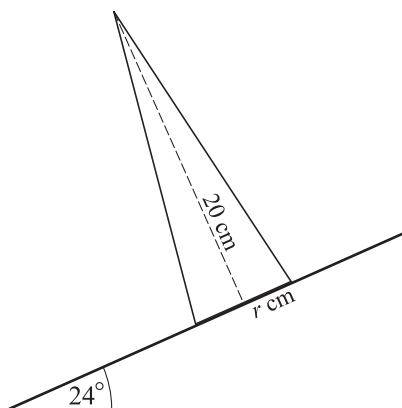OCR Maths M2

Past Paper Pack

2005-2014

1

A uniform solid cone has vertical height 20 cm and base radius r cm. It is placed with its axis vertical on a rough horizontal plane. The plane is slowly tilted until the cone topples when the angle of inclination is 24° (see diagram).

  (i) Find r, correct to 1 decimal place.   [4]

A uniform solid cone of vertical height 20 cm and base radius 2.5 cm is placed on the plane which is inclined at an angle of 24°.

  (ii) State, with justification, whether this cone will topple.   [1]

2   A particle is projected horizontally with a speed of $6\,\text{m s}^{-1}$ from a point 10 m above horizontal ground. The particle moves freely under gravity. Calculate the speed and direction of motion of the particle at the instant it hits the ground.   [6]

3
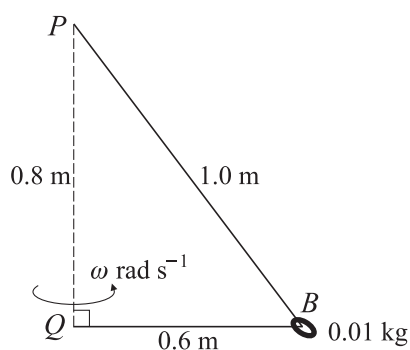
One end of a light inextensible string of length 1.6 m is attached to a point P. The other end is attached to the point Q, vertically below P, where $PQ = 0.8\,\text{m}$. A small smooth bead B, of mass 0.01 kg, is threaded on the string and moves in a horizontal circle, with centre Q and radius 0.6 m. QB rotates with constant angular speed $\omega\,\text{rad s}^{-1}$ (see diagram).

  (i) Show that the tension in the string is 0.1225 N.   [3]

  (ii) Find ω.   [3]

  (iii) Calculate the kinetic energy of the bead.   [2]

June 2005

4
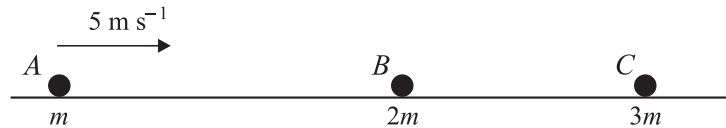
Three smooth spheres A, B and C, of equal radius and of masses m kg, $2m$ kg and $3m$ kg respectively, lie in a straight line and are free to move on a smooth horizontal table. Sphere A is moving with speed $5\,\text{m s}^{-1}$ when it collides directly with sphere B which is stationary. As a result of the collision B starts to move with speed $2\,\text{m s}^{-1}$.
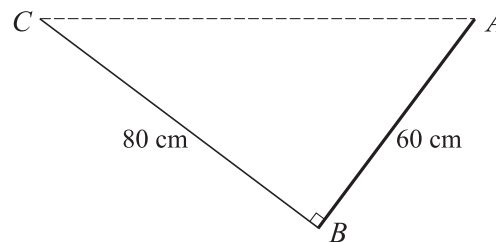
(i) Find the coefficient of restitution between A and B. [4]

(ii) Find, in terms of m, the magnitude of the impulse that A exerts on B, and state the direction of this impulse. [2]

Sphere B subsequently collides with sphere C which is stationary. As a result of this impact B and C coalesce.

(iii) Show that there will be another collision. [3]

5

A uniform rod AB of length 60 cm and weight 15 N is freely suspended from its end A. The end B of the rod is attached to a light inextensible string of length 80 cm whose other end is fixed to a point C which is at the same horizontal level as A. The rod is in equilibrium with the string at right angles to the rod (see diagram).

(i) Show that the tension in the string is 4.5 N. [4]

(ii) Find the magnitude and direction of the force acting on the rod at A. [6]

6 A car of mass 700 kg is travelling up a hill which is inclined at a constant angle of $5°$ to the horizontal. At a certain point P on the hill the car's speed is $20\,\text{m s}^{-1}$. The point Q is 400 m further up the hill from P, and at Q the car's speed is $15\,\text{m s}^{-1}$.

(i) Calculate the work done by the car's engine as the car moves from P to Q, assuming that any resistances to the car's motion may be neglected. [4]

Assume instead that the resistance to the car's motion between P and Q is a constant force of magnitude 200 N.

(ii) Given that the acceleration of the car at Q is zero, show that the power of the engine as the car passes through Q is 12.0 kW, correct to 3 significant figures. [3]

(iii) Given that the power of the car's engine at P is the same as at Q, calculate the car's retardation at P. [3]

[Turn over

7
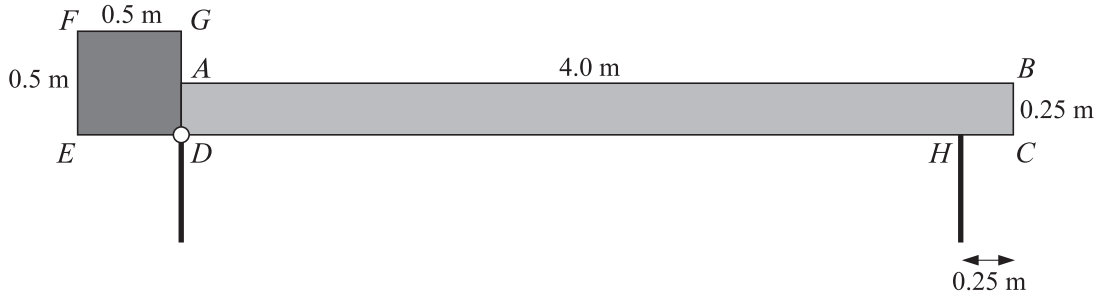
A barrier is modelled as a uniform rectangular plank of wood, $ABCD$, rigidly joined to a uniform square metal plate, $DEFG$. The plank of wood has mass 50 kg and dimensions 4.0 m by 0.25 m. The metal plate has mass 80 kg and side 0.5 m. The plank and plate are joined in such a way that CDE is a straight line (see diagram). The barrier is smoothly pivoted at the point D. In the closed position, the barrier rests on a thin post at H. The distance CH is 0.25 m.

(i) Calculate the contact force at H when the barrier is in the closed position. [3]

In the open position, the centre of mass of the barrier is vertically above D.

(ii) Calculate the angle between AB and the horizontal when the barrier is in the open position. [8]

8 A particle is projected with speed $49 \, \text{m s}^{-1}$ at an angle of elevation θ from a point O on a horizontal plane, and moves freely under gravity. The horizontal and upward vertical displacements of the particle from O at time t seconds after projection are x m and y m respectively.

(i) Express x and y in terms of θ and t, and hence show that

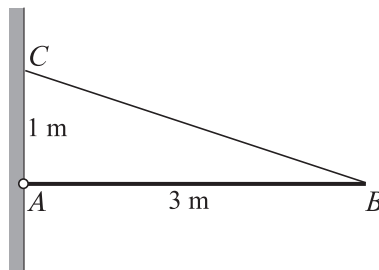$$y = x \tan \theta - \frac{x^2 (1 + \tan^2 \theta)}{490}.$$ [4]
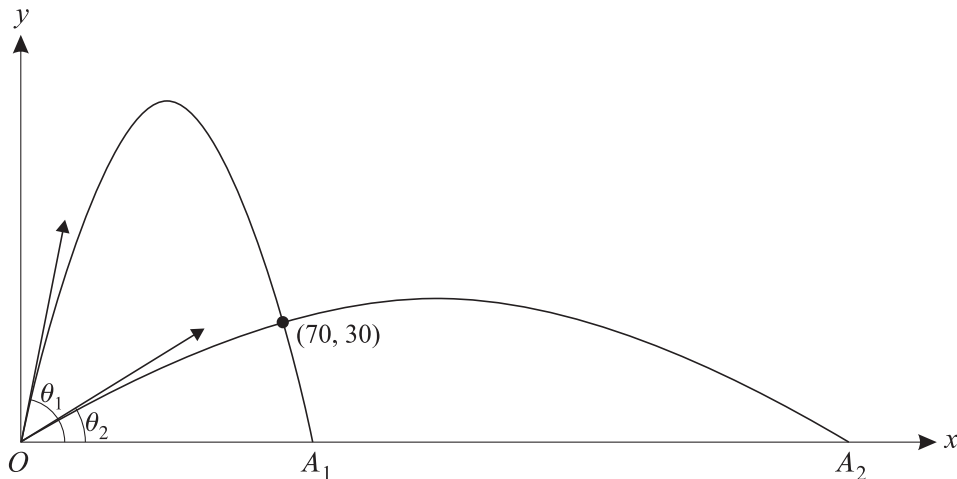
The particle passes through the point where $x = 70$ and $y = 30$. The two possible values of θ are θ_1 and θ_2, and the corresponding points where the particle returns to the plane are A_1 and A_2 respectively (see diagram).

(ii) Find θ_1 and θ_2. [4]

(iii) Calculate the distance between A_1 and A_2. [5]

1

A uniform rod AB has weight $20\,\text{N}$ and length $3\,\text{m}$. The end A is freely hinged to a point on a vertical wall. The rod is held horizontally and in equilibrium by a light inextensible string. One end of the string is attached to the rod at B. The other end of the string is attached to a point C, which is $1\,\text{m}$ directly above A (see diagram). Calculate the tension in the string. [4]

2 A golfer hits a ball from a point O on horizontal ground with a velocity of $50\,\text{m s}^{-1}$ at an angle of $25°$ above the horizontal. The ball first hits the ground at a point A. Assuming that there is no air resistance, calculate

   (i) the time taken for the ball to travel from O to A, [3]

   (ii) the distance OA. [2]

3 A box of mass $50\,\text{kg}$ is dragged along a horizontal floor by a constant force of magnitude $400\,\text{N}$ acting at an angle of α above the horizontal. The total resistance to the motion of the box has magnitude $300\,\text{N}$. The box starts from rest at the point O, and passes the point P, $25\,\text{m}$ from O, with a speed of $2\,\text{m s}^{-1}$.
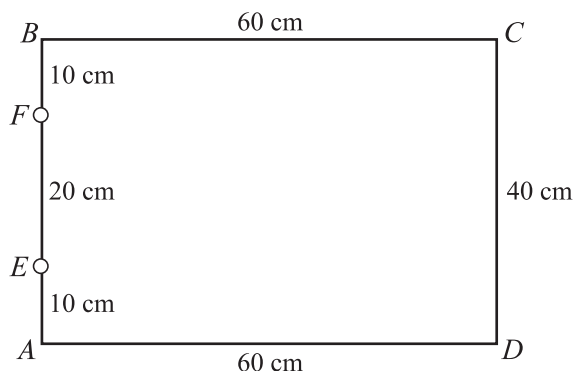
   (i) For the box's motion from O to P, find

      (a) the increase in kinetic energy of the box, [1]

      (b) the work done against the resistance to motion of the box. [1]

   (ii) Hence calculate α. [3]

4

B —————— 60 cm —————— C

10 cm

F○

20 cm   40 cm

E○

10 cm

A —————— 60 cm —————— D

A rectangular frame consists of four uniform metal rods. AB and CD are vertical and each is 40 cm long and has mass 0.2 kg. AD and BC are horizontal and each is 60 cm long. AD has mass 0.7 kg and BC has mass 0.5 kg. The frame is freely hinged at E and F, where E is 10 cm above A, and F is 10 cm below B (see diagram).

  (i) Sketch a diagram showing the directions of the horizontal components of the forces acting on the frame at E and F. [2]

  (ii) Calculate the magnitude of the horizontal component of the force acting on the frame at E. [3]

  (iii) Calculate the distance from AD of the centre of mass of the frame. [3]

5 Three smooth spheres A, B and C, of equal radius and of masses $3m$ kg, $2m$ kg and m kg respectively, are free to move in a straight line on a smooth horizontal table. Spheres B and C are stationary. Sphere A is moving with speed $2\,\text{m s}^{-1}$ when it collides directly with sphere B. The collision is perfectly elastic.

  (i) Find the velocities of A and B after the collision. [6]

  (ii) Find, in terms of m, the magnitude of the impulse that A exerts on B, and state the direction of this impulse. [2]

Sphere B continues its motion and hits C. After the collision, B continues in the same direction with speed $1.0\,\text{m s}^{-1}$ and C moves with speed $2.8\,\text{m s}^{-1}$.

  (iii) Find the coefficient of restitution between B and C. [2]

6 A stone is projected horizontally with speed $7\,\text{m s}^{-1}$ from a point O on the edge of a vertical cliff. The horizontal and upward vertical displacements of the stone from O at any subsequent time, t seconds, are x m and y m respectively. Assume that there is no air resistance.

  (i) Express x and y in terms of t, and hence show that $y = -\frac{1}{10}x^2$. [4]

The stone hits the sea at a point which is 20 m below the level of O.

  (ii) Find the distance between the foot of the cliff and the point where the stone hits the sea. [2]

  (iii) Find the speed and direction of motion of the stone immediately before it hits the sea. [6]

[Turn over

4

7 Marco is riding his bicycle at a constant speed of $12\,\mathrm{m\,s^{-1}}$ along a horizontal road, working at a constant rate of $300\,\mathrm{W}$. Marco and his bicycle have a combined mass of $75\,\mathrm{kg}$.

   (i) Calculate the wind resistance acting on Marco and his bicycle. [2]

Nicolas is riding his bicycle at the same speed as Marco and directly behind him. Nicolas experiences 30% less wind resistance than Marco.

   (ii) Calculate the power output of Nicolas. [2]

The two cyclists arrive at the bottom of a hill which is at an angle of $1°$ to the horizontal. Marco increases his power output to $500\,\mathrm{W}$.

   (iii) Assuming Marco's wind resistance is unchanged, calculate his instantaneous acceleration immediately after starting to climb the hill. [5]

Marco reaches the top of the hill at a speed of $13\,\mathrm{m\,s^{-1}}$. He then freewheels down a hill of length $200\,\mathrm{m}$ which is at a constant angle of $10°$ to the horizontal. He experiences a constant wind resistance of $120\,\mathrm{N}$.

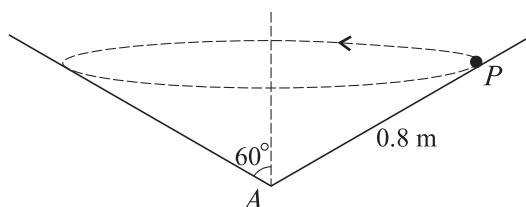   **(iv)** Calculate Marco's speed at the bottom of this hill. [5]

8

Fig. 1

A particle P of mass 0.1 kg is moving with constant angular speed ω rad s^{-1} in a horizontal circle on the smooth inner surface of a cone which is fixed with its axis vertical and its vertex A at its lowest point. The semi-vertical angle of the cone is 60° and the distance AP is 0.8 m (see Fig. 1).

(i) Calculate the magnitude of the force exerted by the cone on the particle. [3]
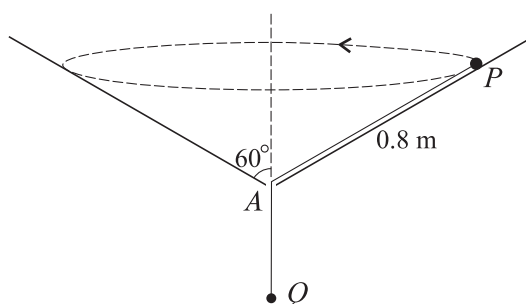
(ii) Calculate ω. [4]

Fig. 2

The particle P is now attached to one end of a light inextensible string which passes through a small smooth hole at A. The lower end of the string is attached to a particle Q of mass 0.2 kg. Q is in equilibrium with the string taut and $AP = 0.8$ m. P moves in a horizontal circle with constant speed v m s^{-1} (see Fig. 2).

