Mark Scheme 4727
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1 **(i)** Attempt to show no closure $3\times3=1,\ 5\times5=1$ *OR* $7\times7=1$	M1 A1	For showing operation table or otherwise For a convincing reason
OR Attempt to show no identity Show $a\times e=a$ has no solution	M1 A1 **2**	For attempt to find identity *OR* for showing operation table For showing identity is not 3, not 5, and not 7 by reference to operation table or otherwise
(ii) (a =) 1	B1 **1**	For value of a stated
(iii) *EITHER*: $\{e, r, r^2, r^3\}$ is cyclic, **(ii)** group is not cyclic	B1*	For a pair of correct statements
OR: $\{e, r, r^2, r^3\}$ has 2 self-inverse elements, **(ii)** group has 4 self-inverse elements	B1*	For a pair of correct statements
OR: $\{e, r, r^2, r^3\}$ has 1 element of order 2 **(ii)** group has 3 elements of order 2	B1*	For a pair of correct statements
OR: $\{e, r, r^2, r^3\}$ has element(s) of order 4 **(ii)** group has no element of order 4	B1*	For a pair of correct statements
Not isomorphic	B1 (dep*) **2** $\boxed{5}$	For correct conclusion
2 *EITHER*: $[3, 1, -2]\times[1, 5, 4]$ $\Rightarrow \mathbf{b}=k[1, -1, 1]$ e.g. put *x OR y OR z* = 0 and solve 2 equations in 2 unknowns Obtain $[0, 2, -1]$ *OR* $[2, 0, 1]$ *OR* $[1, 1, 0]$	M1 A1 M1 M1 A1	For attempt to find vector product of both normals For correct vector identified with **b** For giving a value to one variable For solving the equations in the other variables For a correct vector identified with **a**
OR: Solve $3x+y-2z=4,\ x+5y+4z=6$ e.g. $y+z=1$ *OR* $x-z=1$ *OR* $x+y=2$ Put *x OR y OR z* = t $[x, y, z]=[t, 2-t, -1+t]$ *OR* $[2-t, t, 1-t]$ *OR* $[1+t, 1-t, t]$ Obtain $[0, 2, -1]$ *OR* $[2, 0, 1]$ *OR* $[1, 1, 0]$ Obtain $k[1, -1, 1]$	 M1 M1 M1 A1 A1 **5** $\boxed{5}$	 For eliminating one variable between 2 equations For solving in terms of a parameter For obtaining a parametric solution for x, y, z For a correct vector identified with **a** For correct vector identified with **b**
3 **(i)** $z=\dfrac{6\pm\sqrt{36-144}}{2}$ $z=3\pm3\sqrt{3}\,\mathrm{i}$ Obtain $(r=)\ 6$ Obtain $(\theta=)\ \frac{1}{3}\pi$	M1 A1 A1 A1 **4**	For using quadratic equation formula or completing the square For obtaining cartesian values **AEF** For correct modulus For correct argument
(ii) *EITHER*: 6^{-3} *OR* $\frac{1}{216}$ seen $Z^{-3}=6^{-3}(\cos(-\pi)\pm\mathrm{i}\sin(-\pi))$ Obtain $-\frac{1}{216}$	B1√ M1 A1	f.t. from their r^{-3} For using de Moivre with $n=\pm3$ For correct value
OR: $z^3=6z^2-36z=6(6z-36)-36z$ 216 seen Obtain $-\frac{1}{216}$	M1 B1 A1 **3** $\boxed{7}$	For using equation to find z^3 Ignore any remaining z terms For correct value

