OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education**
**Advanced General Certificate of Education**

# MEI STRUCTURED MATHEMATICS 4754(A)

Applications of Advanced Mathematics (C4)

**Paper A**

Monday **23 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

**TIME** 1 hour 30 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

## NOTE

- This paper will be followed by **Paper B: Comprehension.**

**This question paper consists of 4 printed pages.**

HN/3

**Section A** (36 marks)

**1** Solve the equation $\dfrac{2x}{x-2} - \dfrac{4x}{x+1} = 3$. [5]

**2** A curve is defined parametrically by the equations

$$x = t - \ln t, \qquad y = t + \ln t \qquad (t > 0).$$

Find the gradient of the curve at the point where $t = 2$. [5]

**3** A triangle ABC has vertices $A(-2, 4, 1)$, $B(2, 3, 4)$ and $C(4, 8, 3)$. By calculating a suitable scalar product, show that angle ABC is a right angle. Hence calculate the area of the triangle. [6]

**4** Solve the equation $2\sin 2\theta + \cos 2\theta = 1$, for $0° \leqslant \theta < 360°$. [6]

**5** **(i)** Find the cartesian equation of the plane through the point $(2, -1, 4)$ with normal vector

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}.$$ [3]

**(ii)** Find the coordinates of the point of intersection of this plane and the straight line with equation

$$\mathbf{r} = \begin{pmatrix} 7 \\ 12 \\ 9 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}.$$ [4]

**6** **(i)** Find the first three non-zero terms of the binomial expansion of $\dfrac{1}{\sqrt{4-x^2}}$ for $|x| < 2$. [4]

**(ii)** Use this result to find an approximation for $\displaystyle\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx$, rounding your answer to 4 significant figures. [2]

**(iii)** Given that $\displaystyle\int \frac{1}{\sqrt{4-x^2}}\,dx = \arcsin\left(\tfrac{1}{2}x\right) + c$, evaluate $\displaystyle\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx$, rounding your answer to 4 significant figures. [1]

**Section B** (36 marks)

**7** In a game of rugby, a kick is to be taken from a point P (see Fig. 7). P is a perpendicular distance $y$ metres from the line TOA. Other distances and angles are as shown.

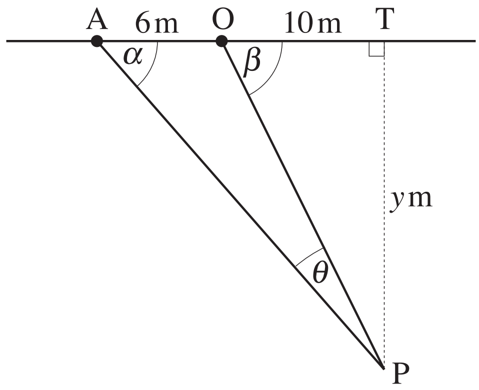


**Fig. 7**

**(i)** Show that $\theta = \beta - \alpha$, and hence that $\tan\theta = \dfrac{6y}{160 + y^2}$.

Calculate the angle $\theta$ when $y = 6$. [8]

**(ii)** By differentiating implicitly, show that $\dfrac{d\theta}{dy} = \dfrac{6(160 - y^2)}{(160 + y^2)^2}\cos^2\theta$. [5]

**(iii)** Use this result to find the value of $y$ that maximises the angle $\theta$. Calculate this maximum value of $\theta$. [You need not verify that this value is indeed a maximum.] [4]

**[Question 8 is printed overleaf.]**

**8** Some years ago an island was populated by red squirrels and there were no grey squirrels. Then grey squirrels were introduced.

The population $x$, in thousands, of red squirrels is modelled by the equation

$$x = \frac{a}{1 + kt},$$

where $t$ is the time in years, and $a$ and $k$ are constants. When $t = 0, x = 2.5$.

**(i)** Show that $\frac{dx}{dt} = -\frac{kx^2}{a}$. [3]

**(ii)** Given that the initial population of 2.5 thousand red squirrels reduces to 1.6 thousand after one year, calculate $a$ and $k$. [3]

**(iii)** What is the long-term population of red squirrels predicted by this model? [1]

The population $y$, in thousands, of grey squirrels is modelled by the differential equation

$$\frac{dy}{dt} = 2y - y^2.$$

When $t = 0, y = 1$.