(iii) State the tension in the string. [1]

(iv) Find v. [6]

1 A child of mass 35 kg runs up a flight of stairs in 10 seconds. The vertical distance climbed is 4 m. Assuming that the child's speed is constant, calculate the power output. [4]

2 A small sphere of mass 0.3 kg is dropped from rest at a height of 2 m above horizontal ground. It falls vertically, hits the ground and rebounds vertically upwards, coming to instantaneous rest at a height of 1.4 m above the ground. Ignoring air resistance, calculate the magnitude of the impulse which the ground exerts on the sphere when it rebounds. [5]

3
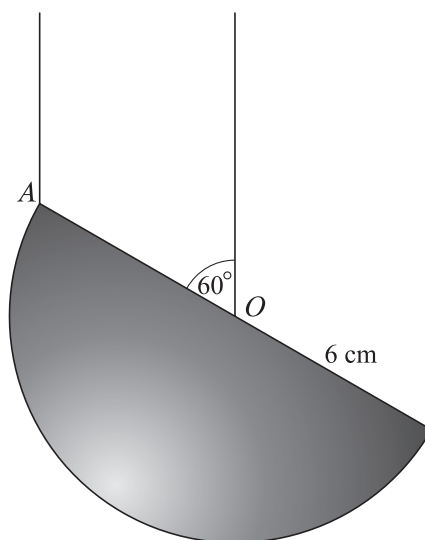
A uniform solid hemisphere of weight 12 N and radius 6 cm is suspended by two vertical strings. One string is attached to the point O, the centre of the plane face, and the other string is attached to the point A on the rim of the plane face. The hemisphere hangs in equilibrium and OA makes an angle of $60°$ with the vertical (see diagram).

(i) Find the horizontal distance from the centre of mass of the hemisphere to the vertical through O. [2]

(ii) Calculate the tensions in the strings. [5]

<u>June 2006</u>

4 A car of mass 900 kg is travelling at a constant speed of 30 m s^{-1} on a level road. The total resistance to motion is 450 N.

  (i) Calculate the power output of the car's engine. [1]

A roof box of mass 50 kg is mounted on the roof of the car. The total resistance to motion of the vehicle increases to 500 N.

  (ii) The car's engine continues to work at the same rate. Calculate the maximum speed of the car on the level road. [2]

The power output of the car's engine increases to 15 000 W. The resistance to motion of the car, with roof box, remains 500 N.

  (iii) Calculate the instantaneous acceleration of the car on the level road when its speed is 25 m s^{-1}. [3]

  (iv) The car climbs a hill which is at an angle of 5° to the horizontal. Calculate the instantaneous retardation of the car when its speed is 26 m s^{-1}. [3]

5
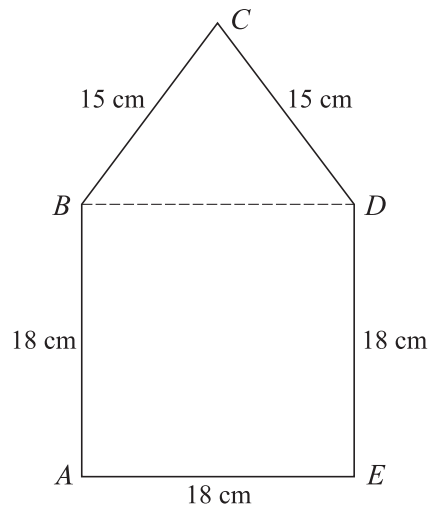
A uniform lamina *ABCDE* consists of a square and an isosceles triangle. The square has sides of 18 cm and *BC* = *CD* = 15 cm (see diagram).

  (i) Taking *x*- and *y*-axes along *AE* and *AB* respectively, find the coordinates of the centre of mass of the lamina. [7]

  (ii) The lamina is freely suspended from *B*. Calculate the angle that *BD* makes with the vertical. [2]
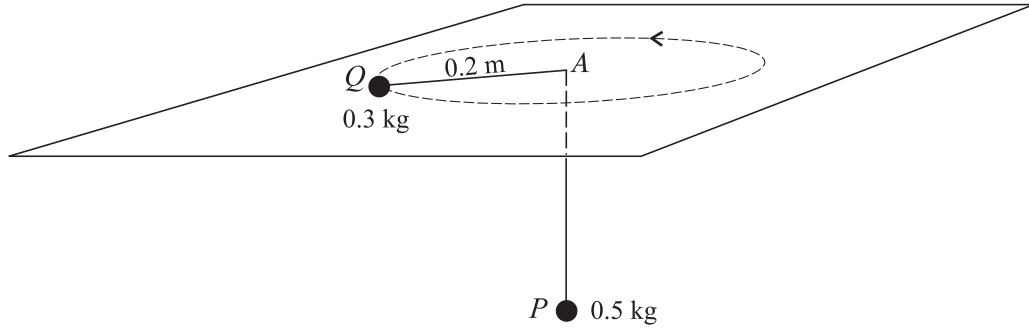
[Turn over

6

Fig. 1

A light inextensible string of length 1 m passes through a small smooth hole A in a fixed smooth horizontal plane. One end of the string is attached to a particle P, of mass 0.5 kg, which hangs in equilibrium below the plane. The other end of the string is attached to a particle Q, of mass 0.3 kg, which rotates with constant angular speed in a circle of radius 0.2 m on the surface of the plane (see Fig. 1).

(i) Calculate the tension in the string and hence find the angular speed of Q. [4]
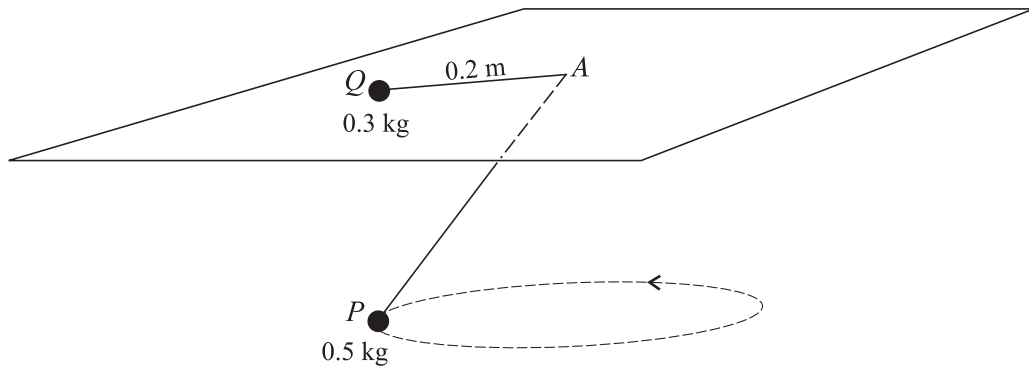
Fig. 2

The particle Q on the plane is now fixed to a point 0.2 m from the hole at A and the particle P rotates in a horizontal circle of radius 0.2 m (see Fig. 2).

(ii) Calculate the tension in the string. [4]

(iii) Calculate the speed of P. [3]

7 A small ball is projected at an angle of $50°$ above the horizontal, from a point A, which is 2 m above ground level. The highest point of the path of the ball is 15 m above the ground, which is horizontal. Air resistance may be ignored.

  (i) Find the speed with which the ball is projected from A. [3]

The ball hits a net at a point B when it has travelled a horizontal distance of 45 m.

  (ii) Find the height of B above the ground. [6]

  (iii) Find the speed of the ball immediately before it hits the net. [4]

8 Two uniform smooth spheres, A and B, have the same radius. The mass of A is 2 kg and the mass of B is m kg. Sphere A is travelling in a straight line on a smooth horizontal surface, with speed $5\,\text{m s}^{-1}$, when it collides directly with sphere B, which is at rest. As a result of the collision, sphere A continues in the same direction with a speed of $2\,\text{m s}^{-1}$.

  (i) Find the greatest possible value of m. [3]

It is given that $m = 1$.

  (ii) Find the coefficient of restitution between A and B. [3]

On another occasion A and B are travelling towards each other, each with speed $5\,\text{m s}^{-1}$, when they collide directly.

  (iii) Find the kinetic energy lost due to the collision. [8]

1 A uniform solid cylinder has height $20 \, \text{cm}$ and diameter $12 \, \text{cm}$. It is placed with its axis vertical on a rough horizontal plane. The plane is slowly tilted until the cylinder topples when the angle of inclination is α. Find α. [3]

2 Two smooth spheres A and B, of equal radius and of masses $0.2 \, \text{kg}$ and $0.1 \, \text{kg}$ respectively, are free to move on a smooth horizontal table. A is moving with speed $4 \, \text{m s}^{-1}$ when it collides directly with B, which is stationary. The collision is perfectly elastic. Calculate the speed of A after the impact. [4]

3 A small sphere of mass $0.2 \, \text{kg}$ is projected vertically downwards with speed $21 \, \text{m s}^{-1}$ from a point at a height of $40 \, \text{m}$ above horizontal ground. It hits the ground and rebounds vertically upwards, coming to instantaneous rest at its initial point of projection. Ignoring air resistance, calculate

  (i) the coefficient of restitution between the sphere and the ground, [6]

  (ii) the magnitude of the impulse which the ground exerts on the sphere. [2]

4 A skier of mass $80 \, \text{kg}$ is pulled up a slope which makes an angle of $20°$ with the horizontal. The skier is subject to a constant frictional force of magnitude $70 \, \text{N}$. The speed of the skier increases from $2 \, \text{m s}^{-1}$ at the point A to $5 \, \text{m s}^{-1}$ at the point B, and the distance AB is $25 \, \text{m}$.

  (i) By modelling the skier as a small object, calculate the work done by the pulling force as the skier moves from A to B. [5]

  (ii)
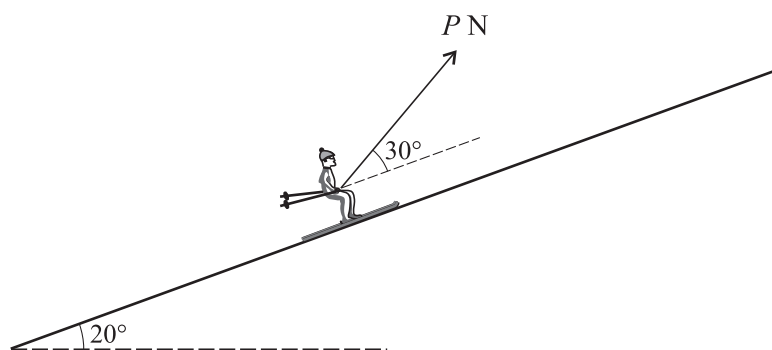
It is given that the pulling force has constant magnitude $P \, \text{N}$, and that it acts at a constant angle of $30°$ above the slope (see diagram). Calculate P. [3]

5   A model train has mass $100\,\text{kg}$. When the train is moving with speed $v\,\text{m s}^{-1}$ the resistance to its motion is $3v^2\,\text{N}$ and the power output of the train is $\dfrac{3000}{v}\,\text{W}$.

     (i) Show that the driving force acting on the train is $120\,\text{N}$ at an instant when the train is moving with speed $5\,\text{m s}^{-1}$.   [2]

     (ii) Find the acceleration of the train at an instant when it is moving horizontally with speed $5\,\text{m s}^{-1}$.   [2]

    The train moves with constant speed up a straight hill inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{98}$.

     (iii) Calculate the speed of the train.   [5]

6
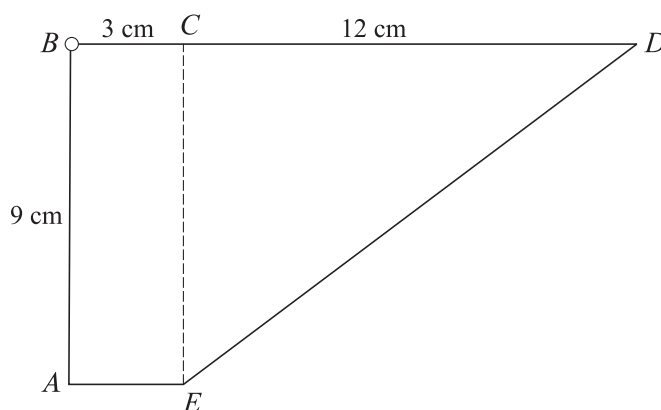
    A uniform lamina $ABCDE$ of weight $30\,\text{N}$ consists of a rectangle and a right-angled triangle. The dimensions are as shown in the diagram.

     (i) Taking x- and y-axes along AE and AB respectively, find the coordinates of the centre of mass of the lamina.   [8]

    The lamina is freely suspended from a hinge at B.

     (ii) Calculate the angle that AB makes with the vertical.   [2]

    The lamina is now held in a position such that BD is horizontal. This is achieved by means of a string attached to D and to a fixed point $15\,\text{cm}$ directly above the hinge at B.

     (iii) Calculate the tension in the string.   [3]

[Questions 7 and 8 are printed overleaf.]

7
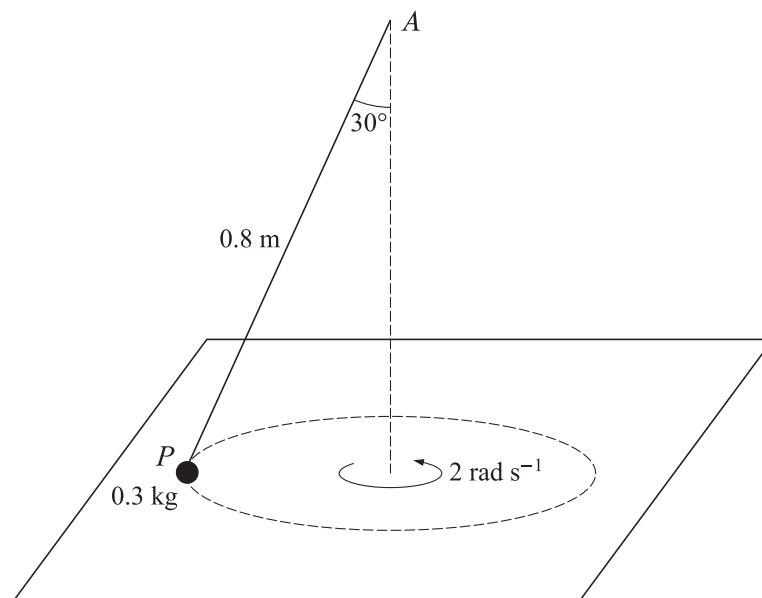
One end of a light inextensible string of length 0.8 m is attached to a fixed point A which lies above a smooth horizontal table. The other end of the string is attached to a particle P, of mass 0.3 kg, which moves in a horizontal circle on the table with constant angular speed $2\,\text{rad s}^{-1}$. AP makes an angle of $30°$ with the vertical (see diagram).

  (i) Calculate the tension in the string. [4]

  (ii) Calculate the normal contact force between the particle and the table. [3]

The particle now moves with constant speed $v\,\text{m s}^{-1}$ and is on the point of leaving the surface of the table.

  (iii) Calculate v. [6]

8   A missile is projected with initial speed $42\,\text{m s}^{-1}$ at an angle of $30°$ above the horizontal. Ignoring air resistance, calculate

  (i) the maximum height of the missile above the level of the point of projection, [3]

  (ii) the distance of the missile from the point of projection at the instant when it is moving **downwards** at an angle of $10°$ to the horizontal. [11]

---

1   A man drags a sack at constant speed in a straight line along horizontal ground by means of a rope attached to the sack. The rope makes an angle of $35°$ with the horizontal and the tension in the rope is 40 N. Calculate the work done in moving the sack 100 m. [3]

2   Calculate the range on a horizontal plane of a small stone projected from a point on the plane with speed $12\,\text{m s}^{-1}$ at an angle of elevation of $27°$. [4]

3   A rocket of mass 250 kg is moving in a straight line in space. There is no resistance to motion, and the mass of the rocket is assumed to be constant. With its motor working at a constant rate of 450 kW the rocket's speed increases from $100\,\text{m s}^{-1}$ to $150\,\text{m s}^{-1}$ in a time t seconds.

   (i) Calculate the value of t. [4]

   (ii) Calculate the acceleration of the rocket at the instant when its speed is $120\,\text{m s}^{-1}$. [4]

4   A ball is projected from a point O on the edge of a vertical cliff. The horizontal and vertically upward components of the initial velocity are $7\,\text{m s}^{-1}$ and $21\,\text{m s}^{-1}$ respectively. At time t seconds after projection the ball is at the point (x, y) referred to horizontal and vertically upward axes through O. Air resistance may be neglected.

   (i) Express x and y in terms of t, and hence show that $y = 3x - \frac{1}{10}x^2$. [5]

   The ball hits the sea at a point which is 25 m below the level of O.

   (ii) Find the horizontal distance between the cliff and the point where the ball hits the sea. [3]

5   A cyclist and her bicycle have a combined mass of 70 kg. The cyclist ascends a straight hill AB of constant slope, starting from rest at A and reaching a speed of $4\,\text{m s}^{-1}$ at B. The level of B is 6 m above the level of A. For the cyclist's motion from A to B, find

   (i) the increase in kinetic energy, [2]

   (ii) the increase in gravitational potential energy. [2]

   During the ascent the resistance to motion is constant and has magnitude 60 N. The work done by the cyclist in moving from A to B is 8000 J.

