Mark Scheme 4727
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1 Directions $[1, 1, -1]$ and $[2, -3, 1]$	B1	For identifying both directions (may be implied by working)
$\theta = \cos^{-1}\dfrac{\lvert[1, 1, -1] \cdot [2, -3, 1]\rvert}{\sqrt{3}\quad\sqrt{14}}$	M1	For using scalar product of their direction vectors
$= \cos^{-1}\dfrac{\lvert-2\rvert}{\sqrt{42}}$	M1	For completely correct process for their angle
$= 72.0^\circ, 72^\circ$ or 1.26 rad	A1 **4** $\boxed{4}$	For correct answer
2 (i) Identities b, 6	B1 B1	For correct identities
Subgroups $\{b, d\}$, $\{6, 4\}$	B1 B1 **4**	For correct subgroups
(ii) $\{a, b, c, d\} \leftrightarrow \{2, 6, 8, 4\}$ or $\{8, 6, 2, 4\}$	B1 B1	For $b \leftrightarrow 6$, $d \leftrightarrow 4$
	B1 **3**	For $a, c \leftrightarrow 2, 8$ in either order
	$\boxed{7}$	**SR** If B0 B0 B0 then M1 A1 may be awarded for stating the orders of all elements in G and H
3 (i) $3y^2\dfrac{dy}{dx} = \dfrac{dz}{dx}$	M1	For differentiating substitution
$\Rightarrow \dfrac{dz}{dx} + 2xz = e^{-x^2}$	A1	For resulting equation in z and x
Integrating factor $\left(e^{\int 2x\,dx} =\right) e^{x^2}$	B1 √	For correct IF f.t. for an equation in suitable form
$\Rightarrow \dfrac{d}{dx}\left(ze^{x^2}\right)$ *OR* $\dfrac{d}{dx}\left(y^3 e^{x^2}\right) = 1$	M1	For using IF correctly
$\Rightarrow z\,e^{x^2}$ *OR* $y^3 e^{x^2} = x\,(+c)$	A1	For correct integration ($+c$ not required here)
$\Rightarrow y = (x+c)^{\frac{1}{3}}\, e^{-\frac{1}{3}x^2}$	A1 **6**	For correct answer **AEF**
(ii) As $x \to \infty$, $y \to 0$	B1 **1** $\boxed{7}$	For correct statement
4 (i) $\cos\theta = \frac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$, $\sin\theta = \frac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$	B1	For either expression, seen or implied z may be used for $e^{i\theta}$ throughout
$\Rightarrow \cos^2\theta\sin^4\theta = \frac{1}{4}\left(e^{i\theta} + e^{-i\theta}\right)^2 \frac{1}{16}\left(e^{i\theta} - e^{-i\theta}\right)^4$		
$= \frac{1}{4}\left(e^{2i\theta} + 2 + e^{-2i\theta}\right).\frac{1}{16}\left(e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}\right)$	M1 A1 A1	For expanding terms For the 2 correct expansions **SR** Allow A1 A0 for $k\left(e^{2i\theta} + 2 + e^{-2i\theta}\right)\left(e^{4i\theta} - 4e^{2i\theta} + 6 - 4e^{-2i\theta} + e^{-4i\theta}\right)$, $k \neq \frac{1}{64}$
$= \frac{1}{64}\left(\left(e^{6i\theta} + e^{-6i\theta}\right) - 2\left(e^{4i\theta} + e^{-4i\theta}\right) - \left(e^{2i\theta} + e^{-2i\theta}\right) + 4\right)$	M1	For grouping terms and using multiple angles
$= \frac{1}{32}(\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2)$ **AG**	A1 **6**	For answer obtained correctly

