

Mark Scheme 4753
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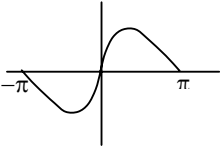
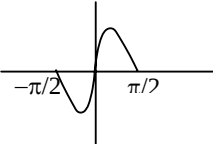
Section A

1 $y = (1 + 6x)^{1/3}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{3}(1 + 6x)^{-2/3} \cdot 6$ $= 2(1 + 6x)^{-2/3}$ $= 2[(1 + 6x)^{1/3}]^{-2}$ $= \frac{2}{y^2} *$	M1 B1 A1 E1	Chain rule $\frac{1}{3}(1 + 6x)^{-2/3}$ or $\frac{1}{3}u^{-2/3}$ any correct expression for the derivative www
or $y^3 = 1 + 6x$ $\Rightarrow x = (y^3 - 1)/6$ $\Rightarrow dx/dy = 3y^2/6 = y^2/2$ $\Rightarrow dy/dx = 1/(dx/dy) = 2/y^2 *$	M1 A1 B1 E1	Finding x in terms of y $y^2/2$ o.e.
or $y^3 = 1 + 6x$ $\Rightarrow 3y^2 dy/dx = 6$ $\Rightarrow dy/dx = 6/3y^2 = 2/y^2 *$	M1 A1 A1 E1 [4]	together with attempt to differentiate implicitly $3y^2 dy/dx = 6$
2 (i) When $t = 0$, $P = 5 + a = 8$ $\Rightarrow a = 3$ When $t = 1$, $5 + 3e^{-b} = 6$ $\Rightarrow e^{-b} = 1/3$ $\Rightarrow -b = \ln 1/3$ $\Rightarrow b = \ln 3 = 1.10$ (3 s.f.) (ii) 5 million	M1 A1 M1 M1 A1ft B1 [6]	substituting $t = 0$ into equation Forming equation using their a Taking lns on correct re-arrangement (ft their a) or $P = 5$
3 (i) $\ln(3x^2)$ (ii) $\ln 3x^2 = \ln(5x + 2)$ $\Rightarrow 3x^2 = 5x + 2$ $\Rightarrow 3x^2 - 5x - 2 = 0*$ (iii) $(3x + 1)(x - 2) = 0$ $\Rightarrow x = -1/3$ or 2 $x = -1/3$ is not valid as $\ln(-1/3)$ is not defined	B1 B1 M1 E1 M1 A1cao B1ft [7]	$2\ln x = \ln x^2$ $\ln x^2 + \ln 3 = \ln 3x^2$ Anti-logging Factorising or quadratic formula ft on one positive and one negative root

4 (i) $\frac{dV}{dt} = 2$ (ii) $\tan 30 = 1/\sqrt{3}$ $= r/h$ $\Rightarrow h = \sqrt{3} r$ $\Rightarrow V = \frac{1}{3} \pi r^2 \cdot \sqrt{3} r = \frac{\sqrt{3}}{3} \pi r^3 *$ $\frac{dV}{dr} = \sqrt{3} \pi r^2$ (iii) When $r = 2$, $dV/dr = 4\sqrt{3}\pi$ $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$ $\Rightarrow 2 = 4\sqrt{3}\pi \frac{dr}{dt}$ $\Rightarrow \frac{dr}{dt} = 1/(2\sqrt{3}\pi)$ or 0.092 cm s^{-1}	B1 M1 E1 B1 M1 M1 A1cao [7]	Correct relationship between r and h in any form From exact working only o.e. e.g. $(3\sqrt{3}/3)\pi r^2$ or $\frac{dr}{dt} = \frac{dr}{dV} \cdot \frac{dV}{dt}$ substituting 2 for dV/dt and r = 2 into their dV/dr
5(i) $y^3 = 2xy + x^2$ $\Rightarrow 3y^2 \frac{dy}{dx} = 2x \frac{dy}{dx} + 2y + 2x$ $\Rightarrow (3y^2 - 2x) \frac{dy}{dx} = 2y + 2x$ $\Rightarrow \frac{dy}{dx} = \frac{2(x+y)}{3y^2 - 2x} *$ (ii) $\frac{dx}{dy} = \frac{3y^2 - 2x}{2(x+y)}$	B1 B1 M1 E1 B1cao [5]	$3y^2 \frac{dy}{dx} =$ $2x \frac{dy}{dx} + 2y + 2x$ collecting dy/dx terms on one side www
6(i) $y = 1 + 2\sin x \quad y \leftrightarrow x$ $\Rightarrow x = 1 + 2\sin y$ $\Rightarrow x - 1 = 2 \sin y$ $\Rightarrow (x - 1)/2 = \sin y$ $\Rightarrow y = \arcsin(\frac{x-1}{2}) *$ Domain is $-1 \leq x \leq 3$ (ii) A is $(\pi/2, 3)$ B is $(1, 0)$ C is $(3, \pi/2)$	M1 A1 E1 B1 B1cao B1cao B1ft [7]	Attempt to invert Allow $\pi/2 = 1.57$ or better ft on their A