   (iii) Calculate the distance AB. [4]

6
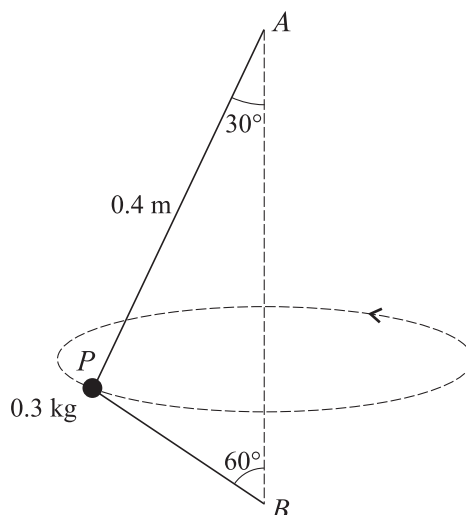
A particle P of mass 0.3 kg is attached to one end of each of two light inextensible strings. The other end of the longer string is attached to a fixed point A and the other end of the shorter string is attached to a fixed point B, which is vertically below A. AP makes an angle of 30° with the vertical and is 0.4 m long. PB makes an angle of 60° with the vertical. The particle moves in a horizontal circle with constant angular speed and with both strings taut (see diagram). The tension in the string AP is 5 N.

Calculate

  (i) the tension in the string PB, [3]

  (ii) the angular speed of P, [3]

  (iii) the kinetic energy of P. [3]

7   Two small spheres A and B, with masses 0.3 kg and m kg respectively, lie at rest on a smooth horizontal surface. A is projected directly towards B with speed $6\,\text{m s}^{-1}$ and hits B. The direction of motion of A is reversed in the collision. The speeds of A and B after the collision are $1\,\text{m s}^{-1}$ and $3\,\text{m s}^{-1}$ respectively. The coefficient of restitution between A and B is e.

  (i) Show that $m = 0.7$. [2]

  (ii) Find e. [2]

B continues to move at $3\,\text{m s}^{-1}$ and strikes a vertical wall at right angles. The coefficient of restitution between B and the wall is f.

  (iii) Find the range of values of f for which there will be a second collision between A and B. [2]

  (iv) Find, in terms of f, the magnitude of the impulse that the wall exerts on B. [3]

  (v) Given that $f = \frac{3}{4}$, calculate the final speeds of A and B, correct to 1 decimal place. [7]

[Question 8 is printed overleaf.]
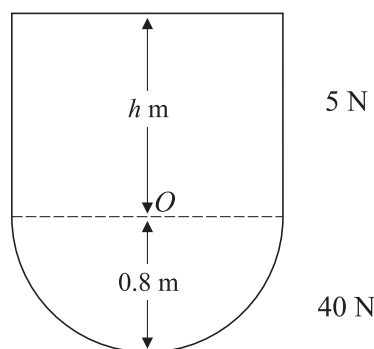
Fig. 1

An object consists of a uniform solid hemisphere of weight 40 N and a uniform solid cylinder of weight 5 N. The cylinder has height h m. The solids have the same base radius 0.8 m and are joined so that the hemisphere's plane face coincides with one of the cylinder's faces. The centre of the common face is the point O (see Fig. 1). The centre of mass of the object lies inside the hemisphere and is at a distance of 0.2 m from O.

(i) Show that $h = 1.2$. [6]
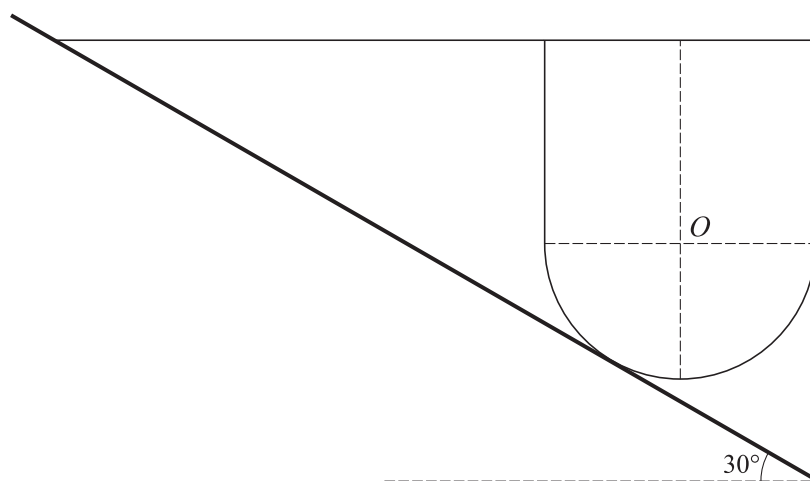
Fig. 2

One end of a light inextensible string is attached to a point on the circumference of the upper face of the cylinder. The string is horizontal and its other end is tied to a fixed point on a rough plane. The object rests in equilibrium on the plane with its axis of symmetry vertical. The plane makes an angle of $30°$ with the horizontal (see Fig. 2). The tension in the string is T N and the frictional force acting on the object is F N.

(ii) By taking moments about O, express F in terms of T. [4]

(iii) Find another equation connecting T and F. Hence calculate the tension and the frictional force. [6]

1   A ball is projected with speed $12\,\text{m s}^{-1}$ at an angle of elevation of $55°$ above the horizontal. At the instant when the ball reaches its greatest height, it hits a vertical wall, which is perpendicular to the ball's path. The coefficient of restitution between the ball and the wall is 0.65. Calculate the speed of the ball

   (i) immediately before its impact with the wall, [2]

   (ii) immediately after its impact with the wall. [2]

2   A particle of mass $m\,\text{kg}$ is projected directly up a rough plane with a speed of $5\,\text{m s}^{-1}$. The plane makes an angle of $30°$ with the horizontal and the coefficient of friction is 0.2. Calculate the distance the particle travels up the plane before coming instantaneously to rest. [6]

3
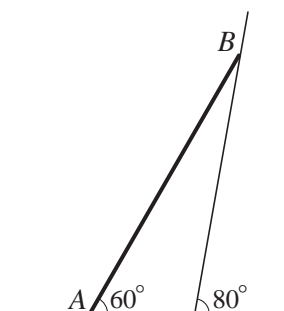
   A uniform rod AB, of weight $25\,\text{N}$ and length $1.6\,\text{m}$, rests in equilibrium in a vertical plane with the end A in contact with rough horizontal ground and the end B resting against a smooth wall which is inclined at $80°$ to the horizontal. The rod is inclined at $60°$ to the horizontal (see diagram). Calculate the magnitude of the force acting on the rod at B. [6]

4   A car of mass $1200\,\text{kg}$ has a maximum speed of $30\,\text{m s}^{-1}$ when travelling on a horizontal road. The car experiences a resistance of $kv\,\text{N}$, where $v\,\text{m s}^{-1}$ is the speed of the car and k is a constant. The maximum power of the car's engine is $45\,000\,\text{W}$.

   (i) Show that $k = 50$. [2]

   (ii) Find the maximum possible acceleration of the car when it is travelling at $20\,\text{m s}^{-1}$ on a horizontal road. [3]

   (iii) The car climbs a hill, which is inclined at an angle of $10°$ to the horizontal, at a constant speed of $15\,\text{m s}^{-1}$. Calculate the power of the car's engine. [3]

5   A particle P of mass $2m$ is moving on a smooth horizontal surface with speed u when it collides directly with a particle Q of mass km whose speed is $3u$ in the opposite direction. As a result of the collision, the directions of motion of both particles are reversed and the speed of P is halved.

   (i) Find, in terms of u and k, the speed of Q after the collision. Hence write down the range of possible values of k. [4]

   (ii) Calculate the magnitude of the impulse which Q exerts on P. [2]

   (iii) Given that $k = \frac{1}{2}$, calculate the coefficient of restitution between P and Q. [3]
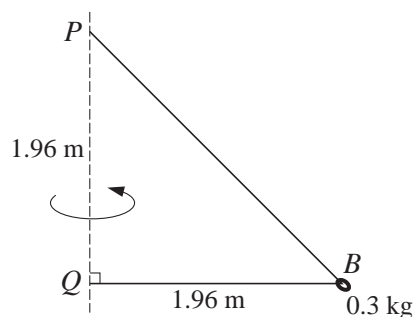
Jan 2008

6 (i)

Fig. 1

One end of a light inextensible string is attached to a point P. The other end is attached to a point Q, 1.96 m vertically below P. A small smooth bead B, of mass 0.3 kg, is threaded on the string and moves in a horizontal circle with centre Q and radius 1.96 m. B rotates about Q with constant angular speed ω rad s^{-1} (see Fig. 1).

(a) Show that the tension in the string is 4.16 N, correct to 3 significant figures. [2]

(b) Calculate ω. [3]

(ii)
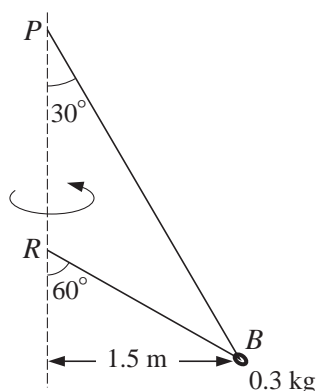
Fig. 2

The lower part of the string is now attached to a point R, vertically below P. PB makes an angle 30° with the vertical and RB makes an angle 60° with the vertical. The bead B now moves in a horizontal circle of radius 1.5 m with constant speed v m s^{-1} (see Fig. 2).

(a) Calculate the tension in the string. [3]

(b) Calculate v. [3]

7 A missile is projected from a point O on horizontal ground with speed 175 m s^{-1} at an angle of elevation θ. The horizontal lower surface of a cloud is 650 m above the ground.

(i) Find the value of θ for which the missile just reaches the cloud. [3]

It is given that $\theta = 55°$.

(ii) Find the length of time for which the missile is above the lower surface of the cloud. [5]

(iii) Find the speed of the missile at the instant it enters the cloud. [4]

8   **(i)** A uniform semicircular lamina has radius 4 cm. Show that the distance from its centre to its centre of mass is 1.70 cm, correct to 3 significant figures.   [2]

   (ii)
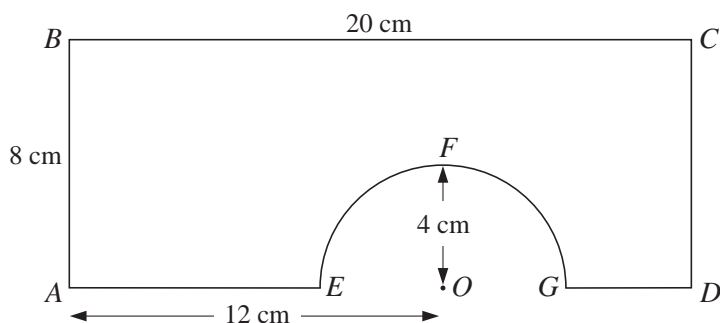
Fig. 1

A model bridge is made from a uniform rectangular board, *ABCD*, with a semicircular section, *EFG*, removed. *O* is the mid-point of *EG*. *AB* = 8 cm, *BC* = 20 cm, *AO* = 12 cm and the radius of the semicircle is 4 cm (see Fig. 1).

   (a) Show that the distance from *AB* to the centre of mass of the model is 9.63 cm, correct to 3 significant figures.   [5]

   (b) Calculate the distance from *AD* to the centre of mass of the model.   [4]

   (iii)
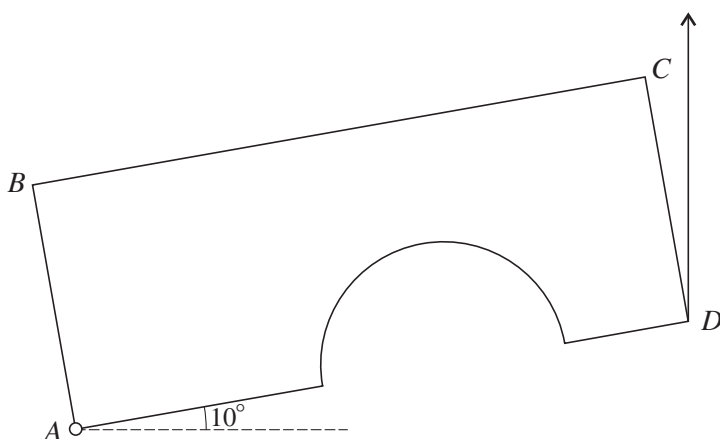
Fig. 2

The model bridge is smoothly pivoted at *A* and is supported in equilibrium by a vertical wire attached to *D*. The weight of the model is 15 N and *AD* makes an angle of 10° with the horizontal (see Fig. 2). Calculate the tension in the wire.   [5]

1 A car is pulled at constant speed along a horizontal straight road by a force of 200 N inclined at 35° to the horizontal. Given that the work done by the force is 5000 J, calculate the distance moved by the car. [3]

2 A bullet of mass 9 grams passes horizontally through a fixed vertical board of thickness 3 cm. The speed of the bullet is reduced from 250 m s^{-1} to 150 m s^{-1} as it passes through the board. The board exerts a constant resistive force on the bullet. Calculate the magnitude of this resistive force. [4]

3 The resistance to the motion of a car of mass 600 kg is kv N, where v m s^{-1} is the car's speed and k is a constant. The car ascends a hill of inclination α, where $\sin \alpha = \frac{1}{10}$. The power exerted by the car's engine is 12 000 W and the car has constant speed 20 m s^{-1}.

   (i) Show that $k = 0.6$. [3]

The power exerted by the car's engine is increased to 16 000 W.

   (ii) Calculate the maximum speed of the car while ascending the hill. [3]

The car now travels on horizontal ground and the power remains 16 000 W.

   (iii) Calculate the acceleration of the car at an instant when its speed is 32 m s^{-1}. [3]

4 A golfer hits a ball from a point O on horizontal ground with a velocity of 35 m s^{-1} at an angle of θ above the horizontal. The horizontal range of the ball is R metres and the time of flight is t seconds.

   (i) Express t in terms of θ, and hence show that $R = 125 \sin 2\theta$. [5]

The golfer hits the ball so that it lands 110 m from O.

   (ii) Calculate the two possible values of t. [5]

5

0.01 kg ●

0.02 kg | 10 cm

3 cm

0.5 kg
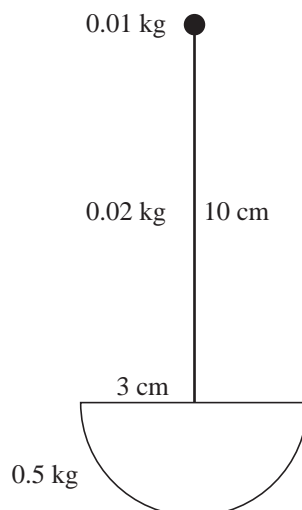
Fig. 1

A toy is constructed by attaching a small ball of mass 0.01 kg to one end of a uniform rod of length 10 cm whose other end is attached to the centre of the plane face of a uniform solid hemisphere with radius 3 cm. The rod has mass 0.02 kg, the hemisphere has mass 0.5 kg and the rod is perpendicular to the plane face of the hemisphere (see Fig. 1).

(i) Show that the distance from the ball to the centre of mass of the toy is 10.7 cm, correct to 1 decimal place. [4]

(ii)
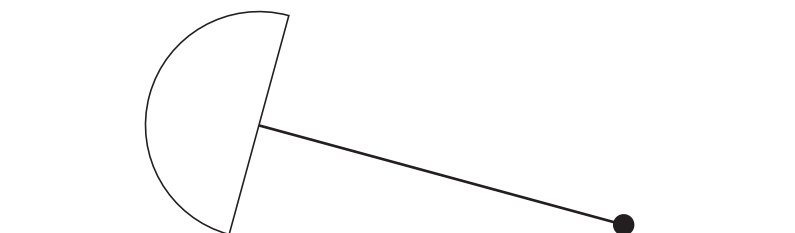
Fig. 2

The toy lies on horizontal ground in a position such that the ball is touching the ground (see Fig. 2). Determine whether the toy is lying in equilibrium or whether it will move to a position where the rod is vertical. [4]

6
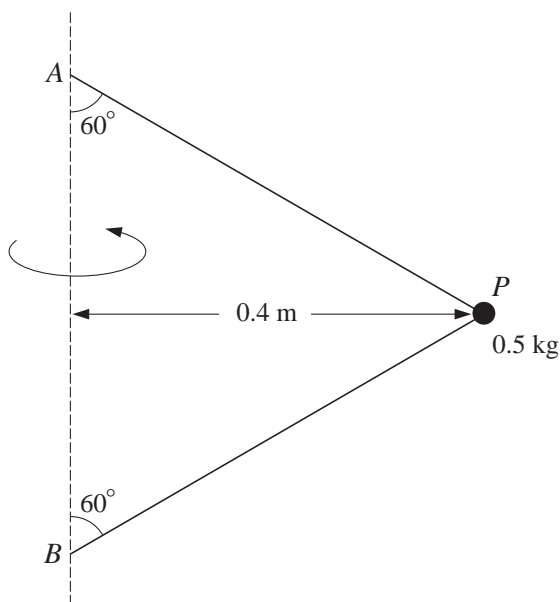
A particle P of mass $0.5\,\text{kg}$ is attached to points A and B on a fixed vertical axis by two light inextensible strings of equal length. Both strings are taut and each is inclined at $60°$ to the vertical (see diagram). The particle moves with constant speed $3\,\text{m s}^{-1}$ in a horizontal circle of radius $0.4\,\text{m}$.

(i) Calculate the tensions in the two strings. [7]

The particle now moves with constant angular speed $\omega\,\text{rad s}^{-1}$ and the string BP is on the point of becoming slack.