(ii) $\int_0^{\frac{1}{3}\pi} \cos^2\theta \sin^4\theta \, d\theta =$ $= \frac{1}{32}\left[\frac{1}{6}\sin 6\theta - \frac{1}{2}\sin 4\theta - \frac{1}{2}\sin 2\theta + 2\theta\right]_0^{\frac{1}{3}\pi}$ $= \frac{1}{32}\left[0 + \frac{1}{4}\sqrt{3} - \frac{1}{4}\sqrt{3} + \frac{2}{3}\pi - 0\right] = \frac{1}{48}\pi$	M1 A1 A1 **3** $\boxed{9}$	For integrating answer to **(i)** For all terms correct For correct answer
5 (i) *EITHER* $z = \sqrt{8}\,\mathrm{cis}(2k+1)\frac{\pi}{4},\ k = 0, 1, 2, 3$ *OR* $z = \sqrt{8}\,e^{(2k+1)\frac{\pi}{4}i}, k = 0, 1, 2, 3$	B1 B1 **2**	For correct modulus **AEF** For correct arguments **AEF**
(ii) $z = 2\sqrt{2}\left\{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i, -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i, -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i, \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right\}$ $z = 2 + 2i, -2 + 2i, -2 - 2i, 2 - 2i$ $(z - \alpha), (z - \beta), (z - \gamma), (z - \delta)$	B1 B1 B1 B1 √ **4**	For any of $\pm\frac{1}{\sqrt{2}} \pm \frac{1}{\sqrt{2}}i$ For any one value of z correct For all values of z correct **AEFcartesian** (may be implied from symmetry or factors) f.t., where $\alpha, \beta, \gamma, \delta$ are answers above
(iii) *EITHER* $(z - (2 + 2i))(z - (2 - 2i))$ $\times (z - (-2 + 2i))(z - (-2 - 2i))$ $= (z^2 + 4z + 8)(z^2 - 4z + 8)$	M1 M1 A1	For combining factors from **(ii)** in pairs Use of complex conjugate pairs For correct answer
OR $z^4 + 64 = (z^2 + az + b)(z^2 + cz + d)$ $\Rightarrow a + c = 0, b + ac + d = 0, ad + bc = 0, bd = 64$ Obtain $(z^2 + 4z + 8)(z^2 - 4z + 8)$	M1 M1 A1 **3** $\boxed{9}$	For equating coefficients For solving equations For correct answer
6 (i) $\mathbf{MB} = [2, 1, -2]$, $\mathbf{OF} = [4, 1, 2]$ $\mathbf{MB} \times \mathbf{OF}$ $= [4, -12, -2]$ *OR* $k[2, -6, -1]$	B1 M1 A1 **3**	For either vector correct (allow multiples) For finding vector product of their **MB** and **OF** For correct vector
(ii) *EITHER* Find vector product of any two of $\pm[2, -1, 2]$, $\pm[0, 0, 2]$, $\pm[2, -1, 0]$ and any two of $\pm[4, 0, 2]$, $\pm[4, -1, 0]$, $\pm[0, 1, 2]$ Obtain $k[1, 2, 0]$ Obtain $k[1, 4, -2]$ $x + 2y = 2$ and $x + 4y - 2z = 0$	M1 A1 A1 M1 A1	For finding two relevant vector products For correct LHS of plane *CMG* For correct LHS of plane *OEG* For substituting a point into each equation For both equations correct **AEF**
OR Use $ax + by + cz = d$ with coordinates of *C*, *M*, *G* *OR* *O*, *E*, *G* substituted Obtain $a : b : c = 1 : 2 : 0$ for *CMG* Obtain $a : b : c = 1 : 4 : -2$ for *OEG* $x + 2y = 2$ and $x + 4y - 2z = 0$	M1 A1 A1 M1 A1 **5**	For use of cartesian equation of plane For correct ratio For correct ratio For substituting a point into each equation For both equations correct **AEF**

(iii) *EITHER* Put x, y *OR* $z = t$ in planes *OR* evaluate $k[1, 2, 0] \times k[1, 4, -2]$	M1	For solving plane equations in terms of a parameter *OR* for finding vector product of normals to planes from **(ii)**