Section B

7(i) $2x - x \ln x = 0$ $\Rightarrow x(2 - \ln x) = 0$ $\Rightarrow (x = 0) \text{ or } \ln x = 2$ $\Rightarrow \text{at A, } x = e^2$	M1 A1 [2]	Equating to zero
(ii) $\frac{dy}{dx} = 2 - x \cdot \frac{1}{x} - \ln x \cdot 1$ $= 1 - \ln x$ $\frac{dy}{dx} = 0 \Rightarrow 1 - \ln x = 0$ $\Rightarrow \ln x = 1, x = e$ When $x = e, y = 2e - e \ln e = e$ So B is (e, e)	M1 B1 A1 M1 A1cao B1ft [6]	Product rule for $x \ln x$ $d/dx (\ln x) = 1/x$ $1 - \ln x$ o.e. equating their derivative to zero $x = e$ $y = e$
(iii) At A, $\frac{dy}{dx} = 1 - \ln e^2 = 1 - 2$ $= -1$ At C, $\frac{dy}{dx} = 1 - \ln 1 = 1$ $1 \times -1 = -1 \Rightarrow$ tangents are perpendicular	M1 A1cao E1 [3]	Substituting $x=1$ or their e^2 into their derivative -1 and 1 www
(iv) Let $u = \ln x, dv/dx = x$ $\Rightarrow v = \frac{1}{2} x^2 \int x \ln x dx = \frac{1}{2} x^2 \ln x - \int \frac{1}{2} x^2 \cdot \frac{1}{x} dx$ $= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x dx$ $= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + c$ * $A = \int_1^e (2x - x \ln x) dx$ $= \left[x^2 - \frac{1}{2} x^2 \ln x + \frac{1}{4} x^2 \right]_1^e$ $= (e^2 - \frac{1}{2} e^2 \ln e + \frac{1}{4} e^2) - (1 - \frac{1}{2} 1^2 \ln 1 + \frac{1}{4} 1^2)$ $= \frac{3}{4} e^2 - \frac{5}{4}$	M1 A1 E1 B1 B1 M1 A1 cao [7]	Parts: $u = \ln x, dv/dx = x \Rightarrow v = \frac{1}{2} x^2$ correct integral and limits $\left[x^2 - \frac{1}{2} x^2 \ln x + \frac{1}{4} x^2 \right]$ o.e. substituting limits correctly

8 (i) $f(-x) = \frac{\sin(-x)}{2 - \cos(-x)}$ $= \frac{-\sin(x)}{2 - \cos(x)}$ $= -f(x)$ 	M1 A1 B1 [3]	substituting $-x$ for x in $f(x)$ Graph completed with rotational symmetry about O.
(ii) $f'(x) = \frac{(2 - \cos x) \cos x - \sin x \cdot \sin x}{(2 - \cos x)^2}$ $= \frac{2 \cos x - \cos^2 x - \sin^2 x}{(2 - \cos x)^2}$ $= \frac{2 \cos x - 1}{(2 - \cos x)^2} *$ $f'(x) = 0$ when $2 \cos x - 1 = 0$ $\Rightarrow \cos x = \frac{1}{2}, x = \pi/3$ When $x = \pi/3, y = \frac{\sin(\pi/3)}{2 - \cos(\pi/3)} = \frac{\sqrt{3}/2}{2 - 1/2}$ $= \frac{\sqrt{3}}{3}$ So range is $-\frac{\sqrt{3}}{3} \leq y \leq \frac{\sqrt{3}}{3}$	M1 A1 E1 M1 A1 M1 A1 B1ft [8]	Quotient or product rule consistent with their derivatives Correct expression numerator = 0 Substituting their $\pi/3$ into y o.e. but exact ft their $\frac{\sqrt{3}}{3}$
(iii) $\int_0^\pi \frac{\sin x}{2 - \cos x} dx$ let $u = 2 - \cos x$ $\Rightarrow du/dx = \sin x$ When $x = 0, u = 1$; when $x = \pi, u = 3$ $= \int_1^3 \frac{1}{u} du$ $= [\ln u]_1^3$ $= \ln 3 - \ln 1 = \ln 3$	M1 B1 A1ft A1cao	$\int \frac{1}{u} du$ $u = 1$ to 3 $[\ln u]$
or $= [\ln(2 - \cos x)]_0^\pi$ $= \ln 3 - \ln 1 = \ln 3$	M2 A1 A1 cao [4]	$[k \ln(2 - \cos x)]$ $k = 1$
(iv) 	B1ft [1]	Graph showing evidence of stretch s.f. $\frac{1}{2}$ in x - direction
(v) Area is stretched with scale factor $\frac{1}{2}$ So area is $\frac{1}{2} \ln 3$	M1 A1ft [2]	soi $\frac{1}{2}$ their $\ln 3$