(ii) Calculate ω. [5]

7

Two small spheres A and B of masses $2\,\text{kg}$ and $3\,\text{kg}$ respectively lie at rest on a smooth horizontal platform which is fixed at a height of $4\,\text{m}$ above horizontal ground (see diagram). Sphere A is given an impulse of $6\,\text{N s}$ towards B, and A then strikes B directly. The coefficient of restitution between A and B is $\frac{2}{3}$.

(i) Show that the speed of B after it has been hit by A is $2\,\text{m s}^{-1}$. [6]

Sphere B leaves the platform and follows the path of a projectile.

(ii) Calculate the speed and direction of motion of B at the instant when it hits the ground. [7]

5

8 **(i)**
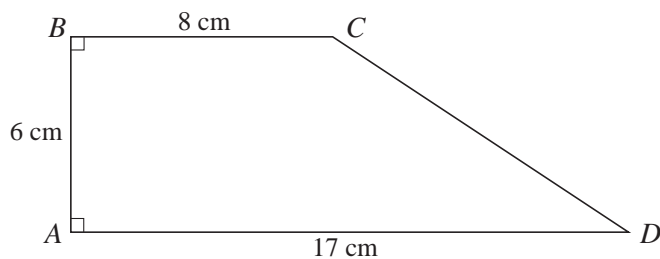
Fig. 1

A uniform lamina *ABCD* is in the form of a right-angled trapezium. *AB* = 6 cm, *BC* = 8 cm and *AD* = 17 cm (see Fig. 1). Taking *x*- and *y*-axes along *AD* and *AB* respectively, find the coordinates of the centre of mass of the lamina. [8]

(ii)
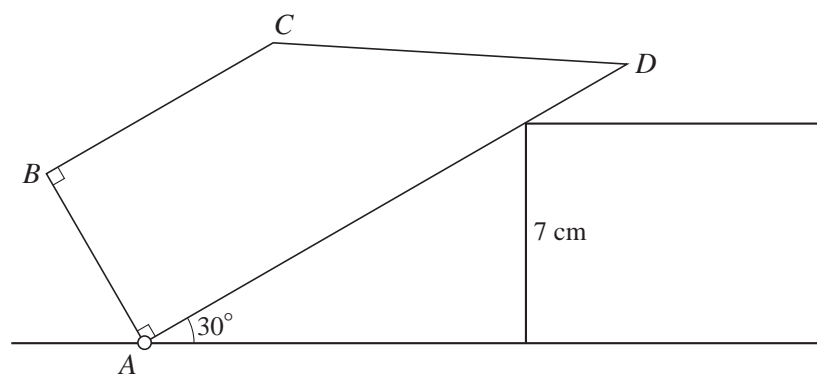
Fig. 2

The lamina is smoothly pivoted at *A* and it rests in a vertical plane in equilibrium against a fixed smooth block of height 7 cm. The mass of the lamina is 3 kg. *AD* makes an angle of 30° with the horizontal (see Fig. 2). Calculate the magnitude of the force which the block exerts on the lamina. [5]

1 A stone is projected from a point on level ground with speed $20\,\mathrm{m\,s}^{-1}$ at an angle of elevation of $\theta°$ above the horizontal. When the stone is at its greatest height it just passes over the top of a tree that is 17 m high. Calculate θ. **[4]**

2
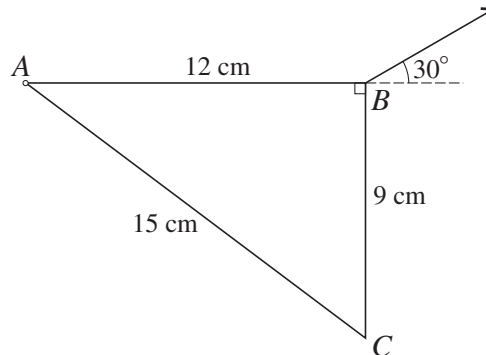
A uniform right-angled triangular lamina ABC with sides $AB = 12\,\mathrm{cm}$, $BC = 9\,\mathrm{cm}$ and $AC = 15\,\mathrm{cm}$ is freely suspended from a hinge at its vertex A. The lamina has mass 2 kg and is held in equilibrium with AB horizontal by means of a string attached to B. The string is at an angle of $30°$ to the horizontal (see diagram). Calculate the tension in the string. **[4]**

3
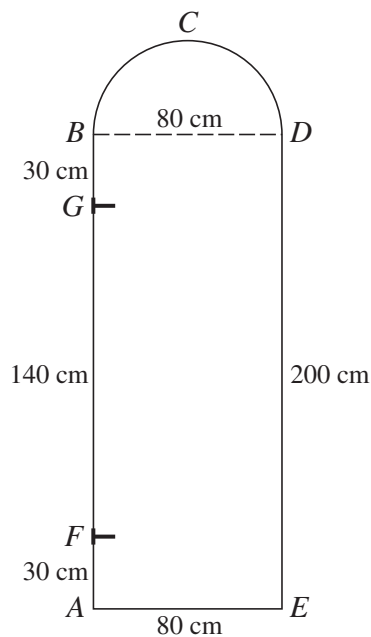
A door is modelled as a lamina $ABCDE$ consisting of a uniform rectangular section $ABDE$ of weight 60 N and a uniform semicircular section BCD of weight 10 N and radius 40 cm. AB is 200 cm and AE is 80 cm. The door is freely hinged at F and G, where G is 30 cm below B and F is 30 cm above A (see diagram).

  (i) Find the magnitudes and directions of the horizontal components of the forces on the door at each of F and G. **[4]**

  (ii) Calculate the distance from AE to the centre of mass of the door. **[6]**

4 A car of mass $800\,\text{kg}$ experiences a resistance of magnitude $kv^2\,\text{N}$, where k is a constant and $v\,\text{m}\,\text{s}^{-1}$ is the car's speed. The car's engine is working at a constant rate of $P\,\text{W}$. At an instant when the car is travelling on a horizontal road with speed $20\,\text{m}\,\text{s}^{-1}$ its acceleration is $0.75\,\text{m}\,\text{s}^{-2}$. At an instant when the car is ascending a hill of constant slope $12°$ to the horizontal with speed $10\,\text{m}\,\text{s}^{-1}$ its acceleration is $0.25\,\text{m}\,\text{s}^{-2}$.

 (i) Show that $k = 0.900$, correct to 3 decimal places, and find P. **[7]**

The power is increased to $1.5P\,\text{W}$.

 (ii) Calculate the maximum steady speed of the car on a horizontal road. **[3]**

5
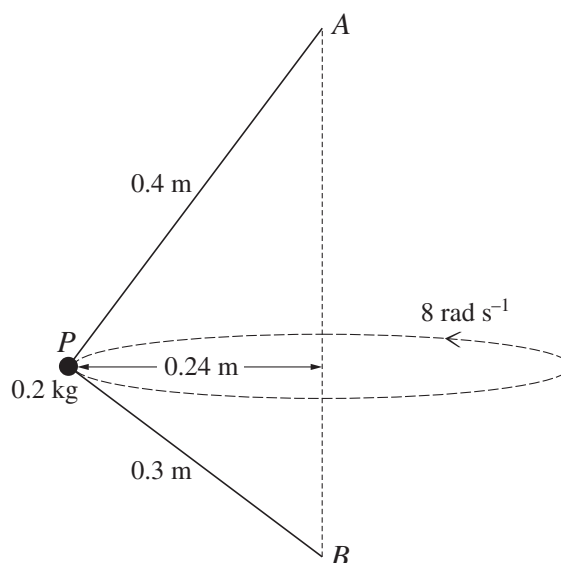
A particle P of mass $0.2\,\text{kg}$ is attached to one end of each of two light inextensible strings, one of length $0.4\,\text{m}$ and one of length $0.3\,\text{m}$. The other end of the longer string is attached to a fixed point A, and the other end of the shorter string is attached to a fixed point B, which is vertically below A. The particle moves in a horizontal circle of radius $0.24\,\text{m}$ at a constant angular speed of $8\,\text{rad}\,\text{s}^{-1}$ (see diagram). Both strings are taut, the tension in AP is $S\,\text{N}$ and the tension in BP is $T\,\text{N}$.

 (i) By resolving vertically, show that $4S = 3T + 9.8$. **[4]**

 (ii) Find another equation connecting S and T and hence calculate the tensions, correct to 1 decimal place. **[8]**

[Questions 6 and 7 are printed overleaf.]

6 A particle is projected from a point O with speed $v\,\mathrm{m\,s^{-1}}$ at an angle of elevation θ above the horizontal and it moves freely under gravity. The horizontal and upward vertical displacements of the particle from O at any subsequent time, t seconds, are x m and y m respectively.

  (i) Express x and y in terms of θ and t, and hence show that

$$y = x\tan\theta - \frac{4.9x^2}{v^2\cos^2\theta}.$$
     [4]
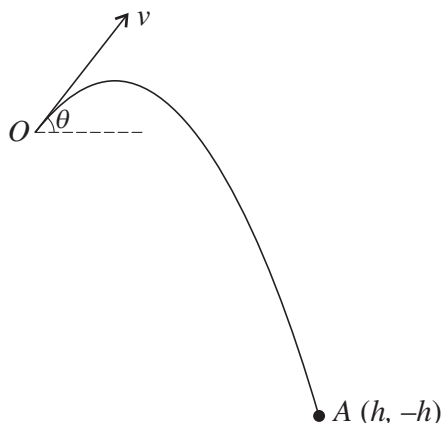
The particle subsequently passes through the point A with coordinates $(h,\ -h)$ as shown in the diagram. It is given that $v = 14$ and $\theta = 30°$.

  (ii) Calculate h.   **[4]**

  (iii) Calculate the direction of motion of the particle at A.   **[5]**

  (iv) Calculate the speed of the particle at A.   **[2]**

7
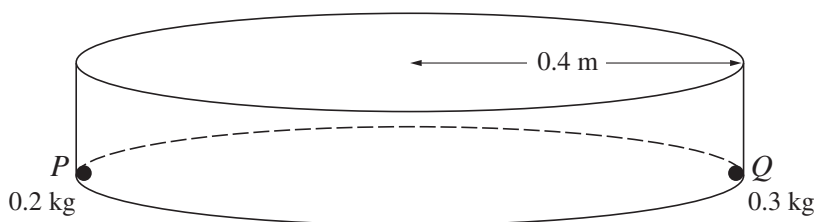
Two small spheres, P and Q, are free to move on the inside of a smooth hollow cylinder, in such a way that they remain in contact with both the curved surface and the base of the cylinder. The mass of P is $0.2\,\mathrm{kg}$, the mass of Q is $0.3\,\mathrm{kg}$ and the radius of the cylinder is $0.4\,\mathrm{m}$. P and Q are stationary at opposite ends of a diameter of the base of the cylinder (see diagram). The coefficient of restitution between P and Q is 0.5. P is given an impulse of magnitude $0.8\,\mathrm{N\,s}$ in a tangential direction.

  (i) Calculate the speeds of the particles after P's first impact with Q.   **[8]**

Q subsequently catches up with P and there is a second impact.

  (ii) Calculate the speeds of the particles after this second impact.   **[7]**

  (iii) Calculate the magnitude of the force exerted on Q by the curved surface of the cylinder after the second impact.   **[2]**

1 A boy on a sledge slides down a straight track of length 180 m which descends a vertical distance of 40 m. The combined mass of the boy and the sledge is 75 kg. The initial speed is $3\,\text{m s}^{-1}$ and the final speed is $12\,\text{m s}^{-1}$. The magnitude, $R\,\text{N}$, of the resistance to motion is constant. By considering the change in energy, calculate R. **[5]**

2 A car of mass 1100 kg has maximum power of 44 000 W. The resistive forces have constant magnitude 1400 N.

   (i) Calculate the maximum steady speed of the car on the level. **[2]**

The car is moving on a hill of constant inclination α to the horizontal, where $\sin \alpha = 0.05$.

   (ii) Calculate the maximum steady speed of the car when ascending the hill. **[3]**

   (iii) Calculate the acceleration of the car when it is descending the hill at a speed of $10\,\text{m s}^{-1}$ working at half the maximum power. **[3]**

3
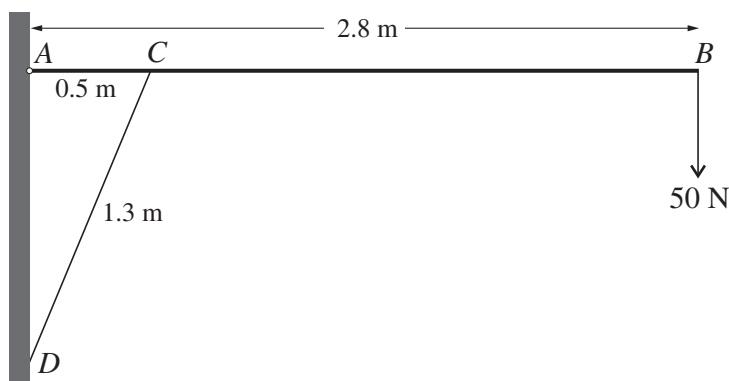
A uniform beam AB has weight 70 N and length 2.8 m. The beam is freely hinged to a wall at A and is supported in a horizontal position by a strut CD of length 1.3 m. One end of the strut is attached to the beam at C, 0.5 m from A, and the other end is attached to the wall at D, vertically below A. The strut exerts a force on the beam in the direction DC. The beam carries a load of weight 50 N at its end B (see diagram).

   (i) Calculate the magnitude of the force exerted by the strut on the beam. **[4]**

   (ii) Calculate the magnitude of the force acting on the beam at A. **[6]**

4 A light inextensible string of length 0.6 m has one end fixed to a point A on a smooth horizontal plane. The other end of the string is attached to a particle B, of mass 0.4 kg, which rotates about A with constant angular speed $2\,\text{rad s}^{-1}$ on the surface of the plane.

   (i) Calculate the tension in the string. **[2]**

A particle P of mass 0.1 kg is attached to the mid-point of the string. The line APB is straight and rotation continues at $2\,\text{rad s}^{-1}$.

   (ii) Calculate the tension in the section of the string AP. **[4]**

   (iii) Calculate the total kinetic energy of the system. **[5]**

5 **(i)**
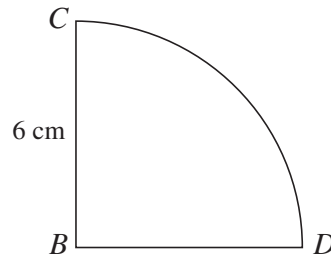
Fig. 1

Fig. 1 shows a uniform lamina *BCD* in the shape of a quarter circle of radius 6 cm. Show that the distance of the centre of mass of the lamina from *B* is 3.60 cm, correct to 3 significant figures. **[2]**

A uniform rectangular lamina *ABDE* has dimensions *AB* = 12 cm and *AE* = 6 cm. A single plane object is formed by attaching the rectangular lamina to the lamina *BCD* along *BD* (see Fig. 2). The mass of *ABDE* is 3 kg and the mass of *BCD* is 2 kg.
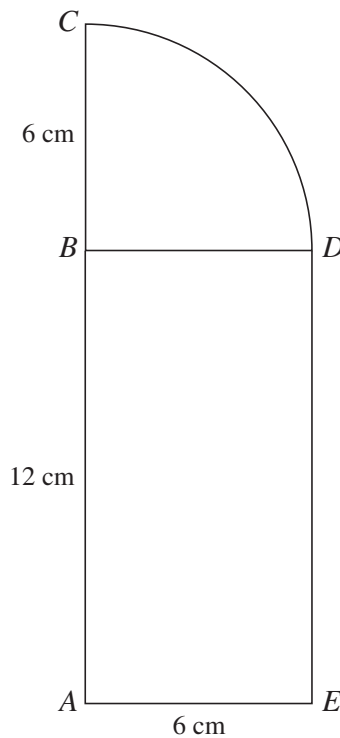

Fig. 2

(ii) Taking *x*- and *y*-axes along *AE* and *AB* respectively, find the coordinates of the centre of mass of the object. **[7]**

The object is freely suspended at *C* and rests in equilibrium.

(iii) Calculate the angle that *AC* makes with the vertical. **[2]**

6 Two uniform spheres, A and B, have the same radius. The mass of A is $0.4\,\text{kg}$ and the mass of B is $0.2\,\text{kg}$. The spheres A and B are travelling in the same direction in a straight line on a smooth horizontal surface, A with speed $5\,\text{m s}^{-1}$, and B with speed $v\,\text{m s}^{-1}$, where $v < 5$. A collides directly with B and the impulse between them has magnitude $0.9\,\text{N s}$. Immediately after the collision, the speed of B is $6\,\text{m s}^{-1}$.

  (i) Calculate v. **[3]**

B subsequently collides directly with a stationary sphere C of mass $0.1\,\text{kg}$ and the same radius as A and B. The coefficient of restitution between B and C is 0.6.