4 (i) $(y = xz \Rightarrow)\ \frac{dy}{dx} = x\frac{dz}{dx} + z$	B1	For a correct statement
$x\frac{dz}{dx} + z = \frac{x^2(1-z^2)}{x^2 z} = \frac{1}{z} - z$	M1	For substituting into differential equation and attempting to simplify to a variables separable form
$x\frac{dz}{dx} = \frac{1}{z} - 2z = \frac{1-2z^2}{z}$	A1 **3**	For correct equation **AG**
(ii) $\int \frac{z}{1-2z^2}\,dz = \int \frac{1}{x}\,dx \Rightarrow -\frac{1}{4}\ln(1-2z^2) = \ln cx$	M1 M1* A1	For separating variables and writing integrals For integrating both sides to ln forms For correct result (*c* not required here)
$1-2z^2 = (cx)^{-4}$	A1√	For exponentiating their ln equation including a constant (this may follow the next M1)
$\frac{x^2-2y^2}{x^2} = \frac{c^{-4}}{x^4}$	M1 (dep*)	For substituting $z = \frac{y}{x}$
$x^2(x^2-2y^2) = k$	A1 **6**	For correct solution properly obtained, including dealing with any necessary change of constant to *k* as given **AG**
	$\boxed{9}$	
5 (i) (a) $e,\ p,\ p^2$	B1	For correct elements
(b) $e,\ q,\ q^2$	B1 **2**	For correct elements **SR** If the answers to parts **(i)** and **(iv)** are reversed, full credit may be earned for both parts
(ii) $p^3 = q^3 = e \Rightarrow (pq)^3 = p^3q^3 = e$	M1	For finding $(pq)^3$ or $(pq^2)^3$
$\Rightarrow$ order 3	A1	For correct order
$(pq^2)^3 = p^3q^6 = p^3(q^3)^2 = e \Rightarrow$ order 3	A1 **3**	For correct order **SR** For answer(s) only allow B1 for either or both
(iii) 3	B1 **1**	For correct order and no others
(iv)	B1	For stating *e* and either *pq* or p^2q^2
$e,\ pq,\ p^2q^2$ *OR* $e,\ pq,\ (pq)^2$	B1	For all 3 elements and no more
	B1	For stating *e* and either pq^2 or p^2q
$e,\ pq^2,\ p^2q$ *OR* $e,\ pq^2,\ (pq^2)^2$ *OR* $e,\ p^2q,\ (p^2q)^2$	B1 **4**	For all 3 elements and no more
	$\boxed{10}$	

6 **(i)** (CF $m=-3\Rightarrow$) Ae^{-3x}	B1 **1**	For correct CF
(ii) ($y=$) $px+q$	B1	For stating linear form for PI (may be implied)
$\Rightarrow p+3(px+q)=2x+1$	M1	For substituting PI into DE (needs y and $\frac{dy}{dx}$)
$\Rightarrow p=\frac{2}{3},\quad q=\frac{1}{9}$	A1 A1	For correct values
$\Rightarrow$ GS $y=Ae^{-3x}+\frac{2}{3}x+\frac{1}{9}$	A1√	For correct GS. f.t. from their CF + PI
		SR Integrating factor method may be used, but CF must be stated somewhere to earn the mark in **(i)**
I.F. e^{3x} $\Rightarrow \frac{d}{dx}\left(ye^{3x}\right)=(2x+1)e^{3x}$	B1	For stating integrating factor
$\Rightarrow ye^{3x}=\frac{1}{3}e^{3x}(2x+1)-\int\frac{2}{3}e^{3x}dx$	M1	For attempt at integrating by parts the right way round
$\Rightarrow ye^{3x}=\frac{2}{3}xe^{3x}+\frac{1}{3}e^{3x}-\frac{2}{9}e^{3x}+A$	A2 *	For correct integration, including constant Award A1 for any 2 algebraic terms correct