**(iv)** Express $\frac{1}{2y - y^2}$ in partial fractions. [4]

**(v)** Hence show by integration that $\ln\left(\frac{y}{2 - y}\right) = 2t$.

Show that $y = \frac{2}{1 + e^{-2t}}$. [7]

**(vi)** What is the long-term population of grey squirrels predicted by this model? [1]

| Candidate Name | Centre Number | Candidate Number |
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# MEI STRUCTURED MATHEMATICS 4754(B)

Applications of Advanced Mathematics (C4)

## Paper B: Comprehension

Monday **23 JANUARY 2006** Afternooon Up to 1 hour

Additional materials:
Rough paper
MEI Examination Formulae and Tables (MF2)

**TIME** Up to 1 hour

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces at the top of this page.
- Answer **all** the questions.
- Write your answers in the spaces provided on the question paper.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this section is 18.

| For Examiner's Use | |
| --- | --- |
| Qu. | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Total | |

**This question paper consists of 5 printed pages, 3 blank pages and an insert.**

For Examiner's Use

**1** Line 59 says "Again Party G just misses out; if there had been 7 seats G would have got the last one."

Where is the evidence for this in the article? [1]

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**2** 6 parties, P, Q, R, S, T and U take part in an election for 7 seats. Their results are shown in the table below.

| **Party** | **Votes (%)** |
|---|---|
| P | 30.2 |
| Q | 11.4 |
| R | 22.4 |
| S | 14.8 |
| T | 10.9 |
| U | 10.3 |

**(i)** Use the Trial-and-Improvement method, starting with values of 10% and 14%, to find an acceptance percentage for 7 seats, and state the allocation of the seats. [4]

| **Acceptance percentage, *a*%** | | **10%** | **14%** | | | |
|---|---|---|---|---|---|---|
| **Party** | **Votes (%)** | **Seats** | **Seats** | **Seats** | **Seats** | **Seats** |
| P | 30.2 | | | | | |
| Q | 11.4 | | | | | |
| R | 22.4 | | | | | |
| S | 14.8 | | | | | |
| T | 10.9 | | | | | |
| U | 10.3 | | | | | |
| **Total seats** | | | | | | |

Seat Allocation P …. Q …. R …. S …. T …. U ….

For Examiner's Use

**(ii)** Now apply the d'Hondt Formula to the same figures to find the allocation of the seats. [5]

| | **Round** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Party** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **Residual** |
| P | 30.2 | | | | | | | |
| Q | 11.4 | | | | | | | |
| R | 22.4 | | | | | | | |
| S | 14.8 | | | | | | | |
| T | 10.9 | | | | | | | |
| U | 10.3 | | | | | | | |
| **Seat allocated to** | | | | | | | | |

Seat Allocation P …. Q …. R …. S …. T …. U ….

**3** In this question, use the figures for the example used in Table 5 in the article, the notation described in the section "Equivalence of the two methods" and the value of 11 found for $a$ in Table 4.

Treating Party E as Party 5, verify that $\dfrac{V_5}{N_5 + 1} < a \leqslant \dfrac{V_5}{N_5}$. [2]

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**[Turn over**

For Examiner's Use

**4** Some of the intervals illustrated by the lines in the graph in Fig. 8 are given in this table.

| Seats | Interval | Seats | Interval |
|---|---|---|---|
| 1 | $22.2 < a \leqslant 27.0$ | 5 | |
| 2 | $16.6 < a \leqslant 22.2$ | 6 | $10.6 < a \leqslant 11.1$ |
| 3 | | 7 | |
| 4 | | | |

**(i)** Describe briefly, giving an example, the relationship between the end-points of these intervals and the values in Table 5, which is reproduced below. [2]

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**(ii)** Complete the table *above*. [1]

| | Round | | | | | | |
|---|---|---|---|---|---|---|---|
| **Party** | **1** | **2** | **3** | **4** | **5** | **6** | **Residual** |
| A | 22.2 | 22.2 | 11.1 | 11.1 | 11.1 | 11.1 | 7.4 |
| B | 6.1 | 6.1 | 6.1 | 6.1 | 6.1 | 6.1 | 6.1 |
| C | 27.0 | 13.5 | 13.5 | 13.5 | 9.0 | 9.0 | 9.0 |
| D | 16.6 | 16.6 | 16.6 | 8.3 | 8.3 | 8.3 | 8.3 |
| E | 11.2 | 11.2 | 11.2 | 11.2 | 11.2 | 5.6 | 5.6 |
| F | 3.7 | 3.7 | 3.7 | 3.7 | 3.7 | 3.7 | 3.7 |
| G | 10.6 | 10.6 | 10.6 | 10.6 | 10.6 | 10.6 | 10.6 |
| H | 2.6 | 2.6 | 2.6 | 2.6 | 2.6 | 2.6 | 2.6 |
| **Seat allocated to** | **C** | **A** | **D** | **C** | **E** | **A** | |

**Table 5**

**5** The ends of the vertical lines in Fig. 8 are marked with circles. Those at the tops of the lines are filled in, e.g. ●, whereas those at the bottom are not, e.g. ○.

**(i)** Relate this distinction to the use of inequality signs. [1]

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**(ii)** Show that the inequality on line 102 can be rearranged to give $0 \leqslant V_k - N_k a < a$. [1]

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**(iii)** Hence justify the use of the inequality signs in line 102. [1]

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