OCR Maths FP1

Past Paper Pack

2005-2013



OXFORD CAMBRIDGE AND RSA EXAMINATIONS

7 JUNE 2005

Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MATHEMATICS

Further Pure Mathematics 1

Tuesday

ay

Afternoon

1 hour 30 minutes

4725

Additional materials: Answer booklet Graph paper List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- You are reminded of the need for clear presentation in your answers.

1 Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers *n*,

$$\sum_{r=1}^{n} (6r^2 + 2r + 1) = n(2n^2 + 4n + 3).$$
 [6]

- 2 The matrices **A** and **I** are given by $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ respectively.
 - (i) Find \mathbf{A}^2 and verify that $\mathbf{A}^2 = 4\mathbf{A} \mathbf{I}$. [4]
 - (ii) Hence, or otherwise, show that $\mathbf{A}^{-1} = 4\mathbf{I} \mathbf{A}$. [2]

3 The complex numbers 2 + 3i and 4 - i are denoted by z and w respectively. Express each of the following in the form x + iy, showing clearly how you obtain your answers.

- (i) z + 5w, [2]
- (ii) z^*w , where z^* is the complex conjugate of z, [3] (iii) $\frac{1}{w}$. [2]
- 4 Use an algebraic method to find the square roots of the complex number 21 20i. [6]
- 5 (i) Show that

$$\frac{r+1}{r+2} - \frac{r}{r+1} = \frac{1}{(r+1)(r+2)}.$$
[2]

(ii) Hence find an expression, in terms of *n*, for

$$\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{(n+1)(n+2)}.$$
[4]

(iii) Hence write down the value of
$$\sum_{r=1}^{\infty} \frac{1}{(r+1)(r+2)}.$$
 [1]

6 The loci C_1 and C_2 are given by

|z - 2i| = 2 and |z + 1| = |z + i|

respectively.

- (i) Sketch, on a single Argand diagram, the loci C_1 and C_2 . [5]
- (ii) Hence write down the complex numbers represented by the points of intersection of C_1 and C_2 . [2]

7 The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} a & 1 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$.

- (i) Given that **B** is singular, show that $a = -\frac{2}{3}$. [3]
- (ii) Given instead that **B** is non-singular, find the inverse matrix \mathbf{B}^{-1} . [4]
- (iii) Hence, or otherwise, solve the equations

$$-x + y + 3z = 1,$$

$$2x + y - z = 4,$$

$$y + 2z = -1.$$
[3]

- 8 (a) The quadratic equation $x^2 2x + 4 = 0$ has roots α and β .
 - (i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]
 - (ii) Show that $\alpha^2 + \beta^2 = -4$. [2]
 - (iii) Hence find a quadratic equation which has roots α^2 and β^2 . [3]
 - (b) The cubic equation $x^3 12x^2 + ax 48 = 0$ has roots p, 2p and 3p.
 - (i) Find the value of *p*. [2]
 - (ii) Hence find the value of *a*. [2]

9 (i) Write down the matrix C which represents a stretch, scale factor 2, in the *x*-direction. [2]

- (ii) The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$. Describe fully the geometrical transformation represented by **D**. [2]
- (iii) The matrix **M** represents the combined effect of the transformation represented by **C** followed by the transformation represented by **D**. Show that

$$\mathbf{M} = \begin{pmatrix} 2 & 3\\ 0 & 1 \end{pmatrix}.$$
 [2]

(iv) Prove by induction that
$$\mathbf{M}^n = \begin{pmatrix} 2^n & 3(2^n - 1) \\ 0 & 1 \end{pmatrix}$$
, for all positive integers *n*. [6]



OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MATHEMATICS

Further Pure Mathematics 1

Wednesday

nesday 10

18 JANUARY 2006

Afternoon

1 hour 30 minutes

4725

Additional materials: 8 page answer booklet Graph paper List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

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(i) Express (1 + 8i)(2 - i) in the form x + iy, showing clearly how you obtain your answer. 1 [2]

(ii) Hence express
$$\frac{1+8i}{2+i}$$
 in the form $x + iy$. [3]

2 Prove by induction that, for
$$n \ge 1$$
, $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$. [5]

The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$. 3

- (i) Find the value of the determinant of M.
- (ii) State, giving a brief reason, whether M is singular or non-singular. [1]
- Use the substitution x = u + 2 to find the exact value of the real root of the equation 4

$$x^3 - 6x^2 + 12x - 13 = 0.$$
 [5]

Use the standard results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers *n*, 5

$$\sum_{r=1}^{n} (8r^3 - 6r^2 + 2r) = 2n^3(n+1).$$
 [6]

- The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix}$. 6
 - (i) Find C^{-1} . [2]
 - (ii) Given that $\mathbf{C} = \mathbf{AB}$, where $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, find \mathbf{B}^{-1} . [5]
- (a) The complex number 3 + 2i is denoted by w and the complex conjugate of w is denoted by w^* . 7 Find
 - (i) the modulus of w, [1]
 - (ii) the argument of w^* , giving your answer in radians, correct to 2 decimal places. [3]
 - (b) Find the complex number u given that $u + 2u^* = 3 + 2i$. [4]
 - (c) Sketch, on an Argand diagram, the locus given by |z + 1| = |z|. [2]

[3]

- 8 The matrix **T** is given by $\mathbf{T} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$.
 - (i) Draw a diagram showing the unit square and its image under the transformation represented by T. [3]
 - (ii) The transformation represented by matrix T is equivalent to a transformation A, followed by a transformation B. Give geometrical descriptions of possible transformations A and B, and state the matrices that represent them.

(i) Show that
$$\frac{1}{r} - \frac{1}{r+2} = \frac{2}{r(r+2)}$$
. [2]

(ii) Hence find an expression, in terms of n, for

$$\frac{2}{1\times 3} + \frac{2}{2\times 4} + \ldots + \frac{2}{n(n+2)}.$$
 [5]

(iii) Hence find the value of

9

(a)
$$\sum_{r=1}^{\infty} \frac{2}{r(r+2)}$$
, [1]

(b)
$$\sum_{r=n+1}^{\infty} \frac{2}{r(r+2)}$$
. [2]

10 The roots of the equation

$$x^3 - 9x^2 + 27x - 29 = 0$$

are denoted by α , β and γ , where α is real and β and γ are complex.