  (ii) Determine whether there will be a further collision between A and B. **[10]**

7
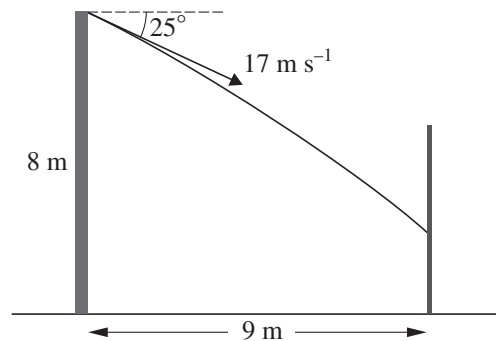
A ball is projected with an initial speed of $17\,\text{m s}^{-1}$ at an angle of $25°$ below the horizontal from a point on the top of a vertical wall. The point of projection is $8\,\text{m}$ above horizontal ground. The ball hits a vertical fence which is at a horizontal distance of $9\,\text{m}$ from the wall (see diagram).

  (i) Calculate the height above the ground of the point where the ball hits the fence. **[5]**

  (ii) Calculate the direction of motion of the ball immediately before it hits the fence. **[5]**

  (iii) It is given that 30% of the kinetic energy of the ball is lost when it hits the fence. Calculate the speed of the ball immediately after it hits the fence. **[4]**

1 Find the average power exerted by a climber of mass 75 kg when climbing a vertical distance of 40 m in 2 minutes. **[3]**

2 A small sphere of mass 0.2 kg is dropped from rest at a height of 3 m above horizontal ground. It falls vertically, hits the ground and rebounds vertically upwards, coming to instantaneous rest at a height of 1.8 m above the ground.

  (i) Calculate the magnitude of the impulse which the ground exerts on the sphere. **[5]**

  (ii) Calculate the coefficient of restitution between the sphere and the ground. **[2]**

3
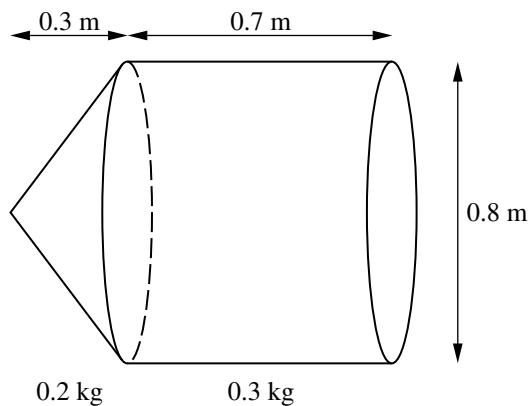
Fig. 1

A uniform conical shell has mass 0.2 kg, height 0.3 m and base diameter 0.8 m. A uniform hollow cylinder has mass 0.3 kg, length 0.7 m and diameter 0.8 m. The conical shell is attached to the cylinder, with the circumference of its base coinciding with one end of the cylinder (see Fig. 1).

  (i) Show that the distance of the centre of mass of the combined object from the vertex of the conical shell is 0.47 m. **[4]**
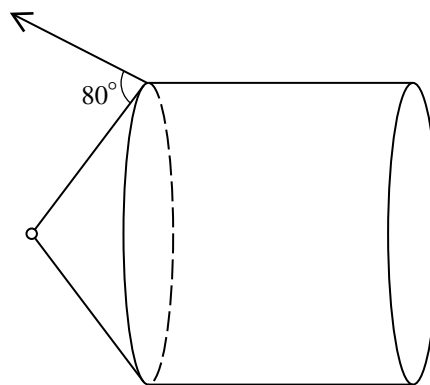
Fig. 2

The combined object is freely suspended from its vertex and is held with its axis horizontal. This is achieved by means of a wire attached to a point on the circumference of the base of the conical shell. The wire makes an angle of 80° with the slant edge of the conical shell (see Fig. 2).

  (ii) Calculate the tension in the wire. **[4]**

4   A car of mass $700\,\text{kg}$ is moving along a horizontal road against a constant resistance to motion of $400\,\text{N}$. At an instant when the car is travelling at $12\,\text{m}\,\text{s}^{-1}$ its acceleration is $0.5\,\text{m}\,\text{s}^{-2}$.

    (i) Find the driving force of the car at this instant. **[2]**

    (ii) Find the power at this instant. **[2]**

The maximum steady speed of the car on a horizontal road is $35\,\text{m}\,\text{s}^{-1}$.

    (iii) Find the maximum power of the car. **[2]**

The car now moves at maximum power against the same resistance up a slope of constant angle $\theta°$ to the horizontal. The maximum steady speed up the slope is $12\,\text{m}\,\text{s}^{-1}$.

    (iv) Find θ. **[4]**

5   Two spheres of the same radius with masses $2\,\text{kg}$ and $3\,\text{kg}$ are moving directly towards each other on a smooth horizontal plane with speeds $8\,\text{m}\,\text{s}^{-1}$ and $4\,\text{m}\,\text{s}^{-1}$ respectively. The spheres collide and the kinetic energy lost is $81\,\text{J}$. Calculate the speed and direction of motion of each sphere after the collision. **[12]**

6
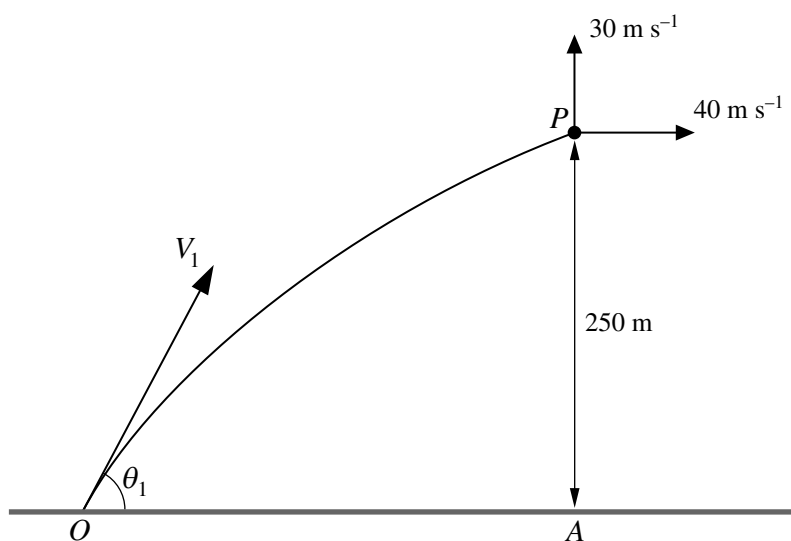
A particle P is projected with speed $V_1\,\text{m}\,\text{s}^{-1}$ at an angle of elevation θ_1 from a point O on horizontal ground. When P is vertically above a point A on the ground its height is $250\,\text{m}$ and its velocity components are $40\,\text{m}\,\text{s}^{-1}$ horizontally and $30\,\text{m}\,\text{s}^{-1}$ vertically upwards (see diagram).

    (i) Show that $V_1 = 86.0$ and $\theta_1 = 62.3°$, correct to 3 significant figures. **[5]**

At the instant when P is vertically above A, a second particle Q is projected from O with speed $V_2\,\text{m}\,\text{s}^{-1}$ at an angle of elevation θ_2. P and Q hit the ground at the same time and at the same place.

    (ii) Calculate the total time of flight of P and the total time of flight of Q. **[4]**

    (iii) Calculate the range of the particles and hence calculate V_2 and θ_2. **[8]**
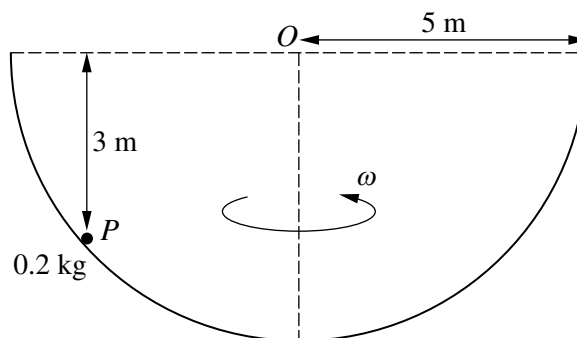
Fig. 1

A particle P of mass 0.2 kg is moving on the smooth inner surface of a fixed hollow hemisphere which has centre O and radius 5 m. P moves with constant angular speed ω in a horizontal circle at a vertical distance of 3 m below the level of O (see Fig. 1).

(i) Calculate the magnitude of the force exerted by the hemisphere on P. **[3]**
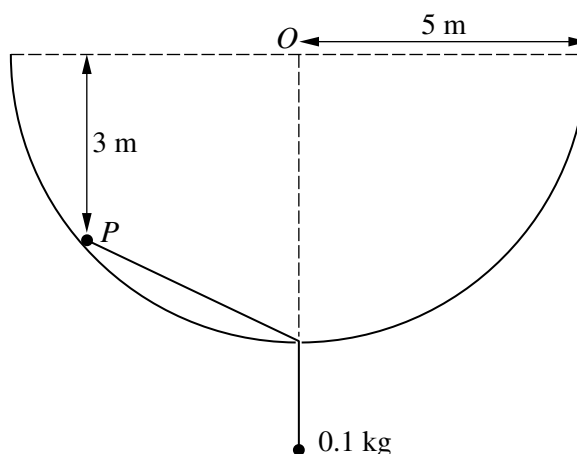
(ii) Calculate ω. **[4]**

Fig. 2

A light inextensible string is now attached to P. The string passes through a small smooth hole at the lowest point of the hemisphere and a particle of mass 0.1 kg hangs in equilibrium at the end of the string. P moves in the same horizontal circle as before (see Fig. 2).

(iii) Calculate the new angular speed of P. **[8]**

June 2010

1 A particle is projected horizontally with a speed of $7\,\mathrm{m\,s^{-1}}$ from a point $10\,\mathrm{m}$ above horizontal ground. The particle moves freely under gravity. Calculate the speed and direction of motion of the particle at the instant it hits the ground. **[6]**
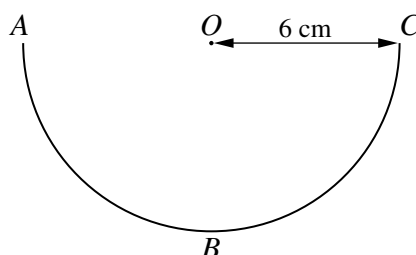
2 (i)

Fig. 1

A uniform piece of wire, ABC, forms a semicircular arc of radius $6\,\mathrm{cm}$. O is the mid-point of AC (see Fig. 1). Show that the distance from O to the centre of mass of the wire is $3.82\,\mathrm{cm}$, correct to 3 significant figures. **[2]**

(ii)
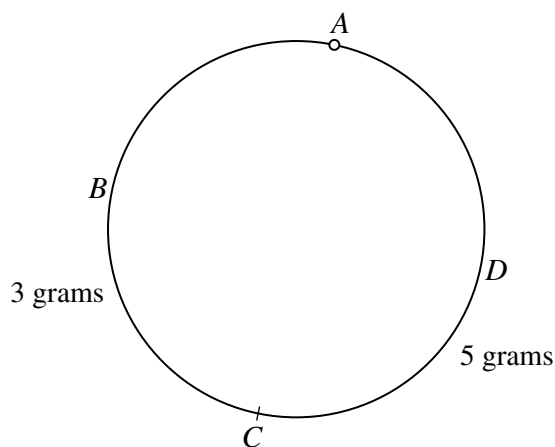
Fig. 2

Two semicircular pieces of wire, ABC and ADC, are joined together at their ends to form a circular hoop of radius $6\,\mathrm{cm}$. The mass of ABC is 3 grams and the mass of ADC is 5 grams. The hoop is freely suspended from A (see Fig. 2). Calculate the angle which the diameter AC makes with the vertical, giving your answer correct to the nearest degree. **[5]**

3 The maximum power produced by the engine of a small aeroplane of mass 2 tonnes is $128\,\mathrm{kW}$. Air resistance opposes the motion directly and the lift force is perpendicular to the direction of motion. The magnitude of the air resistance is proportional to the square of the speed and the maximum steady speed in level flight is $80\,\mathrm{m\,s^{-1}}$.

(i) Calculate the magnitude of the air resistance when the speed is $60\,\mathrm{m\,s^{-1}}$. **[5]**

The aeroplane is climbing at a constant angle of $2°$ to the horizontal.

(ii) Find the maximum acceleration at an instant when the speed of the aeroplane is $60\,\mathrm{m\,s^{-1}}$. **[4]**

4
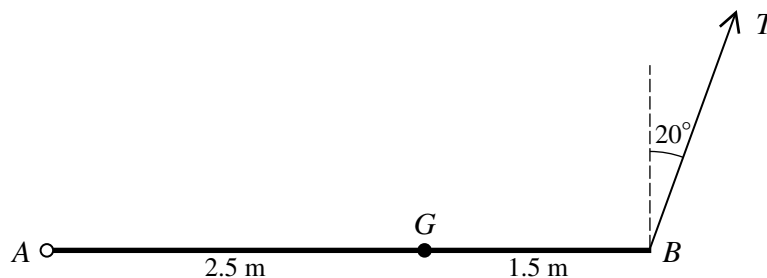
A non-uniform beam AB of length 4 m and mass 5 kg has its centre of mass at the point G of the beam where $AG = 2.5$ m. The beam is freely suspended from its end A and is held in a horizontal position by means of a wire attached to the end B. The wire makes an angle of 20° with the vertical and the tension is T N (see diagram).

(i) Calculate T. **[3]**

(ii) Calculate the magnitude and the direction of the force acting on the beam at A. **[7]**

5
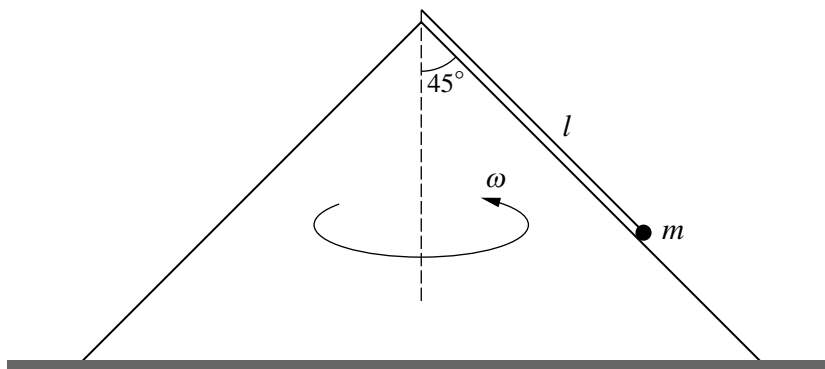
One end of a light inextensible string of length l is attached to the vertex of a smooth cone of semi-vertical angle 45°. The cone is fixed to the ground with its axis vertical. The other end of the string is attached to a particle of mass m which rotates in a horizontal circle in contact with the outer surface of the cone. The angular speed of the particle is ω (see diagram). The tension in the string is T and the contact force between the cone and the particle is R.

(i) By resolving horizontally and vertically, find two equations involving T and R and hence show that $T = \frac{1}{2}m(\sqrt{2}g + l\omega^2)$. **[6]**

(ii) When the string has length 0.8 m, calculate the greatest value of ω for which the particle remains in contact with the cone. **[4]**

[Questions 6 and 7 are printed overleaf.]

4

6 A particle A of mass $2m$ is moving with speed u on a smooth horizontal surface when it collides with a stationary particle B of mass m. After the collision the speed of A is v, the speed of B is $3v$ and the particles move in the same direction.

    (i) Find v in terms of u. **[3]**

    (ii) Show that the coefficient of restitution between A and B is $\frac{4}{5}$. **[2]**

B subsequently hits a vertical wall which is perpendicular to the direction of motion. As a result of the impact, B loses $\frac{3}{4}$ of its kinetic energy.

    (iii) Show that the speed of B after hitting the wall is $\frac{3}{5}u$. **[4]**

    (iv) B then hits A. Calculate the speeds of A and B, in terms of u, after this collision and state their directions of motion. **[8]**

7
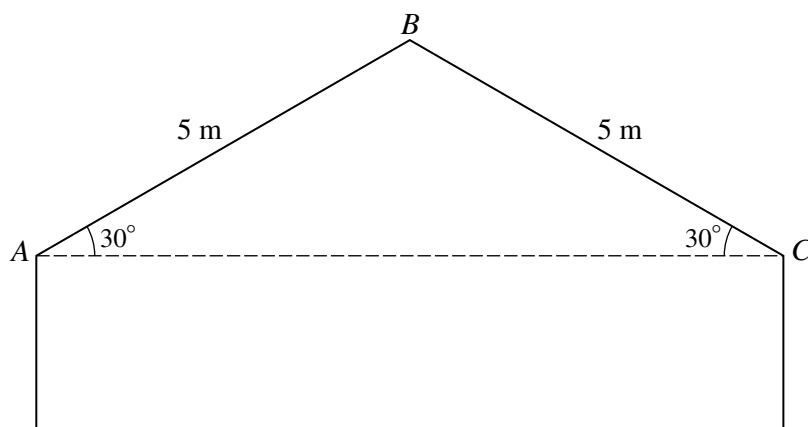
A small ball of mass $0.2\,\text{kg}$ is projected with speed $11\,\text{m s}^{-1}$ up a line of greatest slope of a roof from a point A at the bottom of the roof. The ball remains in contact with the roof and moves up the line of greatest slope to the top of the roof at B. The roof is rough and the coefficient of friction is $\frac{1}{2}$. The distance AB is $5\,\text{m}$ and AB is inclined at $30°$ to the horizontal (see diagram).

