

Mark Scheme 4726
January 2006

4726 FP2

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Final Draft

1(i) Use standard $\ln(1+3x) = 3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3}$

$$= 3x - 9x^2/2 + 9x^3$$

M1 Allow e.g. $3x^2$, $2!$ etc.M1 Attempt to simplify $(3x)^2$ etc.

A1 cao

(ii) Produce $(1 + x + x^2/2)$

B1

M1 Mult. 2 reasonable attempts, each of 3 terms (non-zero)

$$\text{Get } 3x - 3x^2/2 + 6x^3$$

A1✓ From their series

SC M1 Reasonable attempt at diff. and replace $x = 0$ (2 correct)M1✓ Put their values into correct Maclaurin expansion

A1 cao

(Applies to either/both parts)

2 Write as $f(x) = \pm(x - e^{-x})$

So $f'(x) = \pm(1 + e^{-x})$

Use $x_{n+1} = x_n - f(x_n)/f'(x_n)$ with $x_0 = 0.5$

$$\text{Get } x_1 = 0.56631, x_2 = 0.56714$$

$$\text{Get } x_3 = 0.567(1)$$

B1 Or equivalent

B1 Correct from their $f(x)$ M1 Clear evidence of N-R on their f, f'

A1✓ At least one to 4d.p.

A1 cao to 3 d.p.

3 Use $A/x + (Bx + C)/(x^2 + 2)$

Equate $x+6$ to $A(x^2 + 2) + (Bx+C)x$ (or equiv.)

B1

M1✓ Equate to their P.F. (e.g. if $B = 0$ or $C = 0$ used)

Use $x = 0$ or equiv. for A (or equate coeff.etc.)

M1✓ Include cover-up

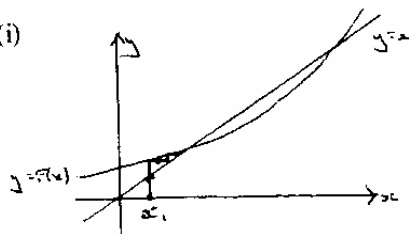
Correctly find one of B,C

A1

Get $A=3, B=-3, C=1$

A1

4(i)

B1 Line from x_1 to curve

B1 Then to line

B1 Clear explanation; allow use of step/staircase

(ii)(a) Converges to $x=a$

B1, B1

(b) Diverges (does not give either root)

B1

5 (i) Give $x = -2$

B1

Attempt to divide out

M1 Giving $y = x+k$; allow $k = 0$ here

$$\text{Get } y = x + 1$$

A1 Must be =

(ii) Write as quad. $x^2 + x(3-y) + (3-2y) = 0$

M1 SC Differentiate M1

Use for real x , $b^2 - 4ac \geq 0$

M1 Solve $dy/dx = 0$ M1

Produce quad. inequality in y

M1 Get 2 x, y values correct A1

Attempt to solve quad. inequality

M1 Attempt at max/min M1

Get A.G. clearly e.g. graph

A1 Justify, e.g. graph, constraints on y A1

- 6 (i) Use parts to $(-e^{-x}.x^n - \int -e^{-x}.nx^{n-1} dx)$ M1 Reasonable attempt e.g. $+e^{-x}$
 Use limits to get e^{-1} A1 cao
 Tidy correctly to A.G. B1 Allow \pm
 A1
- (ii) Use $I_3 = 3I_2 - e^{-1}$ B1 One such seen
 $I_2 = 2I_1 - e^{-1}$
 $I_1 = I_0 - e^{-1}$
 Work out $I_0 = 1 - e^{-1}$ or $I_1 = 1 - 2e^{-1}$ M1,A1
 Get $6 - 16e^{-1}$ A1
- 7 (i) Area under graph $= \int \sqrt{x} dx$ B1 Explain RHS (limits need not be specified)
 $>$ Sum of areas of rectangles from 1 to $N+1$ B1
 Area of each rect. = Width x Height = $1 \times \sqrt{x}$ B1
- (ii) Similarly, area under curve from 0 to N B1
 $<$ sum of areas of rect. from 0 to N B1
 Clear explanation of A.G. B1
- (iii) Integrate $x^{0.5}$ and use 2 different sets of limits M1,M1
 Get area between $\frac{2}{3}((N+1)^{1.5}-1)$ and
 $\frac{2}{3}N^{1.5}$ A1
- 8 (i) Max. $r = 2$ at $\theta = 0$ and π B1,B1 Two θ needed (rads only);
 ignore θ out of range
- (ii) Solve $r = 0$ for θ , giving $\theta = \frac{1}{2}\pi$ and $\frac{3}{2}\pi$ M1,A1 Two θ needed (rads only);
 ignore θ out of range
- (iii) Use correct formula with correct r M1
 Expand r M1
 Get $\int A + B \cos 2\theta + C \cos 4\theta d\theta$ M1 $C \neq 0$
 Integrate their expression correctly M1 $\sqrt{}$
 Get $3\pi/8$ A1 cao
- (iv) Express $\cos 2\theta = \cos^2\theta - \sin^2\theta$ or similar M1
 Use $\cos \theta = x/r$ and/or $\sin \theta = y/r$ M1
 Simplify to $(x^2 + y^2)^{1.5} = 2x^2$ or similar M1,A1
- 9 (i) Correct defⁿ of $\cosh x$ and $\sinh x$ B1,B1
 Expand $2 \cdot \frac{1}{2}(e^x - e^{-x}) \cdot \frac{1}{2}(e^x + e^{-x})$ M1 Reasonable attempt
 Clearly get $\frac{1}{2}(e^{2x} - e^{-2x})$ to A.G. A1
- (ii) Attempt to diff. and solve $dy/dx = 0$ M1 Reasonable attempt
 Use (ii) to get $A \cosh x (B \sinh x + C) = 0$ M1
 Clearly see $\cosh x > 0$ or similar for one useable factor only B1
 Attempt to solve $\sinh x = -C/B$ M1 Quote or via e^{-x} correctly
 Get $x = \ln((3+\sqrt{13})/2)$ A1
 Justify one answer only for $\sinh x = -C/B$ B1
 Accurate test for MINIMUM B1 First or second diff^d test
 with numeric evidence
 B1 Correct value(s) for min.