- (i) Write down the value of $\alpha + \beta + \gamma$. [1]
- (ii) It is given that $\beta = p + iq$, where q > 0. Find the value of p, in terms of α . [4]
- (iii) Write down the value of $\alpha\beta\gamma$. [1]
- (iv) Find the value of q, in terms of α only. [5]



OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MATHEMATICS

Further Pure Mathematics 1

Thursday

8 JUNE 2006

Morning

1 hour 30 minutes

4725

Additional materials: 8 page answer booklet Graph paper List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

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- 2
- **1** The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$.
 - (i) Find **A** + 3**B**. [2]
 - (ii) Show that $\mathbf{A} \mathbf{B} = k\mathbf{I}$, where \mathbf{I} is the identity matrix and k is a constant whose value should be stated. [2]
- 2 The transformation S is a shear parallel to the x-axis in which the image of the point (1, 1) is the point (0, 1).
 - (i) Draw a diagram showing the image of the unit square under S. [2]
 - (ii) Write down the matrix that represents S.
- 3 One root of the quadratic equation $x^2 + px + q = 0$, where p and q are real, is the complex number 2-3i.
 - (i) Write down the other root. [1]
 - (ii) Find the values of p and q. [4]
- 4 Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n, $\sum_{r=1}^{n} (r^3 + r^2) = \frac{1}{12}n(n+1)(n+2)(3n+1).$ [5]
- 5 The complex numbers 3 2i and 2 + i are denoted by z and w respectively. Find, giving your answers in the form x + iy and showing clearly how you obtain these answers,
 - (i) 2z 3w, [2]
 - (ii) $(iz)^2$, [3]
 - (iii) $\frac{z}{w}$. [3]
- 6 In an Argand diagram the loci C_1 and C_2 are given by

|z| = 2 and $\arg z = \frac{1}{3}\pi$

respectively.

- (i) Sketch, on a single Argand diagram, the loci C_1 and C_2 . [5]
- (ii) Hence find, in the form x + iy, the complex number representing the point of intersection of C_1 and C_2 . [2]

[2]

7 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

- (i) Find \mathbf{A}^2 and \mathbf{A}^3 . [3]
- (ii) Hence suggest a suitable form for the matrix \mathbf{A}^n . [1]
- (iii) Use induction to prove that your answer to part (ii) is correct. [4]

8 The matrix **M** is given by
$$\mathbf{M} = \begin{pmatrix} a & 4 & 2 \\ 1 & a & 0 \\ 1 & 2 & 1 \end{pmatrix}$$
.

- (i) Find, in terms of *a*, the determinant of **M**. [3]
- (ii) Hence find the values of *a* for which **M** is singular.
- (iii) State, giving a brief reason in each case, whether the simultaneous equations

$$ax + 4y + 2z = 3a,$$

 $x + ay = 1,$
 $x + 2y + z = 3,$

have any solutions when

(a)
$$a = 3$$
,
(b) $a = 2$.
[4]

9 (i) Use the method of differences to show that

$$\sum_{r=1}^{n} \{ (r+1)^3 - r^3 \} = (n+1)^3 - 1.$$
[2]

[3]

[2]

[5]

- (ii) Show that $(r+1)^3 r^3 \equiv 3r^2 + 3r + 1$.
- (iii) Use the results in parts (i) and (ii) and the standard result for $\sum_{r=1}^{n} r$ to show that

$$3\sum_{r=1}^{n} r^2 = \frac{1}{2}n(n+1)(2n+1).$$
 [6]

- 10 The cubic equation $x^3 2x^2 + 3x + 4 = 0$ has roots α , β and γ .
 - (i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

The cubic equation $x^3 + px^2 + 10x + q = 0$, where *p* and *q* are constants, has roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$.

- (ii) Find the value of *p*. [3]
- (iii) Find the value of q.

3



ADVANCED SUBSIDIARY GCE UNIT MATHEMATICS

Further Pure Mathematics 1 THURSDAY 18 JANUARY 2007

Afternoon

4725/01

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages) List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

- **1** The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} a & -1 \\ -3 & -2 \end{pmatrix}$.
 - (i) Given that $2\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$, write down the value of *a*. [1]

(ii) Given instead that
$$\mathbf{AB} = \begin{pmatrix} 7 & -4 \\ 9 & -7 \end{pmatrix}$$
, find the value of *a*. [2]

2 Use an algebraic method to find the square roots of the complex number 15 + 8i. [6]

3 Use the standard results for
$$\sum_{r=1}^{n} r$$
 and $\sum_{r=1}^{n} r^{3}$ to find
$$\sum_{r=1}^{n} r(r-1)(r+1),$$

expressing your answer in a fully factorised form.

- 4 (i) Sketch, on an Argand diagram, the locus given by $|z 1 + i| = \sqrt{2}$. [3]
 - (ii) Shade on your diagram the region given by $1 \le |z 1 + i| \le \sqrt{2}$. [3]

5 (i) Verify that
$$z^3 - 8 = (z - 2)(z^2 + 2z + 4)$$
. [1]

(ii) Solve the quadratic equation $z^2 + 2z + 4 = 0$, giving your answers exactly in the form x + iy. Show clearly how you obtain your answers. [3]

(iii) Show on an Argand diagram the roots of the cubic equation $z^3 - 8 = 0.$ [3]

6 The sequence u_1, u_2, u_3, \ldots is defined by $u_n = n^2 + 3n$, for all positive integers *n*.

- (i) Show that $u_{n+1} u_n = 2n + 4$. [3]
- (ii) Hence prove by induction that each term of the sequence is divisible by 2. [5]
- 7 The quadratic equation $x^2 + 5x + 10 = 0$ has roots α and β .
 - (i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]
 - (ii) Show that $\alpha^2 + \beta^2 = 5$. [2]
 - (iii) Hence find a quadratic equation which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [4]

[6]

8 (i) Show that $(r+2)! - (r+1)! = (r+1)^2 \times r!$.

(ii) Hence find an expression, in terms of *n*, for

$$2^{2} \times 1! + 3^{2} \times 2! + 4^{2} \times 3! + \ldots + (n+1)^{2} \times n!.$$
[4]

(iii) State, giving a brief reason, whether the series

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \dots$$

converges.

9 The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 0 & 3 \\ -1 & 0 \end{pmatrix}$.

(i) Draw a diagram showing the unit square and its image under the transformation represented by C. [2]

The transformation represented by C is equivalent to a rotation, R, followed by another transformation, S.

- (ii) Describe fully the rotation R and write down the matrix that represents R. [3]
- (iii) Describe fully the transformation S and write down the matrix that represents S. [4]

10 The matrix **D** is given by
$$\mathbf{D} = \begin{pmatrix} a & 2 & 0 \\ 3 & 1 & 2 \\ 0 & -1 & 1 \end{pmatrix}$$
, where $a \neq 2$.