    (i) Show that the speed of the ball when it reaches B is $5.44\,\text{m s}^{-1}$, correct to 2 decimal places. **[6]**

The ball leaves the roof at B and moves freely under gravity. The point C is at the lower edge of the roof. The distance BC is $5\,\text{m}$ and BC is inclined at $30°$ to the horizontal.

    (ii) Determine whether or not the ball hits the roof between B and C. **[7]**

2

1

A uniform square frame $ABCD$ has sides of length 0.6 m. The side AD is removed from the frame, and the open frame $ABCD$ is attached at A to a fixed point (see diagram).

   (i) Calculate the distance of the centre of mass of the open frame from A. **[5]**

The open frame rotates about A in the plane $ABCD$ with angular speed $3\,\text{rad s}^{-1}$.

   **(ii)** Calculate the speed of the centre of mass of the open frame. **[2]**

2   The resistance to the motion of a car is $kv^{\frac{3}{2}}\,\text{N}$, where $v\,\text{m s}^{-1}$ is the car's speed and k is a constant. The power exerted by the car's engine is $15\,000\,\text{W}$, and the car has constant speed $25\,\text{m s}^{-1}$ along a horizontal road.

   (i) Show that $k = 4.8$. **[3]**

With the engine operating at a much lower power, the car descends a hill of inclination α, where $\sin\alpha = \frac{1}{15}$. At an instant when the speed of the car is $16\,\text{m s}^{-1}$, its acceleration is $0.3\,\text{m s}^{-2}$.

   (ii) Given that the mass of the car is $700\,\text{kg}$, calculate the power of the engine. **[5]**

3
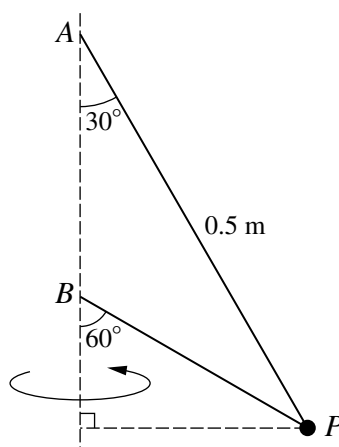
A particle P of mass $0.4\,\text{kg}$ is attached to one end of each of two light inextensible strings which are both taut. The other end of the longer string is attached to a fixed point A, and the other end of the shorter string is attached to a fixed point B, which is vertically below A. The string AP makes an angle of $30°$ with the vertical and is $0.5\,\text{m}$ long. The string BP makes an angle of $60°$ with the vertical. P moves with constant angular speed in a horizontal circle with centre vertically below B (see diagram). The tension in the string AP is twice the tension in the string BP. Calculate

   (i) the tension in each string, **[4]**

   (ii) the angular speed of P. **[4]**

4 A block of mass 25 kg is dragged 30 m up a slope inclined at 5° to the horizontal by a rope inclined at 20° to the slope. The tension in the rope is 100 N and the resistance to the motion of the block is 70 N. The block is initially at rest. Calculate

   **(i)** the work done by the tension in the rope, **[2]**

   (ii) the change in the potential energy of the block, **[2]**

   (iii) the speed of the block after it has moved 30 m up the slope. **[4]**

5 A uniform solid is made of a hemisphere with centre O and radius 0.6 m, and a cylinder of radius 0.6 m and height 0.6 m. The plane face of the hemisphere and a plane face of the cylinder coincide. (The formula for the volume of a sphere is $\frac{4}{3}\pi r^3$.)

   (i) Show that the distance of the centre of mass of the solid from O is 0.09 m. **[5]**

   (ii)
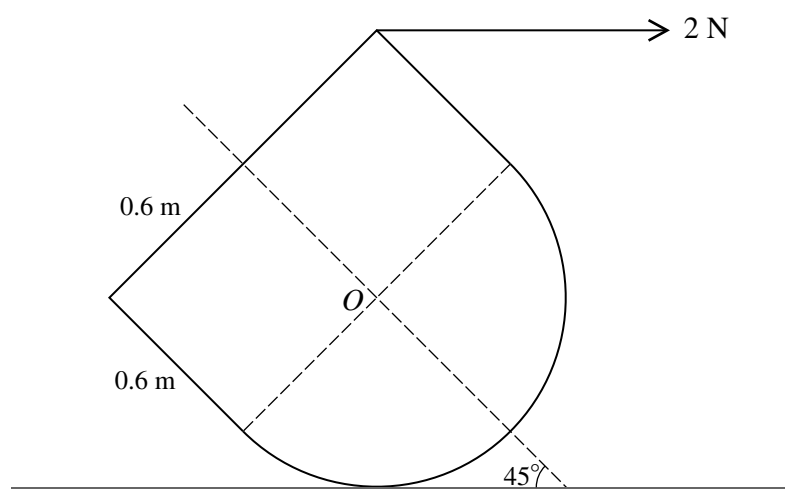
The solid is placed with the curved surface of the hemisphere on a rough horizontal surface and the axis inclined at 45° to the horizontal. The equilibrium of the solid is maintained by a horizontal force of 2 N applied to the highest point on the circumference of its plane face (see diagram). Calculate

   (a) the mass of the solid, **[4]**

   (b) the set of possible values of the coefficient of friction between the surface and the solid. **[3]**

[Questions 6 and 7 are printed overleaf.]

6 A small ball B is projected with speed $14\,\mathrm{m\,s^{-1}}$ at an angle of elevation $30°$ from a point O on a horizontal plane, and moves freely under gravity.

    (i) Calculate the height of B above the plane when moving horizontally. **[2]**

B has mass $0.4\,\mathrm{kg}$. At the instant when B is moving horizontally it receives an impulse of magnitude $I\,\mathrm{N\,s}$ in its direction of motion which immediately increases the speed of B to $15\,\mathrm{m\,s^{-1}}$.

    (ii) Calculate I. **[3]**

For the instant when B returns to the plane, calculate

    (iii) the speed and direction of motion of B, **[4]**

    (iv) the time of flight, and the distance of B from O. **[5]**

7 Three small smooth spheres A, B and C of masses $0.2\,\mathrm{kg}$, $0.7\,\mathrm{kg}$ and $m\,\mathrm{kg}$ respectively are free to move in a straight line on a smooth horizontal table. Initially B and C are stationary and A is moving with velocity $1.8\,\mathrm{m\,s^{-1}}$ directly towards B. The coefficient of restitution for the collision between A and B is e. Immediately after this collision the speed of A is greater than the speed of B.

    (i) Calculate the set of possible values of e. **[9]**

It is now given that the speed of B immediately after the collision with A is $0.75\,\mathrm{m\,s^{-1}}$. B continues its motion and strikes C directly in a perfectly elastic collision. B has speed $0.25\,\mathrm{m\,s^{-1}}$ immediately after its collision with C.

    (ii) Calculate the two possible values of m. **[6]**

1

A sledge with its load has mass 70 kg. It moves down a slope and the resistance to the motion of the sledge is 90 N. The speed of the sledge is controlled by the constant tension in a light rope, which is attached to the sledge and parallel to the slope (see diagram). While travelling 20 m down the slope, the speed of the sledge decreases from $2.1\,\text{m s}^{-1}$ to $1.4\,\text{m s}^{-1}$ and it descends a vertical distance of 3 m.

  (i) Calculate the change in energy of the sledge and its load. **[4]**

  (ii) Calculate the tension in the rope. **[3]**

2   A car of mass 1250 kg travels along a straight road inclined at $2°$ to the horizontal. The resistance to the motion of the car is $kv\,\text{N}$, where $v\,\text{m s}^{-1}$ is the speed of the car and k is a constant. The car travels at a constant speed of $25\,\text{m s}^{-1}$ up the slope and the engine of the car works at a constant rate of 21 kW.

  (i) Calculate the value of k. **[4]**

  (ii) Calculate the constant speed of the car on a horizontal road. **[3]**

3   A uniform lamina $ABCDE$ consists of a square $ACDE$ and an equilateral triangle ABC which are joined along their common edge AC to form a pentagon whose sides are each 8 cm in length.

  (i) Calculate the distance of the centre of mass of the lamina from AC. **[5]**

  (ii) The lamina is freely suspended from A and hangs in equilibrium. Calculate the angle that AC makes with the vertical. **[2]**

4   Two small spheres A and B are moving towards each other along a straight line on a smooth horizontal surface. A has speed $3\,\text{m s}^{-1}$ and B has speed $1.5\,\text{m s}^{-1}$ before they collide directly. The direction of motion of B is reversed in the collision. The speeds of A and B after the collision are $2\,\text{m s}^{-1}$ and $2.9\,\text{m s}^{-1}$ respectively.

  (i) (a) Show that the direction of motion of A is unchanged by the collision. **[2]**

      (b) Calculate the coefficient of restitution between A and B. **[2]**

The mass of B is 0.2 kg.

  (ii) Find the mass of A. **[3]**

B continues to move at $2.9\,\text{m s}^{-1}$ and strikes a vertical wall at right angles. The wall exerts an impulse of magnitude 0.68 N s on B.

  (iii) Calculate the coefficient of restitution between B and the wall. **[4]**

5 A particle is projected with speed $7\,\text{m s}^{-1}$ at an angle of elevation of $30°$ from a point O and moves freely under gravity. The horizontal and vertically upwards displacements of the particle from O at any subsequent time $t\,\text{s}$ are $x\,\text{m}$ and $y\,\text{m}$ respectively.

  (i) Express x and y in terms of t and hence find the equation of the trajectory of the particle. **[4]**

  (ii) Calculate the values of x when $y = 0.6$. **[4]**

  (iii) Find the direction of motion of the particle when $y = 0.6$ and the particle is rising. **[4]**
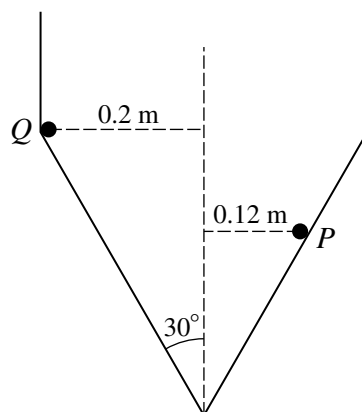
6

Fig. 1

A container is constructed from a hollow cylindrical shell and a hollow cone which are joined along their circumferences. The cylindrical shell has radius $0.2\,\text{m}$, and the cone has semi-vertical angle $30°$. Two identical small spheres P and Q move independently in horizontal circles on the smooth inner surface of the container (see Fig. 1). Each sphere has mass $0.3\,\text{kg}$.

  (i) P moves in a circle of radius $0.12\,\text{m}$ and is in contact with only the conical part of the container. Calculate the angular speed of P. **[5]**
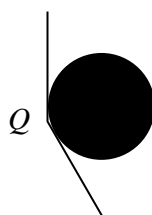
  (ii)

Fig. 2

Q moves with speed $2.1\,\text{m s}^{-1}$ and is in contact with both the cylindrical and conical surfaces of the container (see Fig. 2). Calculate the magnitude of the force which the cylindrical shell exerts on the sphere. **[4]**

  (iii) Calculate the difference between the mechanical energy of P and of Q. **[5]**

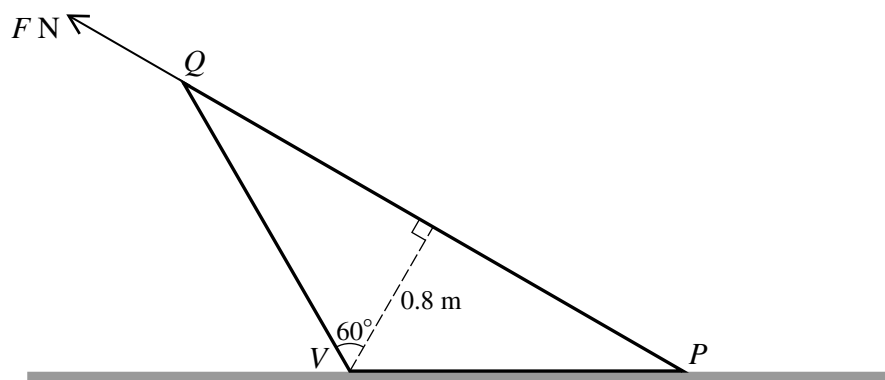**[Question 7 is printed overleaf.]**

7

Fig. 1

A uniform solid cone of height $0.8\,\text{m}$ and semi-vertical angle $60°$ lies with its curved surface on a horizontal plane. The point P on the circumference of the base is in contact with the plane. V is the vertex of the cone and PQ is a diameter of its base. The weight of the cone is $550\,\text{N}$. A force of magnitude $F\,\text{N}$ and line of action PQ is applied to the base of the cone (see Fig. 1). The cone topples about V without sliding.

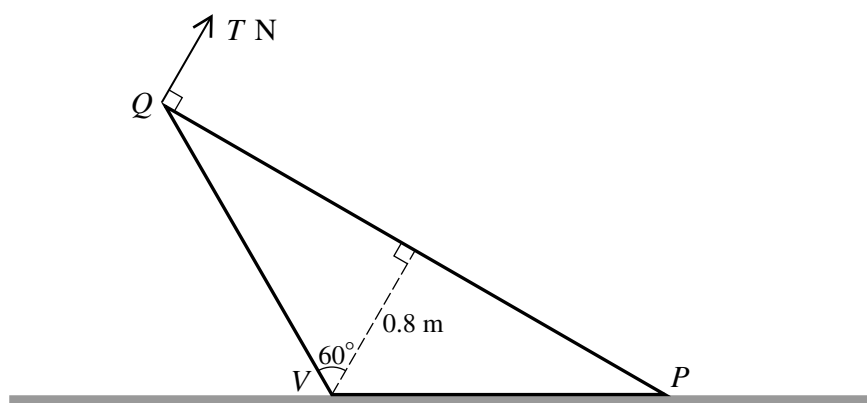 **(i)** Calculate the least possible value of F. **[4]**

Fig. 2

The force of magnitude $F\,\text{N}$ is removed and an increasing force of magnitude $T\,\text{N}$ acting upwards in the vertical plane of symmetry of the cone and perpendicular to PQ is applied to the cone at Q (see Fig. 2). The coefficient of friction between the cone and the horizontal plane is μ.

  (ii) Given that the cone slides before it topples about P, calculate the greatest possible value for μ. **[10]**

1   A particle P is projected with speed $40\,\text{m s}^{-1}$ at an angle of $35°$ above the horizontal from a point O. For the instant $3\,\text{s}$ after projection, calculate the magnitude and direction of the velocity of P. **[5]**
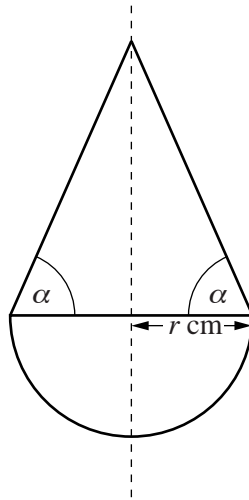
2

Fig. 1

A child's toy is a uniform solid consisting of a hemisphere of radius $r\,\text{cm}$ joined to a cone of base radius $r\,\text{cm}$. The curved surface of the cone makes an angle α with its base. The two shapes are joined at the plane faces with their circumferences coinciding (see Fig. 1). The distance of the centre of mass of the toy above the common circular plane face is $x\,\text{cm}$.

[The volume of a sphere is $\frac{4}{3}\pi r^3$ and the volume of a cone is $\frac{1}{3}\pi r^2 h$.]

    (i) Show that $x = \dfrac{r(\tan^2 \alpha - 3)}{8 + 4\tan \alpha}.$ **[4]**

The toy is placed on a horizontal surface with the hemisphere in contact with the surface. The toy is released from rest from the position in which the common plane circular face is vertical (see Fig. 2).
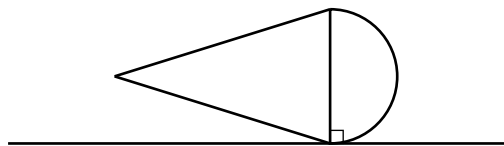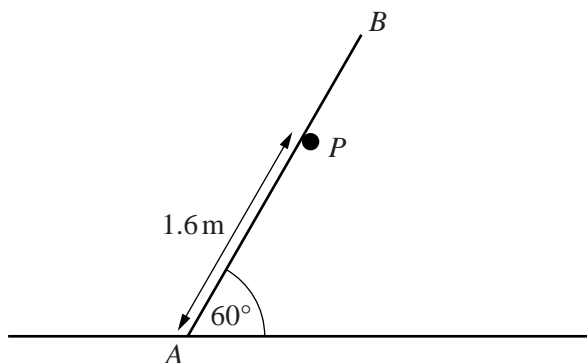

Fig. 2

    (ii) Find the set of values of α such that the toy moves to the upright position. **[3]**

3

A uniform rod AB of mass 10 kg and length 2.4 m rests with A on rough horizontal ground. The rod makes an angle of 60° with the horizontal and is supported by a fixed smooth peg P. The distance AP is 1.6 m (see diagram).