Obtain $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ where $\mathbf{a} = [0, 1, 2], [2, 0, 1]$ *OR* $[4, -1, 0]$	A1	Obtain a correct point **AEF**
$\mathbf{b} = k[-2, 1, 1]$	A1 **3** $\boxed{11}$	Obtain correct direction **AEF**
7 (i) $(x^{-1}ax)^m = (x^{-1}ax)(x^{-1}ax)\ldots(x^{-1}ax)$	M1	For considering powers of $x^{-1}ax$
$= x^{-1}aa\ldots ax$, associativity, $xx^{-1} = e$	A1 A1	For using associativity and inverse properties
$= x^{-1}a^m x = x^{-1}ex$ when $m = n$, not $m < n$	B1	For using order of *a* correctly
$= x^{-1}x$	A1	For using property of identity
$= e \Rightarrow$ order n	A1 **6**	For correct conclusion
(ii) ***EITHER*** $(x^{-1}ax)z = e$	M1	For attempt to solve for *z* **AEF**
$\Rightarrow axz = xe = x \Rightarrow xz = a^{-1}x$	A1	For using pre- or post multiplication
$\Rightarrow z = x^{-1}a^{-1}x$	A1	For correct answer
OR Use $(pq)^{-1} = q^{-1}p^{-1}$ *OR* $(pqr)^{-1} = r^{-1}q^{-1}p^{-1}$	M1	For applying inverse of a product of elements
State $(x^{-1})^{-1} = x$	A1	For stating this property
Obtain $x^{-1}a^{-1}x$	A1 **3**	For correct answer with no incorrect working **SR** correct answer with no working scores B1 only
(iii) $ax = xa \Rightarrow x = a^{-1}xa$	M1	Start from commutative property for *ax*
$\Rightarrow xa^{-1} = a^{-1}x$	A1 **2** $\boxed{11}$	Obtain commutative property for $a^{-1}x$
8 (i) $m^2 + 2km + 4 = 0$	M1	For stating and attempting to solve auxiliary eqn
$\Rightarrow m = -k \pm \sqrt{k^2 - 4}$	A1 **2**	For correct solutions, at any stage **AEF**
(a) $x = \mathrm{e}^{-kt}\left(A\mathrm{e}^{\sqrt{k^2-4}\,t} + B\mathrm{e}^{-\sqrt{k^2-4}\,t}\right)$	M1 A1 **2**	For using $\mathrm{e}^{\mathrm{f}(t)}$ with distinct real roots of aux eqn For correct answer **AEF**
(b) $x = \mathrm{e}^{-kt}\left(A\mathrm{e}^{\mathrm{i}\sqrt{4-k^2}\,t} + B\mathrm{e}^{-\mathrm{i}\sqrt{4-k^2}\,t}\right)$	M1	For using $\mathrm{e}^{\mathrm{f}(t)}$ with complex roots of aux eqn This form may not be seen explicitly but if stated as final answer earns M1 A0
$x = \mathrm{e}^{-kt}\left(A'\cos\sqrt{4-k^2}\,t + B'\sin\sqrt{4-k^2}\,t\right)$ *OR* $x = \mathrm{e}^{-kt}\left(C' \begin{matrix}\cos\\ \sin\end{matrix}\left(\sqrt{4-k^2}\,t + \alpha\right)\right)$	A1 **2**	For correct answer
(c) $x = \mathrm{e}^{-2t}(A'' + B''t)$	M1 A1 **2**	For using $\mathrm{e}^{\mathrm{f}(t)}$ with equal roots of aux eqn For correct answer. Allow *k* for 2

(ii)(a) $x = B'e^{-t}\sin\sqrt{3}\,t$	B1 √	For using $t=0,\ x=0$ correctly. f.t. from **(b)**
$\dot{x} = B'e^{-t}\left(\sqrt{3}\cos\sqrt{3}\,t - \sin\sqrt{3}\,t\right)$	M1 A1 √	For differentiating x For correct expression. f.t. from their x
$t=0,\ \dot{x}=6 \Rightarrow B' = 2\sqrt{3}$, $x = 2\sqrt{3}e^{-t}\sin\sqrt{3}\,t$	A1 **4**	For correct solution **AEF** **SR** √ and **AEF** OK for $x = C'e^{-t}\cos\left(\sqrt{3}\,t + \frac{1}{2}\pi\right)$
(b) $x \to 0$	B1	For correct statement
$e^{-t} \to 0$ and sin( ) is bounded	B1 **2** $\boxed{14}$	For both statements