- (i) Find D^{-1} .
- (ii) Hence, or otherwise, solve the equations

$$ax + 2y = 3,3x + y + 2z = 4,- y + z = 1.$$
 [4]

[1]

[7]

[3]



ADVANCED SUBSIDIARY GCE UNIT MATHEMATICS

Further Pure Mathematics 1

MONDAY 11 JUNE 2007

Afternoon

4725/01

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages) List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
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The complex number a + ib is denoted by z. Given that |z| = 4 and $\arg z = \frac{1}{3}\pi$, find a and b. 1 [4]

2 Prove by induction that, for
$$n \ge 1$$
, $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$. [5]

3 Use the standard results for
$$\sum_{r=1}^{n} r$$
 and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers *n*,

$$\sum_{r=1}^{n} (3r^2 - 3r + 1) = n^3.$$

The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 3 & 5 \end{pmatrix}$. 4

(i) Find
$$A^{-1}$$
. [2]

The matrix \mathbf{B}^{-1} is given by $\mathbf{B}^{-1} = \begin{pmatrix} 1 & 1 \\ 4 & -1 \end{pmatrix}$. [4]

- (ii) Find $(\mathbf{AB})^{-1}$.
- 5 (i) Show that

$$\frac{1}{r} - \frac{1}{r+1} = \frac{1}{r(r+1)}.$$
[1]

[6]

(ii) Hence find an expression, in terms of *n*, for

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{n(n+1)}.$$
[3]

(iii) Hence find the value of
$$\sum_{r=n+1}^{\infty} \frac{1}{r(r+1)}$$
. [3]

The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots α , β and γ . 6

- (i) (a) Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$. [2]
 - **(b)** Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

(ii) (a) Use the substitution $x = \frac{1}{u}$ to find a cubic equation in *u* with integer coefficients. [2]

(**b**) Use your answer to part (**ii**) (**a**) to find the value of
$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$$
. [2]

7 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} a & 4 & 0 \\ 0 & a & 4 \\ 2 & 3 & 1 \end{pmatrix}$.

- (i) Find, in terms of *a*, the determinant of **M**. [3]
- (ii) In the case when a = 2, state whether **M** is singular or non-singular, justifying your answer. [2]
- (iii) In the case when a = 4, determine whether the simultaneous equations

$$ax + 4y = 6,$$

$$ay + 4z = 8,$$

$$2x + 3y + z = 1,$$

have any solutions.

- 8 The loci C_1 and C_2 are given by |z-3| = 3 and $\arg(z-1) = \frac{1}{4}\pi$ respectively.
 - (i) Sketch, on a single Argand diagram, the loci C_1 and C_2 . [6]
 - (ii) Indicate, by shading, the region of the Argand diagram for which

$$|z-3| \leq 3$$
 and $0 \leq \arg(z-1) \leq \frac{1}{4}\pi$. [2]

9 (i) Write down the matrix, **A**, that represents an enlargement, centre (0, 0), with scale factor $\sqrt{2}$. [1]

- (ii) The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}$. Describe fully the geometrical transformation represented by **B**. [3]
- (iii) Given that $\mathbf{C} = \mathbf{AB}$, show that $\mathbf{C} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$. [1]

(iv) Draw a diagram showing the unit square and its image under the transformation represented by C. [2]

- (v) Write down the determinant of C and explain briefly how this value relates to the transformation represented by C.
- 10 (i) Use an algebraic method to find the square roots of the complex number 16 + 30i. [6]
 - (ii) Use your answers to part (i) to solve the equation $z^2 2z (15 + 30i) = 0$, giving your answers in the form x + iy. [5]

[3]



ADVANCED SUBSIDIARY GCE MATHEMATICS

Further Pure Mathematics 1

FRIDAY 11 JANUARY 2008

Morning Time: 1 hour 30 minutes

4725/01

Additional materials: Answer Booklet (8 pages) List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

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© OCR 2008 [T/102/2698]

- 1 The transformation S is a shear with the *y*-axis invariant (i.e. a shear parallel to the *y*-axis). It is given that the image of the point (1, 1) is the point (1, 0).
 - (i) Draw a diagram showing the image of the unit square under the transformation S. [2]

[2]

(ii) Write down the matrix that represents S.

2 Given that
$$\sum_{r=1}^{n} (ar^2 + b) \equiv n(2n^2 + 3n - 2)$$
, find the values of the constants *a* and *b*. [5]

- 3 The cubic equation $2x^3 3x^2 + 24x + 7 = 0$ has roots α , β and γ .
 - (i) Use the substitution $x = \frac{1}{u}$ to find a cubic equation in *u* with integer coefficients. [2]

(ii) Hence, or otherwise, find the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$. [2]

- 4 The complex number 3 4i is denoted by z. Giving your answers in the form x + iy, and showing clearly how you obtain them, find
 - (i) $2z + 5z^*$, [2]
 - (ii) $(z-i)^2$, [3]

(iii)
$$\frac{3}{z}$$
. [3]

- 5 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 2 & 4 & -1 \end{pmatrix}$. Find
 - (i) A 4B, [2]
 - (ii) BC, [4]
 - (iii) CA. [2]

6 The loci C_1 and C_2 are given by

|z| = |z - 4i| and $\arg z = \frac{1}{6}\pi$

respectively.

- (i) Sketch, on a single Argand diagram, the loci C_1 and C_2 . [5]
- (ii) Hence find, in the form x + iy, the complex number represented by the point of intersection of C_1 and C_2 . [3]

- 7 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 3 \\ -2 & 1 \end{pmatrix}$.
 - (i) Given that A is singular, find *a*.
 - (ii) Given instead that A is non-singular, find A^{-1} and hence solve the simultaneous equations

$$ax + 3y = 1,$$

 $-2x + y = -1.$ [5]

[2]

- 8 The sequence u_1, u_2, u_3, \ldots is defined by $u_1 = 1$ and $u_{n+1} = u_n + 2n + 1$.
 - (i) Show that $u_4 = 16$. [2]
 - (ii) Hence suggest an expression for u_n . [1]
 - (iii) Use induction to prove that your answer to part (ii) is correct. [4]
- 9 (i) Show that $\alpha^3 + \beta^3 = (\alpha + \beta)^3 3\alpha\beta(\alpha + \beta)$. [2]
 - (ii) The quadratic equation $x^2 5x + 7 = 0$ has roots α and β . Find a quadratic equation with roots α^3 and β^3 . [6]

10 (i) Show that $\frac{2}{r} - \frac{1}{r+1} - \frac{1}{r+2} = \frac{3r+4}{r(r+1)(r+2)}$. [2]

(ii) Hence find an expression, in terms of *n*, for

$$\sum_{r=1}^{n} \frac{3r+4}{r(r+1)(r+2)}.$$
[6]

(iii) Hence write down the value of
$$\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)(r+2)}.$$
 [1]

(iv) Given that
$$\sum_{r=N+1}^{\infty} \frac{3r+4}{r(r+1)(r+2)} = \frac{7}{10}$$
, find the value of *N*. [4]



4725/01

ADVANCED SUBSIDIARY GCE MATHEMATICS

Further Pure Mathematics 1

MONDAY 2 JUNE 2008

Morning Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages) List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are reminded of the need for clear presentation in your answers.

This document consists of 4 printed pages.