  (i) Calculate the magnitude of the force exerted by the peg on the rod.   **[3]**

 (ii) Find the least value of the coefficient of friction between the rod and the ground needed to maintain equilibrium.   **[5]**

4   A particle P of mass 0.2 kg is attached to one end of a light inextensible string of length 1.2 m. The other end of the string is fixed at a point A which is 0.6 m above a smooth horizontal table. P moves on the table in a circular path whose centre O is vertically below A.

  (i) Given that the angular speed of P is 2.5 rad s^{-1}, find

    (a) the tension in the string,   **[4]**

    (b) the normal reaction between the particle and the table.   **[3]**

 (ii) Find the greatest possible speed of P, given that the particle remains in contact with the table.   **[5]**

5   A car of mass 1500 kg travels up a line of greatest slope of a straight road inclined at 5° to the horizontal. The power of the car's engine is constant and equal to 25 kW and the resistance to the motion of the car is constant and equal to 750 N. The car passes through point A with speed 10 m s^{-1}.

  (i) Find the acceleration of the car at A.   **[5]**

The car later passes through a point B with speed 20 m s^{-1}. The car takes 28 s to travel from A to B.

 (ii) Find the distance AB.   **[7]**

[Questions 6 and 7 are printed overleaf.]

6   A small ball of mass $0.5\,$kg is held at a height of $3.136\,$m above a horizontal floor. The ball is released from rest and rebounds from the floor. The coefficient of restitution between the ball and floor is e.

    (i) Find in terms of e the speed of the ball immediately after the impact with the floor and the impulse that the floor exerts on the ball.   **[4]**

    The ball continues to bounce until it eventually comes to rest.

    (ii) Show that the time between the first bounce and the second bounce is $1.6e$.   **[2]**

    (iii) Write down, in terms of e, the time between

        (a) the second bounce and the third bounce,

        (b) the third bounce and the fourth bounce.   **[2]**

    (iv) Given that the time from the ball being released until it comes to rest is $5\,$s, find the value of e.   **[5]**

7   A particle P is projected horizontally with speed $15\,\text{m\,s}^{-1}$ from the top of a vertical cliff. At the same instant a particle Q is projected from the bottom of the cliff, with speed $25\,\text{m\,s}^{-1}$ at an angle of $\theta°$ above the horizontal. P and Q move in the same vertical plane. The height of the cliff is $60\,$m and the ground at the bottom of the cliff is horizontal.

    (i) Given that the particles hit the ground simultaneously, find the value of θ and find also the distance between the points of impact with the ground.   **[6]**

    (ii) Given instead that the particles collide, find the value of θ, and determine whether Q is rising or falling immediately before this collision.   **[9]**

1 A particle, of mass 0.8 kg, moves along a smooth horizontal surface. It hits a vertical wall, which is at right angles to the direction of motion of the particle, and rebounds. The speed of the particle as it hits the wall is $4\,\mathrm{m\,s^{-1}}$ and the coefficient of restitution between the particle and the wall is 0.3. Find

   (i) the impulse that the wall exerts on the particle, **[3]**

   **(ii)** the kinetic energy lost in the impact. **[2]**

2 A car of mass 1600 kg moves along a straight horizontal road. The resistance to the motion of the car has constant magnitude 800 N and the car's engine is working at a constant rate of 20 kW.

   (i) Find the acceleration of the car at an instant when the car's speed is $20\,\mathrm{m\,s^{-1}}$. **[4]**

The car now moves up a hill inclined at 4° to the horizontal. The car's engine continues to work at 20 kW and the magnitude of the resistance to motion remains at 800 N.

   (ii) Find the greatest steady speed at which the car can move up the hill. **[4]**

3
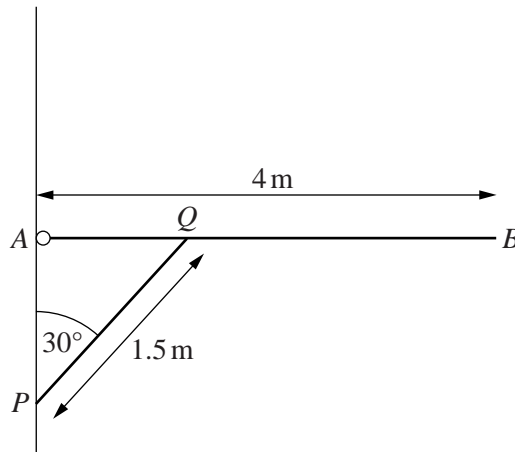
A uniform beam *AB* of mass 15 kg and length 4 m is freely hinged to a vertical wall at *A*. The beam is held in equilibrium in a horizontal position by a light rod *PQ* of length 1.5 m. *P* is fixed to the wall vertically below *A* and *PQ* makes an angle of 30° with the vertical (see diagram). The force exerted on the beam at *Q* by the rod is in the direction *PQ*. Find

   (i) the magnitude of the force exerted on the beam at *Q*, **[3]**

   (ii) the magnitude and direction of the force exerted on the beam at *A*. **[6]**

4   A boy throws a small ball at a vertical wall. The ball is thrown horizontally, from a point O, at a speed of $14.4\,\text{m s}^{-1}$ and it hits the wall at a point which is $0.2\,\text{m}$ below the level of O.

  (i) Find the horizontal distance from O to the wall.   **[4]**

The boy now moves so that he is $6\,\text{m}$ from the wall. He throws the ball at an angle of $15°$ above the horizontal. The ball again hits the wall at a point which is $0.2\,\text{m}$ below the level from which it was thrown.

  (ii) Find the speed at which the ball was thrown.   **[6]**

5   A particle P, of mass $2\,\text{kg}$, is attached to fixed points A and B by light inextensible strings, each of length $2\,\text{m}$. A and B are $3.2\,\text{m}$ apart with A vertically above B. The particle P moves in a horizontal circle with centre at the mid-point of AB.

  (i) Find the tension in each string when the angular speed of P is $4\,\text{rad s}^{-1}$.   **[7]**

  (ii) Find the least possible speed of P.   **[6]**

6   Three particles A, B and C are in a straight line on a smooth horizontal surface. The particles have masses $0.2\,\text{kg}$, $0.4\,\text{kg}$ and $0.6\,\text{kg}$ respectively. B is at rest. A is projected towards B with a speed of $1.8\,\text{m s}^{-1}$ and collides with B. The coefficient of restitution between A and B is $\frac{1}{3}$.
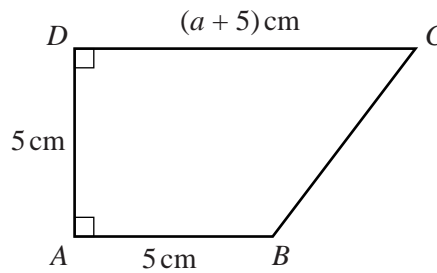
  (i) Show that the speed of B after the collision is $0.8\,\text{m s}^{-1}$ and find the speed of A after the collision.   **[6]**

C is moving with speed $0.2\,\text{m s}^{-1}$ in the same direction as B. Particle B subsequently collides with C. The coefficient of restitution between B and C is e.

  (ii) Find the set of values for e such that B does not collide again with A.   **[7]**

[Question 7 is printed overleaf.]

7

The diagram shows the cross-section through the centre of mass of a uniform solid prism. The cross-section is a trapezium $ABCD$ with AB and CD perpendicular to AD. The lengths of AB and AD are each $5\,\text{cm}$ and the length of CD is $(a + 5)\,\text{cm}$.

(i) Show the distance of the centre of mass of the prism from AD is

$$\frac{a^2 + 15a + 75}{3(a + 10)}\ \text{cm}.$$ **[5]**

The prism is placed with the face containing AB in contact with a horizontal surface.

(ii) Find the greatest value of a for which the prism does not topple. **[3]**

The prism is now placed on an inclined plane which makes an angle $\theta°$ with the horizontal. AB lies along a line of greatest slope with B higher than A.

(iii) Using the value for a found in part **(ii)**, and assuming the prism does not slip down the plane, find the greatest value of θ for which the prism does not topple. **[6]**

1 A block is being pushed in a straight line along horizontal ground by a force of $18\,\text{N}$ inclined at $15°$ below the horizontal. The block moves a distance of $6\,\text{m}$ in $5\,\text{s}$ with constant speed. Find

  (i) the work done by the force, **[3]**

  (ii) the power with which the force is working. **[2]**

2 A car of mass $1500\,\text{kg}$ travels along a straight horizontal road. The resistance to the motion of the car is $kv^{\frac{1}{2}}\,\text{N}$, where $v\,\text{m s}^{-1}$ is the speed of the car and k is a constant. At the instant when the engine produces a power of $15\,000\,\text{W}$, the car has speed $15\,\text{m s}^{-1}$ and is accelerating at $0.4\,\text{m s}^{-2}$.

  (i) Find the value of k. **[4]**

It is given that the greatest steady speed of the car on this road is $30\,\text{m s}^{-1}$.

  (ii) Find the greatest power that the engine can produce. **[3]**

3 A particle A is released from rest from the top of a smooth plane, which makes an angle of $30°$ with the horizontal. The particle A collides $2\,\text{s}$ later with a particle B, which is moving up a line of greatest slope of the plane. The coefficient of restitution between the particles is 0.4 and the speed of B immediately before the collision is $2\,\text{m s}^{-1}$. B has velocity $1\,\text{m s}^{-1}$ down the plane immediately after the collision. Find

  (i) the speed of A immediately after the collision, **[4]**

  (ii) the distance A moves up the plane after the collision. **[2]**

The masses of A and B are $0.5\,\text{kg}$ and $m\,\text{kg}$, respectively.

  (iii) Find the value of m. **[3]**

4
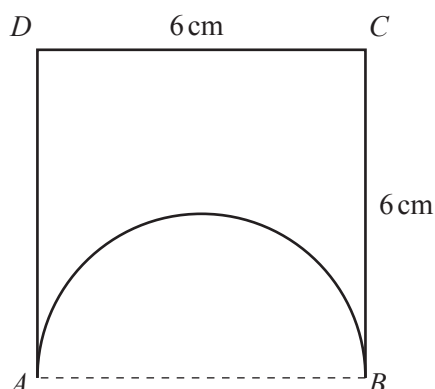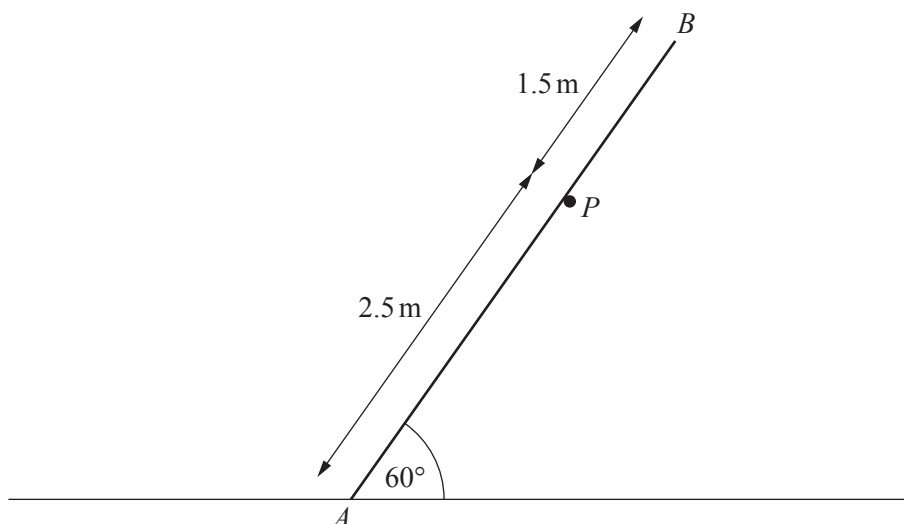

A uniform square lamina $ABCD$ of side $6\,\text{cm}$ has a semicircular piece, with AB as diameter, removed (see diagram).

  (i) Find the distance of the centre of mass of the remaining shape from CD. **[6]**

The remaining shape is suspended from a fixed point by a string attached at C and hangs in equilibrium.

  (ii) Find the angle between CD and the vertical. **[2]**

5

A uniform rod AB, of mass $3\,\text{kg}$ and length $4\,\text{m}$, is in limiting equilibrium with A on rough horizontal ground. The rod is at an angle of $60°$ to the horizontal and is supported by a small smooth peg P, such that the distance AP is $2.5\,\text{m}$ (see diagram). Find

   (i) the force acting on the rod at P,   **[3]**

   (ii) the coefficient of friction between the ground and the rod.   **[5]**

6   A particle of mass $0.5\,\text{kg}$ is held at rest at a point P, which is at the bottom of an inclined plane. The particle is given an impulse of $1.8\,\text{N s}$ directed up a line of greatest slope of the plane.

   (i) Find the speed at which the particle starts to move.   **[2]**

The particle subsequently moves up the plane to a point Q, which is $0.3\,\text{m}$ above the level of P.

   (ii) Given that the plane is smooth, find the speed of the particle at Q.   **[4]**

It is given instead that the plane is rough. The particle is now projected up the plane from P with initial speed $3\,\text{m s}^{-1}$, and comes to rest at a point R which is $0.2\,\text{m}$ above the level of P.

   (iii) Given that the plane is inclined at $30°$ to the horizontal, find the magnitude of the frictional force on the particle.   **[4]**

7 A particle is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle of θ above the horizontal from a point O. At time t s after projection, the horizontal and vertically upwards displacements of the particle from O are $x\,\mathrm{m}$ and $y\,\mathrm{m}$ respectively.

   (i) Express x and y in terms of t and θ and hence obtain the equation of trajectory
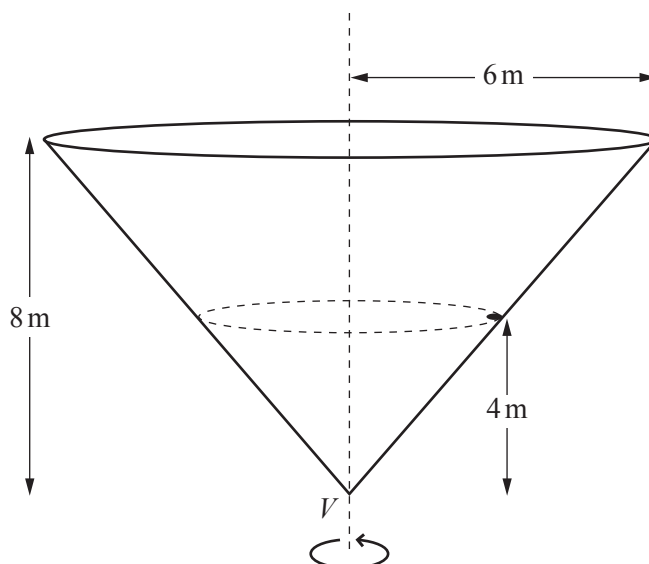
$$y = x\tan\theta - \frac{gx^2\sec^2\theta}{2u^2}.$$

[4]

In a shot put competition, a shot is thrown from a height of $2.1\,\mathrm{m}$ above horizontal ground. It has initial velocity of $14\,\mathrm{m\,s^{-1}}$ at an angle of θ above the horizontal. The shot travels a horizontal distance of $22\,\mathrm{m}$ before hitting the ground.

   (ii) Show that $12.1\tan^2\theta - 22\tan\theta + 10 = 0$, and find the value of θ. **[5]**

   (iii) Find the time of flight of the shot. **[2]**

8

A conical shell has radius $6\,\mathrm{m}$ and height $8\,\mathrm{m}$. The shell, with its vertex V downwards, is rotating about its vertical axis. A particle, of mass $0.4\,\mathrm{kg}$, is in contact with the rough inner surface of the shell. The particle is $4\,\mathrm{m}$ above the level of V (see diagram). The particle and shell rotate with the same constant angular speed. The coefficient of friction between the particle and the shell is μ.

   (i) The frictional force on the particle is $F\,\mathrm{N}$, and the normal force of the shell on the particle is $R\,\mathrm{N}$. It is given that the speed of the particle is $4.5\,\mathrm{m\,s^{-1}}$, which is the smallest possible speed for the particle not to slip.

   (a) By resolving vertically, show that $4F + 3R = 19.6$. **[2]**

   (b) By finding another equation connecting F and R, find the values of F and R and show that $\mu = 0.336$, correct to 3 significant figures. **[6]**

   (ii) Find the largest possible angular speed of the shell for which the particle does not slip. **[6]**

1   *A* and *B* are two points on a line of greatest slope of a smooth inclined plane, with *B* a vertical distance of 8 m below the level of *A*. A particle of mass 0.75 kg is projected down the plane from *A* with a speed of $2 \, \text{m} \, \text{s}^{-1}$. Find

       (i) the loss in potential energy of the particle as it moves from *A* to *B*,   **[2]**

   **(ii)** the speed of the particle when it reaches *B*.   **[4]**

2   The power developed by the engine of a car as it travels at a constant speed of $32 \, \text{m} \, \text{s}^{-1}$ on a horizontal road is 20 kW.

