P and Q equal Use conservation of momentum $5m\text{x}2.8 - m\text{x}2.8 = 5mq + mp$ Use of NEL $p - q = -0.95(-2.8 - 2.8)$ $p = 6.3$ m s^{-1} $q = 0.98$ m s^{-1} Q moves to left	M1 A1 B1ft B1ft M1 A1ft A1 A1 **[8]**	Or with '5m' if for Q soi Ft on velocity Ft on velocity supporting work required forAG direction must be clear	allow g missing for M1. Might see $v^2 = 0.8g$ p is vel of P, q is vel of Q, both to left Allow $\pm e$
	(ii)	By energy for P at top $\frac{1}{2}m6.3^2 = \frac{1}{2}mv^2 + mg \times 1.6$ $v^2 = 8.33$ Use $F = ma$ at top $mg + R = m \times \frac{8.33}{0.8}$ $R = 0.6125m$	M1 A1 A1 M1 A1ft A1CAO **[6]**	must have 3 terms Soi must have 3 terms their v^2 Or $49m/80$	allow g missing, sign error allow g missing, sign error

		Answer	Marks	Guidance	
7	(i)	$mg \times 0.2 = \frac{2.45m \times e}{0.3}$ $e = 0.24$	M1 A1 **[2]**	 No errors; must show all numbers	allow sin/cos, wrong sign, missing g
	(ii)	Use $F = ma$ down slope $mg\sin\alpha - \frac{2.45m(x-0.3)}{0.3} = m\ddot{x}$ $\ddot{x} = -\frac{49}{6}(x - 0.54)$ SHM (about $x = 0.54$) $\omega = 7/\sqrt{6}$ (2.8577) $T = 2.20$ $a = 0.105$ m (0.1049795)	M1 A1 A1 B1 B1CAO B1ft **[6]**	3 terms needed oe Accept 2.45/0.3 for ω^2 Dep M1A1. Must be in correct form, and ω^2 in simplified form Soi AG Need to see $2\pi/\omega$ oe ft their ω $\frac{3\sqrt{6}}{70}$	Allow sign error, sin/cos, missing g or m Could use x in place of $x - 0.3$, leading to $\ddot{x} = -\frac{49}{6}(x - 0.24)$(about $x = 0.24$) Or x + 0.24 in place of x – 0.3 leading to $\ddot{x} = -\frac{49}{6}x$ (about $x = 0$) May see $\omega^2 = 8\frac{1}{6}$ 2.1986568... NB Can find a by energy, leading to ω and T
	(iii)	Use of SHM eqn for distance $x = -0.0956(227...)$ Dist from O is 0.444(377...) (m) Use of SHM equation for velocity $v = -0.124$ $(-0.123949...)$	M1 A1ft A1CAO M1 A1 **[5]**	$x = a\sin\omega t$ Their a $v = a\omega\cos\omega t$ must be clear velocity is towards O	Allow M1 for x = $a\cos\omega t$ Or -0.9553 or -0.09577 Allow M1 for $v = -a\omega\sin\omega t$ if consistent with x eqn for sin/cos, a, ω Use of $v^2 = \omega^2(a^2 - x^2)$ will not gain A1 unless direction is established