2

1 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix}$ and **I** is the 2 × 2 identity matrix. Find (i) $\mathbf{A} - 3\mathbf{I}$,

- (ii) A^{-1} . [2]
- 2 The complex number 3 + 4i is denoted by *a*.
 - (i) Find |a| and $\arg a$. [2]
 - (ii) Sketch on a single Argand diagram the loci given by
 - (a) |z-a| = |a|, [2]
 - **(b)** $\arg(z-3) = \arg a.$ [3]

3 (i) Show that
$$\frac{1}{r!} - \frac{1}{(r+1)!} = \frac{r}{(r+1)!}$$
 [2]

(ii) Hence find an expression, in terms of *n*, for

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \dots + \frac{n}{(n+1)!}.$$
[4]

[2]

4 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \ge 1$,

$$\mathbf{A}^{n} = \begin{pmatrix} 3^{n} & \frac{1}{2}(3^{n} - 1) \\ 0 & 1 \end{pmatrix}.$$
 [6]

5 Find
$$\sum_{r=1}^{n} r^2(r-1)$$
, expressing your answer in a fully factorised form. [6]

- 6 The cubic equation $x^3 + ax^2 + bx + c = 0$, where a, b and c are real, has roots (3 + i) and 2.
 - (i) Write down the other root of the equation. [1]
 - (ii) Find the values of a, b and c. [6]

7 Describe fully the geometrical transformation represented by each of the following matrices:

(i)
$$\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$
, [1]

$$(\mathbf{i}\mathbf{i}) \begin{pmatrix} 0 & 1\\ 1 & 0 \end{pmatrix},$$
 [2]

$$(\mathbf{iii}) \begin{pmatrix} 1 & 0\\ 0 & 6 \end{pmatrix},$$

$$[2]$$

$$(iv) \begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix}.$$
 [2]

8 The quadratic equation $x^2 + kx + 2k = 0$, where k is a non-zero constant, has roots α and β . Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [7]

9 (i) Use an algebraic method to find the square roots of the complex number 5 + 12i. [5] (ii) Find (3 - 2i)². [2]

(iii) Hence solve the quartic equation $x^4 - 10x^2 + 169 = 0.$ [4]

10 The matrix **A** is given by
$$\mathbf{A} = \begin{pmatrix} a & 8 & 10 \\ 2 & 1 & 2 \\ 4 & 3 & 6 \end{pmatrix}$$
. The matrix **B** is such that $\mathbf{AB} = \begin{pmatrix} a & 6 & 1 \\ 1 & 1 & 0 \\ 1 & 3 & 0 \end{pmatrix}$.

(i) Show that **AB** is non-singular. [2]
(ii) Find
$$(\mathbf{AB})^{-1}$$
. [4]

(ii) Find
$$(AB)^{-1}$$
. [4]

(iii) Find \mathbf{B}^{-1} .

[5]



ADVANCED SUBSIDIARY GCE MATHEMATICS

Further Pure Mathematics 1

4725

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required: None Thursday 15 January 2009 Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- This document consists of 4 pages. Any blank pages are indicated.

Express $\frac{2+3i}{5-i}$ in the form x + iy, showing clearly how you obtain your answer. 1 [4]

2 The matrix **A** is given by
$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ a & 5 \end{pmatrix}$$
. Find
(i) \mathbf{A}^{-1} ,

(ii)
$$2\mathbf{A} - \begin{pmatrix} 1 & 2 \\ 0 & 4 \end{pmatrix}$$
. [2]

- Find $\sum_{r=1}^{n} (4r^3 + 6r^2 + 2r)$, expressing your answer in a fully factorised form. 3 [6]
- 4 Given that A and B are 2×2 non-singular matrices and I is the 2×2 identity matrix, simplify

$$\mathbf{B}(\mathbf{A}\mathbf{B})^{-1}\mathbf{A} - \mathbf{I}.$$
 [4]

5 By using the determinant of an appropriate matrix, or otherwise, find the value of k for which the simultaneous equations

$$2x - y + z = 7,3y + z = 4,x + ky + kz = 5,$$

do not have a unique solution for x, y and z.

- (i) The transformation P is represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Give a geometrical description of 6 [2] transformation P.
 - (ii) The transformation Q is represented by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. Give a geometrical description of transformation Q. [2]
 - (iii) The transformation R is equivalent to transformation P followed by transformation Q. Find the matrix that represents R. [2]
 - (iv) Give a geometrical description of the single transformation that is represented by your answer to part (iii). [3]
- It is given that $u_n = 13^n + 6^{n-1}$, where *n* is a positive integer. 7
 - (i) Show that $u_n + u_{n+1} = 14 \times 13^n + 7 \times 6^{n-1}$. [3]
 - (ii) Prove by induction that u_n is a multiple of 7.

4725 Jan09

[5]

[2]

[4]

8 (i) Show that $(\alpha - \beta)^2 \equiv (\alpha + \beta)^2 - 4\alpha\beta$. [2]

The quadratic equation $x^2 - 6kx + k^2 = 0$, where k is a positive constant, has roots α and β , with $\alpha > \beta$.

- (ii) Show that $\alpha \beta = 4\sqrt{2k}$. [4]
- (iii) Hence find a quadratic equation with roots $\alpha + 1$ and $\beta 1$. [4]
- 9 (i) Show that $\frac{1}{2r-3} \frac{1}{2r+1} = \frac{4}{4r^2 4r 3}$. [2]
 - (ii) Hence find an expression, in terms of *n*, for

$$\sum_{r=2}^{n} \frac{4}{4r^2 - 4r - 3}.$$
 [6]

(iii) Show that
$$\sum_{r=2}^{\infty} \frac{4}{4r^2 - 4r - 3} = \frac{4}{3}$$
. [1]

- 10 (i) Use an algebraic method to find the square roots of the complex number $2 + i\sqrt{5}$. Give your answers in the form x + iy, where x and y are exact real numbers. [6]
 - (ii) Hence find, in the form x + iy where x and y are exact real numbers, the roots of the equation

$$z^4 - 4z^2 + 9 = 0.$$
 [4]

- (iii) Show, on an Argand diagram, the roots of the equation in part (ii). [1]
- (iv) Given that α is the root of the equation in part (ii) such that $0 < \arg \alpha < \frac{1}{2}\pi$, sketch on the same Argand diagram the locus given by $|z \alpha| = |z|$. [3]



ADVANCED SUBSIDIARY GCE MATHEMATICS

Further Pure Mathematics 1

4725

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required: None

Friday 5 June 2009 Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- This document consists of 4 pages. Any blank pages are indicated.

1 Evaluate
$$\sum_{r=101}^{250} r^3$$
.

2 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$ and **I** is the 2 × 2 identity matrix. Find the values of the constants *a* and *b* for which $a\mathbf{A} + b\mathbf{B} = \mathbf{I}$. [4]

2

- 3 The complex numbers z and w are given by z = 5 2i and w = 3 + 7i. Giving your answers in the form x + iy and showing clearly how you obtain them, find
 - (i) 4z 3w, [2]

(ii)
$$z^*w$$
.

- 4 The roots of the quadratic equation $x^2 + x 8 = 0$ are p and q. Find the value of $p + q + \frac{1}{p} + \frac{1}{q}$. [4]
- 5 The cubic equation $x^3 + 5x^2 + 7 = 0$ has roots α , β and γ .
 - (i) Use the substitution $x = \sqrt{u}$ to find a cubic equation in *u* with integer coefficients. [3]
 - (ii) Hence find the value of $\alpha^2 \beta^2 + \beta^2 \gamma^2 + \gamma^2 \alpha^2$. [2]
- 6 The complex number 3 3i is denoted by *a*.

