

Mark Scheme 4734
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1	(i) $E(T) = E(X) + \lambda E(Y)$ $\Rightarrow 100 = 45 + 33\lambda$ $\Rightarrow \lambda = 5/3$ AG	M1 A1	Use $E(X + \lambda Y)$ 2	aef
	(ii) $\text{Var}(T) = \text{Var}(X) + (5/3)^2 \text{Var}(Y)$ $= 256$ $T \sim N(100, 256)$	M1 A1 B1✓	3	ft variance
	(iii) Same student for X and Y so independence unlikely.	B1	1	Sensible reason
2	(i) Use $3a/2 = 1$	B1	1	Or similar
	(ii) $y = 2/3x$ $y = 1 - 1/3x$	B1 M1A1	3	M1 for correct gradient B1M1A0 if not $y = \dots$
	(iii) $f(x) = \begin{cases} \frac{2}{3}x & 0 \leq x \leq 1 \\ 1 - \frac{1}{3}x & 1 < x \leq 3. \end{cases}$	B1✓	1	ft (ii)
	(iv) $\int_0^1 \frac{2}{3}x^2 dx + \int_1^3 (x - \frac{1}{3}x^2) dx$ $\left[\frac{2}{9}x^3 \right]_0^1 + \left[\frac{1}{2}x^2 - \frac{1}{9}x^3 \right]_1^3$ $= 4/3$	M1 A1✓A1✓ A1	4	One correct, with limits ft from similar f aef
3	(i) Assumes breaking strengths have normal normal distributions Equal variances	B1 B1	2	
	(ii) $H_0: \mu_T = \mu_U$, $H_1: \mu_T > \mu_U$ where μ_T, μ_U are means for treated and untreated thread. $\bar{x}_T = 18.05$, $\bar{x}_U = 17.26$ $s_T^2 = 0.715$, $s_U^2 = 0.738$ $s^2 = (5 \times 0.715 + 4 \times 0.738)/9$ EITHER: $(18.05 - 17.26)/[s\sqrt{(1/5 + 1/6)}]$ $= 1.532$ Compare correctly with 1.383 Reject H_0 and accept there is sufficient evidence that mean has increased so that the treatment has been successful. OR: $\bar{X}_T - \bar{X}_U \geq ks\sqrt{1/5 + 1/6}; = 0.713$ $0.79 > 0.713$, reject H_0 etc	B1 B1 M1 A1 M1 A1✓ M1A1 M1A1✓	8	For both hypotheses May be implied below by 0.79 Allow biased, 0.596, 0.590 if $s^2 = (6 \times 0.596 + 5 \times 0.590)/9$ With pooled variance est. Conclusion in context. Ft 1.532 Allow $>$ or $=$ Or equivalent. Ft 0.713

4	(i) $s^2 = \frac{1}{11}(2604.4 - 177.6^2/12)$ $= 1.0836\dots$ Use $\bar{x} \pm t\sqrt{\frac{s^2}{12}}$ $t = 2.201$ $\bar{x} = 177.6/12 = 14.8$ (14.14, 15.46), (14.1, 15.5)	M1 A1 M1 B1 A1 A1	aef 6
	(ii) EITHER: $(14.8 - 15.4)/(\sqrt{s^2/12})$ $= -1.997$ Compare correctly with -1.796 Reject H_0 and accept that there is evidence that the mean is less than 15.4 OR: $\bar{X} - 15.4 \leq -k\sqrt{\frac{s^2}{12}}$; $\bar{X} \leq 14.86$ $14.8 < 14.86$, reject H_0 etc	M1 A1 M1 A1 M1A1 M1A1	With their variance In context. Ft -1.997 Allow $<$ or $=$ Or equivalent. Ft 14.86
5	(i) $978/1200 = 0.815$	B1	1
	(ii) Use $\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{1200}}$ $z = 1.645$ $\sqrt{(0.815 \times 0.185/1200)}$ (0.797, 0.833)	M1 B1 A1 A1	Reasonable variance ft $\hat{p}$ Allow 1199 Interval
	(iii) If a large number of such samples were taken, p would be contained in about 90% of the confidence intervals.	B2	2 B1 if idea correct but badly expressed.
	(iv) $1.645\sqrt{(0.815 \times 0.185/n)} = 0.01$ $n = 1.645^2(0.815 \times 0.185)/0.01^2$ $= 4080$	M1 A1 M1 A1	Allow one error; $>$ or $<$ All correct Correct procedure for sim equ Integer rounding to 4100

6	(i) $\int_1^t \frac{3}{x^4} dx$	M1	Any variable	
	$F(t) = \begin{cases} 1 - \frac{1}{t^3} & t \geq 1, \\ 0 & \text{otherwise.} \end{cases}$	A1	2	
	(ii) $G(y) = P(Y \leq y)$	M1		
	$= P(T \leq y^{1/3})$	A1		
	$= F(y^{1/3})$	M1		
	$= 1 - 1/y$	A1 ✓	ft F(t)	
	$g(y) = G'(y)$	M1		
	$= 1/y^2, y \geq 1$ AG	A1	6	
	(iii) EITHER $\int_1^\infty \frac{\sqrt{y}}{y^2} dy$	OR $\int_1^\infty \frac{3t^{2/3}}{t^4} dt$	M1	
	$\left[-2y^{-1/2} \right]_1^\infty$	$\left[-2t^{-3/2} \right]_1^\infty$	B1	
	$= 2$		A1	3
7	(i)(a) H_0 : Eye colour and reaction are not associated.	B1	Or equivalent (independent, or unrelated)	
	H_1 : Eye colour and reaction are associated	B1	2	
	(b) $65 \times 39 / 140$	B1	1	
	(c) $6.11^2 / 18.11 + 5.3^2 / 11.7 + 0.81^2 / 9.19$	M1	Or equivalent ; one correct	
	$2.061 + 2.401 + 0.071$	A1	At least 3 dp here	
	$4.533, 4.53$ AG	A1	3	But accept from 2 dp
	(d) $v = 4$	B1	Stated or implied	
	Use tables to obtain $\alpha = 2\frac{1}{2}$	B1	2	
	(ii) $H_0: p_{BL} = p_{BR} = 0.4, p_O = 0.2$	B1	Or in words, in terms of probs or proportions	
	(H_1 : At least two prob. not as above)			
	E values $\frac{56}{56} \quad \frac{28}{28}$	M1A1		
	$\chi^2 = 9^2/56 + 14^2/56 + 8^2/28$	M1		
	$= 5.839$	A1	Accept 5.84	
	Compare correctly with 5.991	M1	M1A0 if 5.991 seen and	
	Accept that sample is consistent with hypothesis.	A1 ✓	7	no explicit comparison
	SR: If three tests for p then count only $p_{BR} = 0.4$.			
	$(42/140 - 0.4)/\sigma$	M1		
	$\sigma = \sqrt{(0.4 \times 0.6/140)}$; -2.415	A1A1		
	Compare with -1.96; conclusion in context	M1A1	Max 6/7 (with H_0)	the