$\Rightarrow$ GS $y=Ae^{-3x}+\frac{2}{3}x+\frac{1}{9}$	A1√ **5**	For correct GS. f.t. from their * with constant
(iii) *EITHER* $\frac{dy}{dx}=-3Ae^{-3x}+\frac{2}{3}$	M1	For differentiating their GS
$\Rightarrow -3A+\frac{2}{3}=0$	M1	For putting $\frac{dy}{dx}=0$ when $x=0$
$y=\frac{2}{9}e^{-3x}+\frac{2}{3}x+\frac{1}{9}$	A1	For correct solution
OR $\frac{dy}{dx}=0,\ x=0\ \Rightarrow 3y=1$	M1	For using original DE with $\frac{dy}{dx}=0$ and $x=0$ to find y
$\Rightarrow \frac{1}{3}=A+\frac{1}{9}$	M1	For using their GS with y and $x=0$ to find A
$y=\frac{2}{9}e^{-3x}+\frac{2}{3}x+\frac{1}{9}$	A1 **3**	For correct solution
(iv) $y=\frac{2}{3}x+\frac{1}{9}$	B1√ **1** $\boxed{10}$	For correct function. f.t. from linear part of **(iii)**

7 (i) *EITHER*: (**AG** is **r** =) $[6, 4, 8]+t\,k[1, 0, 1]$ *or* $[3, 4, 5]+t\,k[1, 0, 1]$	B1		For a correct equation
Normal to *BCD* is	M1		For finding vector product of any two of $\pm[1, -4, -1], \pm[2, 1, 1], \pm[1, 5, 2]$
$\mathbf{n} = k[1, 1, -3]$	A1		For correct **n**
Equation of *BCD* is $\mathbf{r}.[1, 1, -3] = -6$	A1		For correct equation (or in cartesian form)
Intersect at $(6+t)+4+(-3)(8+t) = -6$	M1		For substituting point on *AG* into plane
$t = -4$ ($t = -1$ using $[3, 4, 5]$) $\Rightarrow$ $\mathbf{OM} = [2, 4, 4]$	A1		For correct position vector of *M* **AG**
OR: (**AG** is **r** =) $[6, 4, 8]+t\,k[1, 0, 1]$ *or* $[3, 4, 5]+t\,k[1, 0, 1]$	B1		For a correct equation
$\mathbf{r} = \mathbf{u}+\lambda\mathbf{v}+\mu\mathbf{w}$, where $\mathbf{u} = [2, 1, 3]$ *or* $[1, 5, 4]$ *or* $[3, 6, 5]$; $\mathbf{v}, \mathbf{w} =$ two of $[1, -4, -1], [1, 5, 2], [2, 1, 1]$	M1 A1		For a correct parametric equation of *BCD*
e.g. $(x =)\ 6+t = 2+\lambda+\mu$; $(y =)\ 4 = 1-4\lambda+5\mu$; $(z =)\ 8+t = 3-\lambda+2\mu$	M1		For forming 3 equations in t, λ, μ from line and plane, and attempting to solve them
$t = -4$ *or* $\lambda = -\frac{1}{3}, \mu = \frac{1}{3}$	A1		For correct value of t *or* λ, μ
$\Rightarrow$ $\mathbf{OM} = [2, 4, 4]$	A1	**6**	For correct position vector of *M* **AG**
(ii) A, G, M have $t = 0, -3, -4$ *OR* $AG = 3\sqrt{2}$, $AM = 4\sqrt{2}$ *OR* $\mathbf{AG} = [-3, 0, -3]$, $\mathbf{AM} = [-4, 0, -4]$ $\Rightarrow AG : AM = 3:4$	B1	**1**	For correct ratio **AEF**
(iii) $\mathbf{OP} = \mathbf{OC}+\frac{4}{3}\mathbf{CG}$	M1		For using given ratio to find position vector of *P*
$= \left[\frac{11}{3}, \frac{11}{3}, \frac{16}{3}\right]$	A1	**2**	For correct vector
(iv) *EITHER*: Normal to *ABD* is	M1		For finding vector product of any two of $\pm[4, 3, 5], \pm[1, 5, 2], \pm[3, -2, 3]$
$\mathbf{n} = k[19, 3, -17]$	A1		For correct **n**