(i) Find
$$|a|$$
 and $\arg a$. [2]

- (ii) Sketch on a single Argand diagram the loci given by
 - (a) $|z-a| = 3\sqrt{2}$, [3]
 - **(b)** $\arg(z-a) = \frac{1}{4}\pi$. **[3]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z-a| \ge 3\sqrt{2}$$
 and $0 \le \arg(z-a) \le \frac{1}{4}\pi$. [3]

7 (i) Use the method of differences to show that

$$\sum_{r=1}^{n} \{ (r+1)^4 - r^4 \} = (n+1)^4 - 1.$$
 [2]

- (ii) Show that $(r+1)^4 r^4 \equiv 4r^3 + 6r^2 + 4r + 1.$ [2]
- (iii) Hence show that

$$4\sum_{r=1}^{n}r^{3} = n^{2}(n+1)^{2}.$$
 [6]

[2]

- 3
- 8 The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$.
 - (i) Draw a diagram showing the image of the unit square under the transformation represented by C. [3]

The transformation represented by C is equivalent to a transformation S followed by another transformation T.

- (ii) Given that S is a shear with the y-axis invariant in which the image of the point (1, 1) is (1, 2), write down the matrix that represents S. [2]
- (iii) Find the matrix that represents transformation T and describe fully the transformation T. [6]
- 9 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 2 \end{pmatrix}$.
 - (i) Find, in terms of *a*, the determinant of **A**.
 - (ii) Hence find the values of *a* for which A is singular.
 - (iii) State, giving a brief reason in each case, whether the simultaneous equations

$$ax + y + z = 2a,$$

 $x + ay + z = -1,$
 $x + y + 2z = -1,$

have any solutions when

- (a) a = 0,
- **(b)** a = 1.

10 The sequence u_1, u_2, u_3, \dots is defined by $u_1 = 3$ and $u_{n+1} = 3u_n - 2$.

- (i) Find u_2 and u_3 and verify that $\frac{1}{2}(u_4 1) = 27$. [3]
- (ii) Hence suggest an expression for u_n . [2]
- (iii) Use induction to prove that your answer to part (ii) is correct. [5]

[3]

[3]

[4]



ADVANCED SUBSIDIARY GCE MATHEMATICS

Further Pure Mathematics 1

4725

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required: None

Wednesday 20 January 2010 Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is **72**.
- This document consists of 4 pages. Any blank pages are indicated.

The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 2 \\ 3 & 4 \end{pmatrix}$ and **I** is the 2 × 2 identity matrix.

- (i) Find A 4I. [2]
- (ii) Given that A is singular, find the value of *a*. [3]
- **2** The cubic equation $2x^3 + 3x 3 = 0$ has roots α , β and γ .
 - (i) Use the substitution x = u 1 to find a cubic equation in u with integer coefficients. [3]
 - (ii) Hence find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. [2]
- 3 The complex number z satisfies the equation $z + 2iz^* = 12 + 9i$. Find z, giving your answer in the form x + iy. [5]
- 4 Find $\sum_{r=1}^{n} r(r+1)(r-2)$, expressing your answer in a fully factorised form. [6]
- 5 (i) The transformation T is represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Give a geometrical description of T. [2]
 - (ii) The transformation T is equivalent to a reflection in the line y = -x followed by another transformation S. Give a geometrical description of S and find the matrix that represents S. [4]
- 6 One root of the cubic equation $x^3 + px^2 + 6x + q = 0$, where p and q are real, is the complex number 5 i.
 - (i) Find the real root of the cubic equation. [3]
 - (ii) Find the values of p and q. [4]

7 (i) Show that
$$\frac{1}{r^2} - \frac{1}{(r+1)^2} \equiv \frac{2r+1}{r^2(r+1)^2}$$
. [1]

(ii) Hence find an expression, in terms of *n*, for $\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2}$. [4]

(iii) Find
$$\sum_{r=2}^{\infty} \frac{2r+1}{r^2(r+1)^2}$$
. [2]

- 8 The complex number *a* is such that $a^2 = 5 12i$.
 - (i) Use an algebraic method to find the two possible values of *a*. [5]
 - (ii) Sketch on a single Argand diagram the two possible loci given by |z a| = |a|. [4]

1

2

9 The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 3 & 1 \\ 1 & 1 & a \end{pmatrix}$, where $a \neq 1$.

- (i) Find \mathbf{A}^{-1} .
- (ii) Hence, or otherwise, solve the equations

$$2x - y + z = 1,
3y + z = 2,
x + y + az = 2.$$
[4]

[7]

10 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(i) Find
$$\mathbf{M}^2$$
 and \mathbf{M}^3 . [3]

- (ii) Hence suggest a suitable form for the matrix Mⁿ.
 (iii) Use induction to prove that your answer to part (ii) is correct.
- (iv) Describe fully the single geometrical transformation represented by \mathbf{M}^{10} . [3]



ADVANCED SUBSIDIARY GCE

MATHEMATICS

Further Pure Mathematics 1

4725

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

• Scientific or graphical calculator

Friday 11 June 2010 Morning

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer all the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Prove by induction that, for
$$n \ge 1$$
, $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$. [5]

2

2 The matrices **A**, **B** and **C** are given by $\mathbf{A} = (1 - 4)$, $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 3 & 0 \\ -2 & 2 \end{pmatrix}$. Find

- (ii) BA 4C. [4]
- 3 Find $\sum_{r=1}^{n} (2r-1)^2$, expressing your answer in a fully factorised form. [6]
- 4 The complex numbers *a* and *b* are given by a = 7 + 6i and b = 1 3i. Showing clearly how you obtain your answers, find
 - (i) |a-2b| and $\arg(a-2b)$, [4]

(ii)
$$\frac{b}{a}$$
, giving your answer in the form $x + iy$. [3]

- 5 (a) Write down the matrix that represents a reflection in the line y = x. [2]
 - (b) Describe fully the geometrical transformation represented by each of the following matrices:

(i)
$$\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$$
, [2]

(ii)
$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$$
. [2]

- 6 (i) Sketch on a single Argand diagram the loci given by
 - (a) |z-3+4i|=5, [2]
 - **(b)** |z| = |z 6|. [2]
 - (ii) Indicate, by shading, the region of the Argand diagram for which

$$|z-3+4i| \le 5$$
 and $|z| \ge |z-6|$. [2]

7 The quadratic equation $x^2 + 2kx + k = 0$, where k is a non-zero constant, has roots α and β . Find a quadratic equation with roots $\frac{\alpha + \beta}{\alpha}$ and $\frac{\alpha + \beta}{\beta}$. [7]

4725 Jun10

8 (i) Show that
$$\frac{1}{\sqrt{r+2} + \sqrt{r}} \equiv \frac{\sqrt{r+2} - \sqrt{r}}{2}$$
. [2]

(ii) Hence find an expression, in terms of *n*, for

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r+2} + \sqrt{r}}.$$
 [6]

(iii) State, giving a brief reason, whether the series $\sum_{r=1}^{\infty} \frac{1}{\sqrt{r+2} + \sqrt{r}}$ converges. [1]

- The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & a & -1 \\ 0 & a & 2 \\ 1 & 2 & 1 \end{pmatrix}$. 9
 - (i) Find, in terms of *a*, the determinant of A.
 - (ii) Three simultaneous equations are shown below.
 - x + 2y + z = 1For each of the following values of a, determine whether the equations are consistent or

ay + 2z = 2a

ax + ay - z = -1

inconsistent. If the equations are consistent, determine whether or not there is a unique solution.

