

**ADVANCED SUBSIDIARY GCE UNIT  
MATHEMATICS**

Further Pure Mathematics 1  
**THURSDAY 18 JANUARY 2007**

**4725/01**

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)  
List of Formulae (MF1)

**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.

**ADVICE TO CANDIDATES**

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

- 1 The matrices  $\mathbf{A}$  and  $\mathbf{B}$  are given by  $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} a & -1 \\ -3 & -2 \end{pmatrix}$ .
- (i) Given that  $2\mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$ , write down the value of  $a$ . [1]
- (ii) Given instead that  $\mathbf{AB} = \begin{pmatrix} 7 & -4 \\ 9 & -7 \end{pmatrix}$ , find the value of  $a$ . [2]
- 2 Use an algebraic method to find the square roots of the complex number  $15 + 8i$ . [6]
- 3 Use the standard results for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^3$  to find
- $$\sum_{r=1}^n r(r-1)(r+1),$$
- expressing your answer in a fully factorised form. [6]
- 4 (i) Sketch, on an Argand diagram, the locus given by  $|z - 1 + i| = \sqrt{2}$ . [3]
- (ii) Shade on your diagram the region given by  $1 \leq |z - 1 + i| \leq \sqrt{2}$ . [3]
- 5 (i) Verify that  $z^3 - 8 = (z - 2)(z^2 + 2z + 4)$ . [1]
- (ii) Solve the quadratic equation  $z^2 + 2z + 4 = 0$ , giving your answers exactly in the form  $x + iy$ . Show clearly how you obtain your answers. [3]
- (iii) Show on an Argand diagram the roots of the cubic equation  $z^3 - 8 = 0$ . [3]
- 6 The sequence  $u_1, u_2, u_3, \dots$  is defined by  $u_n = n^2 + 3n$ , for all positive integers  $n$ .
- (i) Show that  $u_{n+1} - u_n = 2n + 4$ . [3]
- (ii) Hence prove by induction that each term of the sequence is divisible by 2. [5]
- 7 The quadratic equation  $x^2 + 5x + 10 = 0$  has roots  $\alpha$  and  $\beta$ .
- (i) Write down the values of  $\alpha + \beta$  and  $\alpha\beta$ . [2]
- (ii) Show that  $\alpha^2 + \beta^2 = 5$ . [2]
- (iii) Hence find a quadratic equation which has roots  $\frac{\alpha}{\beta}$  and  $\frac{\beta}{\alpha}$ . [4]

- 8 (i) Show that  $(r+2)! - (r+1)! = (r+1)^2 \times r!$ . [3]

(ii) Hence find an expression, in terms of  $n$ , for

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \dots + (n+1)^2 \times n!. \quad [4]$$

(iii) State, giving a brief reason, whether the series

$$2^2 \times 1! + 3^2 \times 2! + 4^2 \times 3! + \dots$$

converges. [1]

- 9 The matrix  $\mathbf{C}$  is given by  $\mathbf{C} = \begin{pmatrix} 0 & 3 \\ -1 & 0 \end{pmatrix}$ .

(i) Draw a diagram showing the unit square and its image under the transformation represented by  $\mathbf{C}$ . [2]

The transformation represented by  $\mathbf{C}$  is equivalent to a rotation,  $\mathbf{R}$ , followed by another transformation,  $\mathbf{S}$ .

(ii) Describe fully the rotation  $\mathbf{R}$  and write down the matrix that represents  $\mathbf{R}$ . [3]

(iii) Describe fully the transformation  $\mathbf{S}$  and write down the matrix that represents  $\mathbf{S}$ . [4]

- 10 The matrix  $\mathbf{D}$  is given by  $\mathbf{D} = \begin{pmatrix} a & 2 & 0 \\ 3 & 1 & 2 \\ 0 & -1 & 1 \end{pmatrix}$ , where  $a \neq 2$ .

(i) Find  $\mathbf{D}^{-1}$ . [7]

(ii) Hence, or otherwise, solve the equations

$$\begin{aligned} ax + 2y &= 3, \\ 3x + y + 2z &= 4, \\ -y + z &= 1. \end{aligned} \quad [4]$$

---

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

OCR is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.