

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

MATHEMATICS

4725

Further Pure Mathematics 1

Wednesday 18 JANUARY 2006 Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

- 1 (i) Express $(1 + 8i)(2 - i)$ in the form $x + iy$, showing clearly how you obtain your answer. [2]
- (ii) Hence express $\frac{1 + 8i}{2 + i}$ in the form $x + iy$. [3]
- 2 Prove by induction that, for $n \geq 1$, $\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$. [5]
- 3 The matrix \mathbf{M} is given by $\mathbf{M} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$.
- (i) Find the value of the determinant of \mathbf{M} . [3]
- (ii) State, giving a brief reason, whether \mathbf{M} is singular or non-singular. [1]
- 4 Use the substitution $x = u + 2$ to find the exact value of the real root of the equation
- $$x^3 - 6x^2 + 12x - 13 = 0. \quad [5]$$
- 5 Use the standard results for $\sum_{r=1}^n r$, $\sum_{r=1}^n r^2$ and $\sum_{r=1}^n r^3$ to show that, for all positive integers n ,
- $$\sum_{r=1}^n (8r^3 - 6r^2 + 2r) = 2n^3(n+1). \quad [6]$$
- 6 The matrix \mathbf{C} is given by $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix}$.
- (i) Find \mathbf{C}^{-1} . [2]
- (ii) Given that $\mathbf{C} = \mathbf{AB}$, where $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, find \mathbf{B}^{-1} . [5]
- 7 (a) The complex number $3 + 2i$ is denoted by w and the complex conjugate of w is denoted by w^* . Find
- (i) the modulus of w , [1]
- (ii) the argument of w^* , giving your answer in radians, correct to 2 decimal places. [3]
- (b) Find the complex number u given that $u + 2u^* = 3 + 2i$. [4]
- (c) Sketch, on an Argand diagram, the locus given by $|z + 1| = |z|$. [2]

8 The matrix \mathbf{T} is given by $\mathbf{T} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$.

(i) Draw a diagram showing the unit square and its image under the transformation represented by \mathbf{T} . [3]

(ii) The transformation represented by matrix \mathbf{T} is equivalent to a transformation A, followed by a transformation B. Give geometrical descriptions of possible transformations A and B, and state the matrices that represent them. [6]

9 (i) Show that $\frac{1}{r} - \frac{1}{r+2} = \frac{2}{r(r+2)}$. [2]

(ii) Hence find an expression, in terms of n , for

$$\frac{2}{1 \times 3} + \frac{2}{2 \times 4} + \dots + \frac{2}{n(n+2)}. \quad [5]$$

(iii) Hence find the value of

(a) $\sum_{r=1}^{\infty} \frac{2}{r(r+2)}$, [1]

(b) $\sum_{r=n+1}^{\infty} \frac{2}{r(r+2)}$. [2]

10 The roots of the equation

$$x^3 - 9x^2 + 27x - 29 = 0$$

are denoted by α , β and γ , where α is real and β and γ are complex.

(i) Write down the value of $\alpha + \beta + \gamma$. [1]

(ii) It is given that $\beta = p + iq$, where $q > 0$. Find the value of p , in terms of α . [4]

(iii) Write down the value of $\alpha\beta\gamma$. [1]

(iv) Find the value of q , in terms of α only. [5]