- (a) a = 0
- **(b)** a = 1
- (c) a = 2

The complex number z, where $0 < \arg z < \frac{1}{2}\pi$, is such that $z^2 = 3 + 4i$. 10

- (i) Use an algebraic method to find z.
 - (ii) Show that $z^3 = 2 + 11i$. [1]

The complex number w is the root of the equation

$$w^6 - 4w^3 + 125 = 0$$

for which $-\frac{1}{2}\pi < \arg w < 0$.

(iii) Find w.

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[6]

[5]

[5]

[3]



ADVANCED SUBSIDIARY GCE

MATHEMATICS

Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:

- Printed answer book 4725
- List of Formulae (MF1)

Other materials required:

• Scientific or graphical calculator

Wednesday 19 January 2011 Afternoon

Duration: 1 hour 30 minutes

4725

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the printed answer book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [] at the end of each question or part question on the question paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

• Do not send this question paper for marking; it should be retained in the centre or destroyed.

- 1 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 2 & 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & -1 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$. Find
 - (i) 2A + B, [2]
 - (ii) AC, [2]
 - (iii) CB. [3]
- 2 The complex numbers z and w are given by z = 4 + 3i and w = 6 i. Giving your answers in the form x + iy and showing clearly how you obtain them, find
 - (i) 3z 4w, [2]

(ii)
$$\frac{z^*}{w}$$
. [4]

3 The sequence u_1, u_2, u_3, \ldots is defined by $u_1 = 2$, and $u_{n+1} = 2u_n - 1$ for $n \ge 1$. Prove by induction that $u_n = 2^{n-1} + 1$. [4]

4 Given that
$$\sum_{r=1}^{n} (ar^3 + br) \equiv n(n-1)(n+1)(n+2)$$
, find the values of the constants *a* and *b*. [6]

5 Given that A and B are non-singular square matrices, simplify

$$AB(A^{-1}B)^{-1}$$
. [3]

- 6 (i) Sketch on a single Argand diagram the loci given by
 - (a) |z| = |z 8|, [2]
 - **(b)** $\arg(z+2i) = \frac{1}{4}\pi$. **[3]**
 - (ii) Indicate by shading the region of the Argand diagram for which

$$|z| \leq |z-8|$$
 and $0 \leq \arg(z+2i) \leq \frac{1}{4}\pi$. [3]

- 7 (i) Write down the matrix, A, that represents a shear with *x*-axis invariant in which the image of the point (1, 1) is (4, 1). [2]
 - (ii) The matrix **B** is given by $\mathbf{B} = \begin{pmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{pmatrix}$. Describe fully the geometrical transformation represented by **B**. [2]
 - (iii) The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 2 & 6 \\ 0 & 2 \end{pmatrix}$.
 - (a) Draw a diagram showing the unit square and its image under the transformation represented by C.
 - (b) Write down the determinant of C and explain briefly how this value relates to the transformation represented by C. [2]

- 8 The quadratic equation $2x^2 x + 3 = 0$ has roots α and β , and the quadratic equation $x^2 px + q = 0$ has roots $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$.
 - (i) Show that $p = \frac{5}{6}$. [4]
 - (ii) Find the value of q.

9 The matrix **M** is given by
$$\mathbf{M} = \begin{pmatrix} a & -a & 1 \\ 3 & a & 1 \\ 4 & 2 & 1 \end{pmatrix}$$
.

- (i) Find, in terms of *a*, the determinant of **M**.
- (ii) Hence find the values of *a* for which \mathbf{M}^{-1} does not exist.
- (iii) Determine whether the simultaneous equations

$$6x - 6y + z = 3k,$$

$$3x + 6y + z = 0,$$

$$4x + 2y + z = k,$$

where k is a non-zero constant, have a unique solution, no solution or an infinite number of solutions, justifying your answer. [3]

- 10 (i) Show that $\frac{1}{r} \frac{2}{r+1} + \frac{1}{r+2} \equiv \frac{2}{r(r+1)(r+2)}$. [2]
 - (ii) Hence find an expression, in terms of *n*, for

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)}.$$
 [6]

(iii) Show that
$$\sum_{r=n+1}^{\infty} \frac{2}{r(r+1)(r+2)} = \frac{1}{(n+1)(n+2)}$$
. [3]

[5]

[3]

[3]



ADVANCED SUBSIDIARY GCE

MATHEMATICS

Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:

- Printed answer book 4725
- List of Formulae (MF1)

Other materials required:

• Scientific or graphical calculator

Thursday 16 June 2011 Afternoon

4725

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the printed answer book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [] at the end of each question or part question on the question paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

• Do not send this question paper for marking; it should be retained in the centre or destroyed.

- 1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & a \\ 0 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & a \\ 4 & 1 \end{pmatrix}$. I denotes the 2 × 2 identity matrix. Find
 - (i) A + 3B 4I, [3]

- 2 Prove by induction that, for $n \ge 1$, $\sum_{r=1}^{n} \frac{1}{r(r+1)} = \frac{n}{n+1}$. [5]
- **3** By using the determinant of an appropriate matrix, find the values of k for which the simultaneous equations

$$kx + 8y = 1,$$

$$2x + ky = 3,$$

do not have a unique solution.

- 4 Find $\sum_{r=1}^{2n} (3r^2 \frac{1}{2})$, expressing your answer in a fully factorised form. [6]
- 5 The complex number $1 + i\sqrt{3}$ is denoted by *a*.
 - (i) Find |a| and $\arg a$. [2]
 - (ii) Sketch on a single Argand diagram the loci given by |z a| = |a| and $\arg(z a) = \frac{1}{2}\pi$. [6]

6 The matrix **C** is given by
$$\mathbf{C} = \begin{pmatrix} a & 1 & 0 \\ 1 & 2 & 1 \\ -1 & 3 & 4 \end{pmatrix}$$
, where $a \neq 1$. Find \mathbf{C}^{-1} . [7]

7 (i) Show that
$$\frac{1}{r-1} - \frac{1}{r+1} \equiv \frac{2}{r^2 - 1}$$
. [1]

(ii) Hence find an expression, in terms of *n*, for $\sum_{r=2}^{n} \frac{2}{r^2 - 1}$. [5]

(iii) Find the value of
$$\sum_{r=1000}^{\infty} \frac{2}{r^2 - 1}$$
. [3]

[3]