Equation of *ABD* is $\mathbf{r}.[19, 3, -17] = -10$	M1		For finding equation (or in cartesian form)
$19.\frac{11}{3}+3.\frac{11}{3}-17.\frac{16}{3} = -10$	A1		For verifying that *P* satisfies equation
OR: Equation of *ABD* is $\mathbf{r} = [6, 4, 8]+\lambda[4, 3, 5]+\mu[1, 5, 2]$ (etc.)	M1		For finding equation in parametric form
$\left[\frac{11}{3}, \frac{11}{3}, \frac{16}{3}\right] = [6, 4, 8]+\lambda[4, 3, 5]+\mu[1, 5, 2]$	M1		For substituting *P* and solving 2 equations for λ, μ
$\lambda = -\frac{2}{3}$, $\mu = \frac{1}{3}$	A1		For correct λ, μ
	A1		For verifying 3rd equation is satisfied
OR: $\mathbf{AP} = \left[-\frac{7}{3}, -\frac{1}{3}, -\frac{8}{3}\right]$	M1 A1		For finding 3 relevant vectors in plane *ABDP*; For correct **AP** *or* **BP** *or* **DP**
$\mathbf{AB} = [-4, -3, -5]$, $\mathbf{AD} = [-3, 2, -3]$ $\Rightarrow \mathbf{AB}+\mathbf{AD} = [-7, -1, -8]$	M1		For finding **AB, AD** *or* **BA, BD** *or* **DB, DA**
$\Rightarrow \mathbf{AP} = \frac{1}{3}(\mathbf{AB}+\mathbf{AD})$	A1	**4**	For verifying linear relationship
		$\boxed{13}$	

8 **(i)** $\cos 4\theta + i\sin 4\theta =$ $c^4 + 4i c^3 s - 6c^2 s^2 - 4i cs^3 + s^4$ $\Rightarrow \sin 4\theta = 4c^3 s - 4cs^3$ and $\cos 4\theta = c^4 - 6c^2 s^2 + s^4$ $\Rightarrow \tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$	M1 A1 M1 A1 **4**	For using de Moivre with $n = 4$ For both expressions For expressing $\dfrac{\sin 4\theta}{\cos 4\theta}$ in terms of c and s For simplifying to correct expression
(ii) $\cot 4\theta = \dfrac{\cot^4\theta - 6\cot^2\theta + 1}{4\cot^3\theta - 4\cot\theta}$	B1 **1**	For inverting **(i)** and using $\cot\theta = \dfrac{1}{\tan\theta}$ or $\tan\theta = \dfrac{1}{\cot\theta}$. **AG**
(iii) $\cot 4\theta = 0$ Put $x = \cot^2\theta$ $\theta = \frac{1}{8}\pi \Rightarrow x^2 - 6x + 1 = 0$ *OR* $x^2 - 6x + 1 = 0 \Rightarrow \theta = \frac{1}{8}\pi$	B1 B1 B1 **3**	For putting $\cot 4\theta = 0$ (can be awarded in **(iv)** if not earned here) For putting $x = \cot^2\theta$ in the numerator of **(ii)** For deducing quadratic from **(ii)** and $\theta = \frac{1}{8}\pi$ *OR* For deducing $\theta = \frac{1}{8}\pi$ from **(ii)** and quadratic
(iv) $4\theta = \frac{3}{2}\pi$ *OR* $\frac{1}{2}(2n+1)\pi$ 2nd root is $x = \cot^2\left(\frac{3}{8}\pi\right)$ $\Rightarrow \cot^2\left(\frac{1}{8}\pi\right) + \cot^2\left(\frac{3}{8}\pi\right) = 6$ $\Rightarrow \operatorname{cosec}^2\left(\frac{1}{8}\pi\right) + \operatorname{cosec}^2\left(\frac{3}{8}\pi\right) = 8$	M1 A1 M1 M1 A1 **5** $\boxed{13}$	For attempting to find another value of θ For the other root of the quadratic For using sum of roots of quadratic For using $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$ For correct value