       (i) Calculate the resistance to the motion of the car.   **[3]**

    The car, of mass 1500 kg, now travels down a straight road inclined at 2° to the horizontal. The resistance to the motion of the car is unchanged.

       (ii) Find the power produced by the engine of the car when the car has speed $32 \, \text{m} \, \text{s}^{-1}$ and is accelerating at $0.1 \, \text{m} \, \text{s}^{-2}$.   **[4]**

3
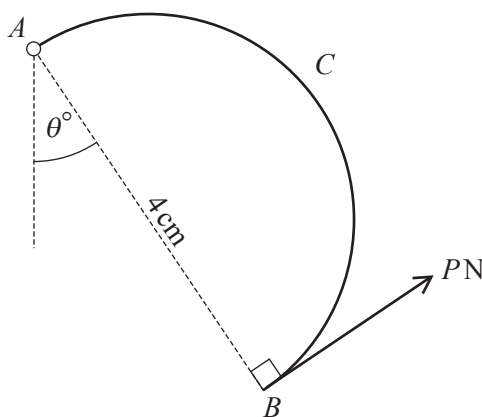
    A uniform semicircular arc *ACB* is freely pivoted at *A*. The arc has mass 0.3 kg and is held in equilibrium by a force of magnitude *P* N applied at *B*. The line of action of this force lies in the same plane as the arc, and is perpendicular to *AB*. The diameter *AB* has length 4 cm and makes an angle of $\theta°$ with the downward vertical (see diagram).

       (i) Given that $\theta = 0$, find the magnitude of the force acting on the arc at *A*.   **[6]**

       (ii) Given instead that $\theta = 30$, find the value of *P*.   **[4]**

4 A solid uniform cone has height $8\,\text{cm}$, base radius $5\,\text{cm}$ and mass $4\,\text{kg}$. A uniform conical shell has height $10\,\text{cm}$, base radius $5\,\text{cm}$ and mass $0.4\,\text{kg}$. The two shapes are joined together so that the circumferences of their circular bases coincide.

    (i) Find the distance of the centre of mass of the shape from the common circular base. **[4]**


$$8\,\text{cm} \qquad 10\,\text{cm}$$
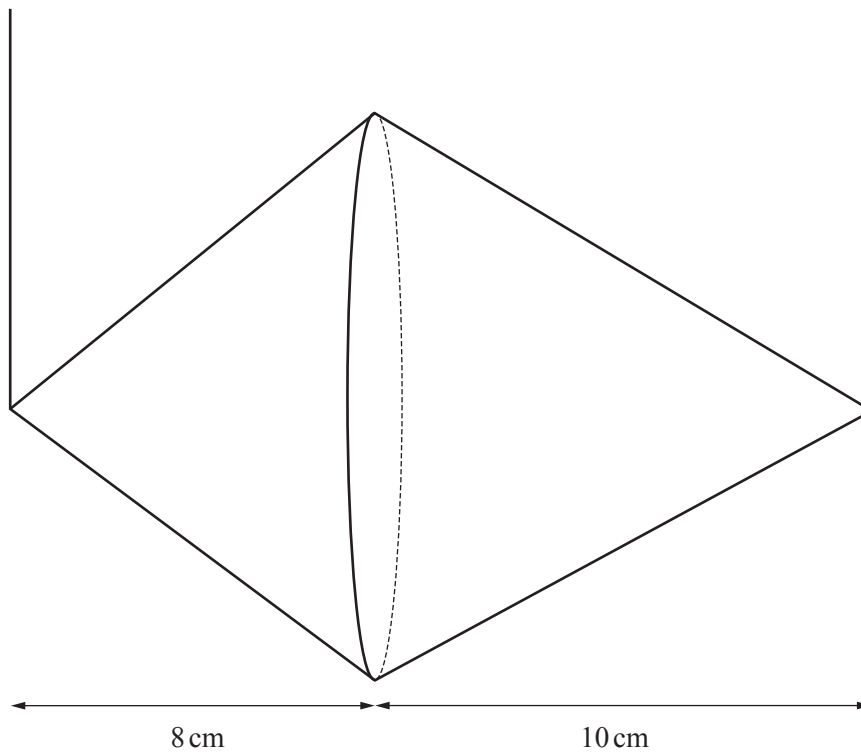
The object is suspended with a string attached to the vertex of the cone and another string attached to the vertex of the conical shell. The object is in equilibrium with the strings vertical and the axis of symmetry of the object horizontal (see diagram).

    (ii) Find the tension in each string. **[4]**

5 A vertical hollow cylinder of radius $0.4\,\text{m}$ is rotating about its axis. A particle P is in contact with the rough inner surface of the cylinder. The cylinder and P rotate with the same constant angular speed. The coefficient of friction between P and the cylinder is μ.

   (i) Given that the angular speed of the cylinder is $7\,\text{rad}\,\text{s}^{-1}$ and P is on the point of moving downwards, find the value of μ. **[5]**

The particle is now attached to one end of a light inextensible string of length $0.5\,\text{m}$. The other end is fixed to a point A on the axis of the cylinder (see diagram).
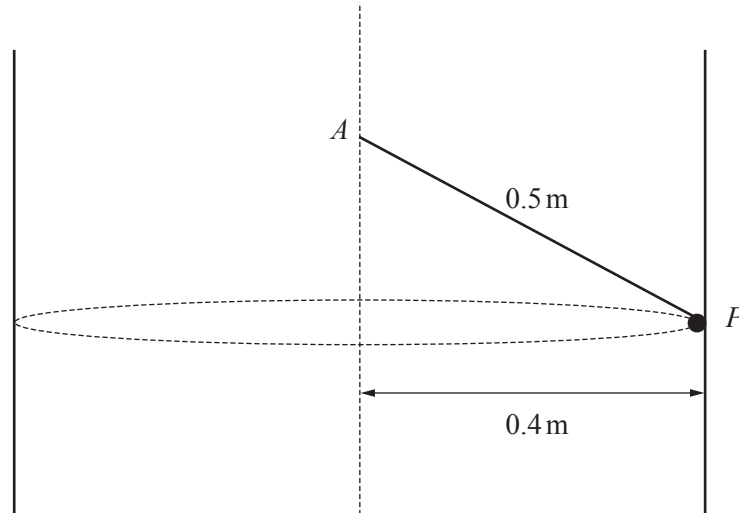

   (ii) Find the angular speed for which the contact force between P and the cylinder becomes zero. **[5]**

6
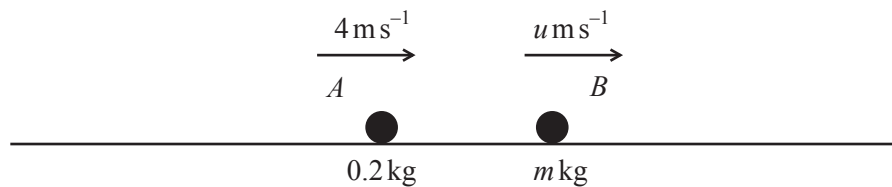
The masses of two particles A and B are $0.2\,\text{kg}$ and $m\,\text{kg}$ respectively. The particles are moving with constant speeds $4\,\text{m}\,\text{s}^{-1}$ and $u\,\text{m}\,\text{s}^{-1}$ in the same horizontal line and in the same direction (see diagram). The two particles collide and the coefficient of restitution between the particles is e. After the collision, A and B continue in the same direction with speeds $4(1 - e + e^2)\,\text{m}\,\text{s}^{-1}$ and $4\,\text{m}\,\text{s}^{-1}$ respectively.

   (i) Find u and m in terms of e. **[6]**

  (ii) Find the value of e for which the speed of A after the collision is least and find, in this case, the total loss in kinetic energy due to the collision. **[5]**

  (iii) Find the possible values of e for which the magnitude of the impulse that B exerts on A is $0.192\,\text{N}\,\text{s}$. **[4]**

7
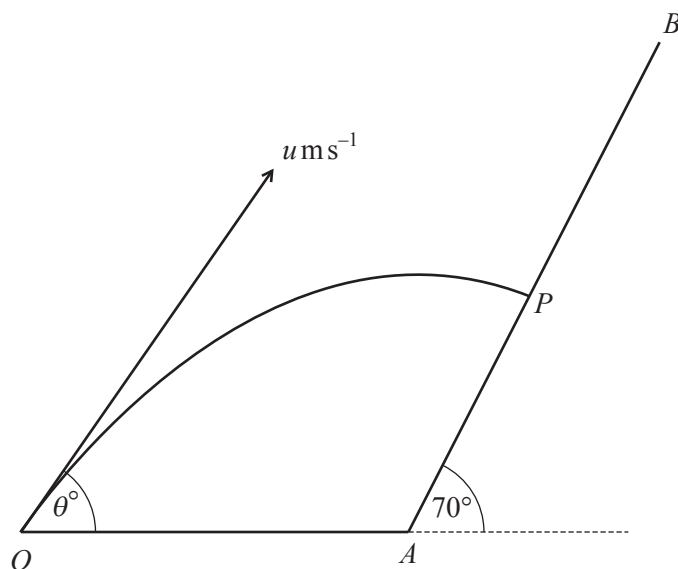
The diagram shows a surface consisting of a horizontal part OA and a plane AB inclined at an angle of $70°$ to the horizontal. A particle is projected from the point O with speed $u\,\mathrm{m\,s}^{-1}$ at an angle of $\theta°$ above the horizontal OA. The particle hits the plane AB at the point P, with speed $14\,\mathrm{m\,s}^{-1}$ and at right angles to the plane, $1.4\,\mathrm{s}$ after projection.

  (i) Show that the value of u is 15.9, correct to 3 significant figures, and find the value of θ.   **[7]**

 (ii) Find the height of P above the level of A.   **[3]**

The particle rebounds with speed $v\,\mathrm{m\,s}^{-1}$. The particle next lands at A.

(iii) Find the value of v.   **[5]**

(iv) Find the coefficient of restitution between the particle and the plane at P.   **[1]**

1 A football is kicked from horizontal ground with speed $20\,\mathrm{m\,s^{-1}}$ at an angle of $\theta°$ above the horizontal. The greatest height the football reaches above ground level is $2.44\,\mathrm{m}$. By modelling the football as a particle and ignoring air resistance, find

    (i) the value of θ, **[2]**

   **(ii)** the range of the football. **[2]**

2 A uniform solid cylinder of height $12\,\mathrm{cm}$ and radius $r\,\mathrm{cm}$ is in equilibrium on a rough inclined plane with one of its circular faces in contact with the plane.

    (i) The cylinder is on the point of toppling when the angle of inclination of the plane to the horizontal is $21°$. Find r. **[3]**

The cylinder is now placed on a different inclined plane with one of its circular faces in contact with the plane. This plane is also inclined at $21°$ to the horizontal. The coefficient of friction between this plane and the cylinder is μ.

    (ii) The cylinder slides down this plane but does not topple. Find an inequality for μ. **[2]**
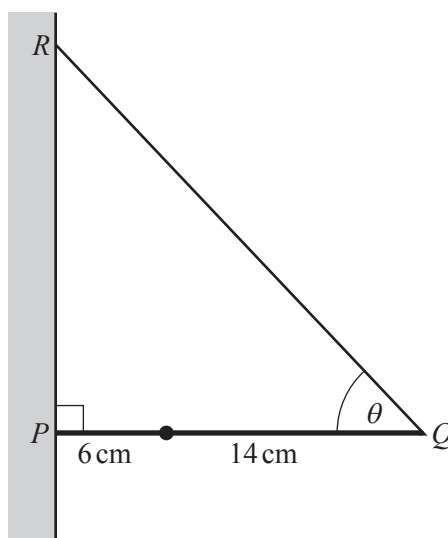
3
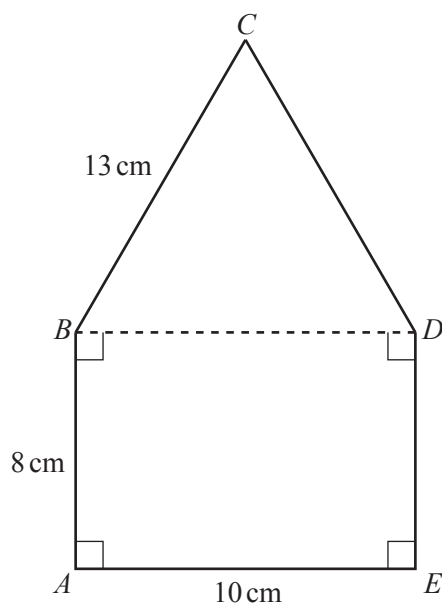
A uniform lamina $ABCDE$ consists of a rectangle $ABDE$ and an isosceles triangle BCD joined along their common edge. $AB = DE = 8\,\mathrm{cm}$, $AE = BD = 10\,\mathrm{cm}$ and $BC = CD = 13\,\mathrm{cm}$ (see diagram).

    (i) Find the distance of the centre of mass of the lamina from AE. **[5]**

   **(ii)** The lamina is freely suspended from B and hangs in equilibrium. Calculate the angle that BD makes with the vertical. **[3]**

4

A uniform rod PQ has weight $18\,\text{N}$ and length $20\,\text{cm}$. The end P rests against a rough vertical wall. A particle of weight $3\,\text{N}$ is attached to the rod at a point $6\,\text{cm}$ from P. The rod is held in a horizontal position, perpendicular to the wall, by a light inextensible string attached to the rod at Q and to a point R on the wall vertically above P, as shown in the diagram. The string is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{3}{5}$. The system is in limiting equilibrium.

(i) Find the tension in the string. **[3]**

(ii) Find the magnitude of the force exerted by the wall on the rod. **[4]**

(iii) Find the coefficient of friction between the wall and the rod. **[2]**

5 **(i)** A car of mass $800\,\text{kg}$ is moving at a constant speed of $20\,\text{m s}^{-1}$ on a straight road down a hill inclined at an angle α to the horizontal. The engine of the car works at a constant rate of $10\,\text{kW}$ and there is a resistance to motion of $1300\,\text{N}$. Show that $\sin\alpha = \frac{5}{49}$. **[4]**

(ii) The car now travels up the same hill and its engine now works at a constant rate of $20\,\text{kW}$. The resistance to motion remains $1300\,\text{N}$. The car starts from rest and its speed is $8\,\text{m s}^{-1}$ after it has travelled a distance of $22.1\,\text{m}$. Calculate the time taken by the car to travel this distance. **[5]**

6 Two small spheres A and B, of masses $2m\,\text{kg}$ and $3m\,\text{kg}$ respectively, are moving in opposite directions along the same straight line towards each other on a smooth horizontal surface. A has speed $4\,\text{m s}^{-1}$ and B has speed $2\,\text{m s}^{-1}$ before they collide. The coefficient of restitution between A and B is 0.4.

(i) Find the speed of each sphere after the collision. **[6]**

(ii) Find, in terms of m, the loss of kinetic energy during the collision. **[4]**

(iii) Given that the magnitude of the impulse exerted on A by B during the collision is $2.52\,\text{N s}$, find m. **[3]**
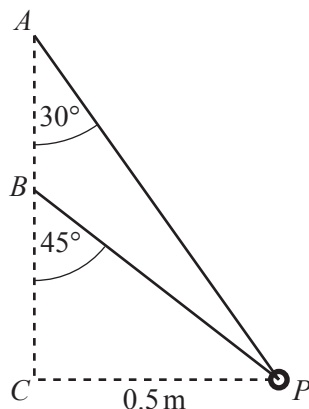
A small smooth ring P of mass 0.4 kg is threaded onto a light inextensible string fixed at A and B as shown in the diagram, with A vertically above B. The string is inclined to the vertical at angles of 30° and 45° at A and B respectively. P moves in a horizontal circle of radius 0.5 m about a point C vertically below B.

(i) Calculate the tension in the string. **[3]**

(ii) Calculate the speed of P. **[3]**

The end of the string at B is moved so both ends of the string are now fixed at A.

(iii) Show that, when the string is taut, AP is now 0.854 m correct to 3 significant figures. **[2]**

P moves in a horizontal circle with angular speed $3.46\,\text{rad s}^{-1}$.

(iv) Find the tension in the string and the angle that the string now makes with the vertical. **[4]**

8   A child is trying to throw a small stone to hit a target painted on a vertical wall. The child and the wall are on horizontal ground. The child is standing a horizontal distance of 8 m from the base of the wall. The child throws the stone from a height of 1 m with speed $12\,\text{m s}^{-1}$ at an angle of 20° above the horizontal.

(i) Find the direction of motion of the stone when it hits the wall. **[6]**

The child now throws the stone with a speed of $V\,\text{m s}^{-1}$ from the same initial position and still at an angle of 20° above the horizontal. This time the stone hits the target which is 2.5 m above the ground.

(ii) Find V. **[6]**