- 8 The matrix **X** is given by $\mathbf{X} = \begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}$.
 - (i) The diagram in the printed answer book shows the unit square OABC. The image of the unit square under the transformation represented by **X** is OA'B'C'. Draw and label OA'B'C'. [3]
 - (ii) The transformation represented by X is equivalent to a transformation A, followed by a transformation B. Give geometrical descriptions of possible transformations A and B and state the matrices that represent them.
- 9 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is 16 30i.
 - (i) Write down the other root of the quadratic equation. [1]
 - (ii) Find the values of *a* and *b*. [4]
 - (iii) Use an algebraic method to solve the quartic equation $y^4 + ay^2 + b = 0.$ [7]
- 10 The cubic equation $x^3 + 3x^2 + 2 = 0$ has roots α , β and γ .
 - (i) Use the substitution $x = \frac{1}{\sqrt{u}}$ to show that $4u^3 + 12u^2 + 9u 1 = 0.$ [5]
 - (ii) Hence find the values of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ and $\frac{1}{\alpha^2 \beta^2} + \frac{1}{\beta^2 \gamma^2} + \frac{1}{\gamma^2 \alpha^2}$. [5]



Friday 20 January 2012 – Afternoon

AS GCE MATHEMATICS

4725 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4725
- List of Formulae (MF1)
 Other materials required:

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

Scientific or graphical calculator

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

• Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 The complex number a + 5i, where a is positive, is denoted by z. Given that |z| = 13, find the value of a and hence find arg z. [4]
- 2 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & 6 \\ 3 & -5 \end{pmatrix}$, and **I** is the 2 × 2 identity matrix. Given that $p\mathbf{A} + q\mathbf{B} = \mathbf{I}$, find the values of the constants *p* and *q*. [5]
- 3 Use an algebraic method to find the square roots of $3 + (6\sqrt{2})i$. Give your answers in the form x + iy, where x and y are exact real numbers. [6]

4 Find
$$\sum_{r=1}^{n} r(r^2 - 3)$$
, expressing your answer in a fully factorised form. [6]

- 5 (a) Find the matrix that represents a reflection in the line y = -x.
 - **(b)** The matrix **C** is given by $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$.
 - (i) Describe fully the geometrical transformation represented by C. [2]
 - (ii) State the value of the determinant of C and describe briefly how this value relates to the transformation represented by C. [2]
- 6 Sketch, on a single Argand diagram, the loci given by $|z \sqrt{3} i| = 2$ and $\arg z = \frac{1}{6}\pi$. [6]
- 7 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$.
 - (i) Show that $\mathbf{M}^4 = \begin{pmatrix} 81 & 0 \\ 80 & 1 \end{pmatrix}$. [3]
 - (ii) Hence suggest a suitable form for the matrix \mathbf{M}^n , where *n* is a positive integer. [2]
 - (iii) Use induction to prove that your answer to part (ii) is correct.

8 (i) Show that
$$\frac{r}{r+1} - \frac{r-1}{r} \equiv \frac{1}{r(r+1)}$$
. [2]

(ii) Hence find an expression, in terms of *n*, for

$$\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \dots + \frac{1}{n(n+1)}.$$
[4]

[2]

[4]

(iii) Hence find
$$\sum_{r=n+1}^{\infty} \frac{1}{r(r+1)}$$
. [2]

9 The matrix **X** is given by $\mathbf{X} = \begin{pmatrix} a & 2 & 9 \\ 2 & a & 3 \\ 1 & 0 & -1 \end{pmatrix}$.

- (i) Find the determinant of **X** in terms of *a*. [3]
- (ii) Hence find the values of a for which **X** is singular. [3]
- (iii) Given that **X** is non-singular, find \mathbf{X}^{-1} in terms of *a*. [4]
- **10** The cubic equation $3x^3 9x^2 + 6x + 2 = 0$ has roots α , β and γ .
 - (i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

The cubic equation $x^3 + ax^2 + bx + c = 0$ has roots α^2 , β^2 and γ^2 .

(ii) Show that
$$c = -\frac{4}{9}$$
 and find the values of *a* and *b*. [9]



Friday 1 June 2012 - Morning

AS GCE MATHEMATICS

4725 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4725
- List of Formulae (MF1)

Duration: 1 hour 30 minutes

Other materials required: • Scientific or graphical calculator

• Scientific of graphical calculator

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 The complex numbers z and w are given by z = 6 i and w = 5 + 4i. Giving your answers in the form x + iy and showing clearly how you obtain them, find
 - (i) z + 3w, [2]
 - (ii) $\frac{z}{w}$.
- 2 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$. Find
 - (i) AB, [2]

[3]

(ii)
$$B^{-1}A^{-1}$$
. [3]

3 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is the complex number 4 – 3i. Find the values of a and b. [4]

4 Find
$$\sum_{r=1}^{n} (3r^2 - 3r + 2)$$
, expressing your answer in a fully factorised form. [7]

5 Prove by induction that, for
$$n \ge 1$$
, $\sum_{r=1}^{n} 4 \times 3^r = 6(3^n - 1)$. [5]

- 6 The quadratic equation $2x^2 + x + 5 = 0$ has roots α and β .
 - (i) Use the substitution $x = \frac{1}{u+1}$ to obtain a quadratic equation in *u* with integer coefficients. [3]
 - (ii) Hence, or otherwise, find the value of $\left(\frac{1}{\alpha} l\right) \left(\frac{1}{\beta} l\right)$. [3]
- 7 The loci C_1 and C_2 are given by |z 3 4i| = 4 and |z| = |z 8i| respectively.
 - (i) Sketch, on a single Argand diagram, the loci C_1 and C_2 . [6]
 - (ii) Hence find the complex numbers represented by the points of intersection of C_1 and C_2 . [2]
 - (iii) Indicate, by shading, the region of the Argand diagram for which

$$|z-3-4i| \le 4 \text{ and } |z| \ge |z-8i|.$$
 [2]

8 (i) Show that
$$\frac{1}{r} - \frac{1}{r+2} \equiv \frac{2}{r(r+2)}$$
. [1]

(ii) Hence find an expression, in terms of *n*, for
$$\sum_{r=1}^{n} \frac{2}{r(r+2)}$$
. [6]

(iii) Given that
$$\sum_{r=N+1}^{\infty} \frac{2}{r(r+2)} = \frac{11}{30}$$
, find the value of N. [4]

9 (i) The matrix **X** is given by $\mathbf{X} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Describe fully the geometrical transformation represented by **X**. [2]

(ii) The matrix **Z** is given by
$$\mathbf{Z} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2}(2+\sqrt{3}) \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2}(1-2\sqrt{3}) \end{pmatrix}$$
. The transformation represented by **Z** is

equivalent to the transformation represented by **X**, followed by another transformation represented by the matrix **Y**. Find **Y**. [5]

- (iii) Describe fully the geometrical transformation represented by **Y**.
- **10** The matrix **D** is given by $\mathbf{D} = \begin{pmatrix} a & 2 & -1 \\ 2 & a & 1 \\ 1 & 1 & a \end{pmatrix}$.
 - (i) Find the determinant of **D** in terms of *a*.
 - (ii) Three simultaneous equations are shown below.

$$ax + 2y - z = 0$$

$$2x + ay + z = a$$

$$x + y + az = a$$

For each of the following values of a, determine whether or not there is a unique solution. If the solution is not unique, determine whether the equations are consistent or inconsistent.

- (a) a = 3
- **(b)** a = 2
- (c) a = 0

[7]

[3]

[2]



Wednesday 23 January 2013 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4725/01
- List of Formulae (MF1) Other materials required:

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

Scientific or graphical calculator

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- The matrix **A** is given by $\mathbf{A} = \begin{pmatrix} a & 1 \\ 1 & 4 \end{pmatrix}$, where $a \neq \frac{1}{4}$, and **I** denotes the 2 × 2 identity matrix. Find 1 (i) 2A - 3I, [3] (ii) A^{-1} .
 - [2]

2 Find
$$\sum_{r=1}^{n} (r-1)(r+1)$$
, giving your answer in a fully factorised form. [6]

- The complex number 2 i is denoted by z. 3
 - (i) Find |z| and arg z. [2]
 - (ii) Given that $az + bz^* = 4 8i$, find the values of the real constants a and b. [5]
- The quadratic equation $x^2 + x + k = 0$ has roots α and β . 4
 - (i) Use the substitution x = 2u + 1 to obtain a quadratic equation in u. [2]
 - (ii) Hence, or otherwise, find the value of $\left(\frac{\alpha-1}{2}\right)\left(\frac{\beta-1}{2}\right)$ in terms of k. [2]
- 5 By using the determinant of an appropriate matrix, find the values of λ for which the simultaneous equations

$$3x + 2y + 4z = 5,$$

$$\lambda y + z = 1,$$

$$x + \lambda y + \lambda z = 4,$$

do not have a unique solution for *x*, *y* and *z*.

[6]

2



3

The diagram shows the unit square *OABC*, and its image *OAB'C'* after a transformation. The points have the following coordinates: A(1, 0), B(1, 1), C(0, 1), B'(3, 2) and C'(2, 2).

(i) Write down the matrix, **X**, for this transformation.

0

A

[2]

- (ii) The transformation represented by X is equivalent to a transformation P followed by a transformation Q. Give geometrical descriptions of a pair of possible transformations P and Q and state the matrices that represent them.
- (iii) Find the matrix that represents transformation Q followed by transformation P. [2]
- 7 (i) Sketch on a single Argand diagram the loci given by
 - (a) |z| = 2, [2]

(b)
$$\arg(z-3-i) = \pi$$
. [3]

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z| \leq 2$$
 and $0 \leq \arg(z-3-i) \leq \pi$. [2]

х

8

(i) Show that
$$\frac{1}{r} - \frac{3}{r+1} + \frac{2}{r+2} \equiv \frac{2-r}{r(r+1)(r+2)}$$
. [2]

(ii) Hence show that
$$\sum_{r=1}^{n} \frac{2-r}{r(r+1)(r+2)} = \frac{n}{(n+1)(n+2)}.$$
 [5]

(iii) Find the value of
$$\sum_{r=2}^{\infty} \frac{2-r}{r(r+1)(r+2)}$$
. [2]

- 9 (i) Show that $(\alpha\beta + \beta\gamma + \gamma\alpha)^2 \equiv \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma).$ [3]
 - (ii) It is given that α , β and γ are the roots of the cubic equation $x^3 + px^2 4x + 3 = 0$, where *p* is a constant. Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ in terms of *p*. [5]
- 10 The sequence u_1, u_2, u_3, \dots is defined by $u_1 = 2$ and $u_{n+1} = \frac{u_n}{1 + u_n}$ for $n \ge 1$.
 - (i) Find u_2 and u_3 , and show that $u_4 = \frac{2}{7}$. [3]
 - (ii) Hence suggest an expression for u_n . [2]

[5]

(iii) Use induction to prove that your answer to part (ii) is correct.



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Monday 10 June 2013 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4725/01
- List of Formulae (MF1) Other materials required:

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

Scientific or graphical calculator

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

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- You are reminded of the need for clear presentation in your answers.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

• Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

- 1 The complex number 3 + ai, where *a* is real, is denoted by *z*. Given that $\arg z = \frac{1}{6}\pi$, find the value of *a* and hence find |z| and $z^* 3$. [6]
- 2 The matrices A, B and C are given by $A = (5 \ 1), B = (2 \ -5) and C = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$. (i) Find 3A - 4B. [2]

(ii) Find CB. Determine whether CB is singular or non-singular, giving a reason for your answer. [5]

- 3 Use an algebraic method to find the square roots of $11 + (12\sqrt{5})i$. Give your answers in the form x + iy, where x and y are exact real numbers. [6]
- 4 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}$. Prove by induction that, for $n \ge 1$,

$$\mathbf{M}^{n} = \begin{pmatrix} 2^{n} & 2^{n+1} - 2\\ 0 & 1 \end{pmatrix}.$$
 [6]

5 Find $\sum_{r=1}^{n} (4r^3 - 3r^2 + r)$, giving your answer in a fully factorised form.



The Argand diagram above shows a half-line *l* and a circle *C*. The circle has centre 3i and passes through the origin.

(i) Write down, in complex number form, the equations of *l* and *C*.

[4]

[6]

(ii) Write down inequalities that define the region shaded in the diagram. [The shaded region includes the boundaries.][3]

6

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(i) Find the matrix that represents a rotation through 90° clockwise about the origin.	[2]
--	-----

- (ii) Find the matrix that represents a reflection in the x-axis.
- (iii) Hence find the matrix that represents a rotation through 90° clockwise about the origin, followed by a reflection in the *x*-axis. [2]
- (iv) Describe a single transformation that is represented by your answer to part (iii). [2]
- 8 The cubic equation $kx^3 + 6x^2 + x 3 = 0$, where k is a non-zero constant, has roots α , β and γ .

Find the value of
$$(\alpha + 1)(\beta + 1) + (\beta + 1)(\gamma + 1) + (\gamma + 1)(\alpha + 1)$$
 in terms of k. [6]

9 (i) Show that
$$\frac{1}{3r-1} - \frac{1}{3r+2} \equiv \frac{3}{(3r-1)(3r+2)}$$
. [2]

(ii) Hence show that
$$\sum_{r=1}^{2n} \frac{1}{(3r-1)(3r+2)} = \frac{n}{2(3n+1)}.$$
 [6]

10 The matrix **A** is given by
$$\mathbf{A} = \begin{pmatrix} a & 2 & 1 \\ 1 & 3 & 2 \\ 4 & 1 & 1 \end{pmatrix}$$
.

- (i) Find the value of *a* for which A is singular.
- (ii) Given that A is non-singular, find A^{-1} and hence solve the equations

axx4x

$$\begin{array}{l} + 2y + z = 1, \\ + 3y + 2z = 2, \\ + y + z = 3. \end{array}$$

[7]

[5]

[2]

7