

**ADVANCED GCE UNIT
MATHEMATICS**

Mechanics 3

WEDNESDAY 10 JANUARY 2007

4730/01

Afternoon

Time: 1 hour 30 minutes

Additional Materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.

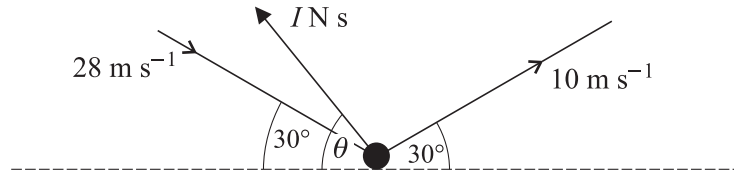
ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

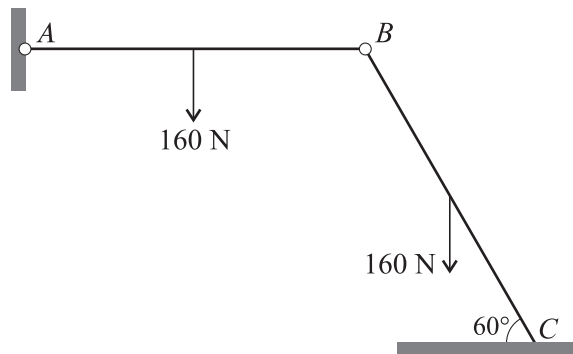
- 1 A particle P of mass 0.6 kg is attached to a fixed point O by a light inextensible string of length 0.4 m . While hanging at a distance 0.4 m vertically below O , P is projected horizontally with speed 5 m s^{-1} and moves in a complete vertical circle. Calculate the tension in the string when P is vertically above O . [6]

2



When a tennis ball of mass 0.057 kg bounces it receives an impulse of magnitude $I \text{ N s}$ at an angle of θ to the horizontal. Immediately before the ball bounces it has speed 28 m s^{-1} in a direction of 30° to the horizontal. Immediately after the ball bounces it has speed 10 m s^{-1} in a direction of 30° to the horizontal (see diagram). Find I and θ . [7]

3



Two identical uniform rods, AB and BC , are freely jointed to each other at B , and A is freely jointed to a fixed point. The rods are in limiting equilibrium in a vertical plane, with C resting on a rough horizontal surface. AB is horizontal, and BC is inclined at 60° to the horizontal. The weight of each rod is 160 N (see diagram).

- (i) By taking moments for AB about A , find the vertical component of the force on AB at B . Hence or otherwise find the magnitude of the vertical component of the contact force on BC at C . [3]
- (ii) Calculate the magnitude of the frictional force on BC at C and state its direction. [4]
- (iii) Calculate the value of the coefficient of friction at C . [2]
- 4 A particle P of mass 0.2 kg is suspended from a fixed point O by a light elastic string of natural length 0.7 m and modulus of elasticity 3.5 N . P is at the equilibrium position when it is projected vertically downwards with speed 1.6 m s^{-1} . At time $t \text{ s}$ after being set in motion P is $x \text{ m}$ below the equilibrium position and has velocity $v \text{ m s}^{-1}$.
- (i) Show that the equilibrium position of P is 1.092 m below O . [3]
- (ii) Prove that P moves with simple harmonic motion, and calculate the amplitude. [5]
- (iii) Calculate x and v when $t = 0.4$. [5]

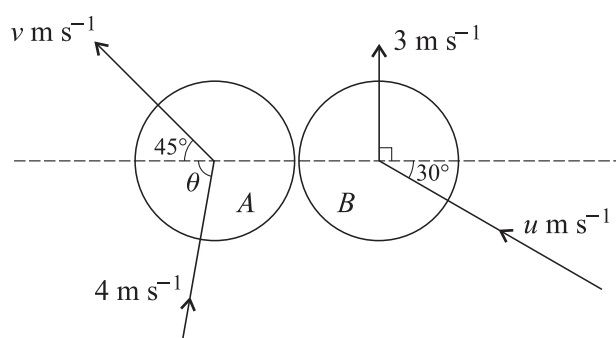
- 5 The pilot of a hot air balloon keeps it at a fixed altitude by dropping sand from the balloon. Each grain of sand has mass m kg and is released from rest. When a grain has fallen a distance x m, it has speed v m s⁻¹. Each grain falls vertically and the only forces acting on it are its weight and air resistance of magnitude mkv^2 N, where k is a positive constant.

(i) Show that $\left(\frac{v}{g - kv^2}\right) \frac{dv}{dx} = 1$. [2]

(ii) Find v^2 in terms of k , g and x . Hence show that, as x becomes large, the limiting value of v is $\sqrt{\frac{g}{k}}$. [7]

- (iii) Given that the altitude of the balloon is 300 m and that each grain strikes the ground at 90% of its limiting velocity, find k . [3]

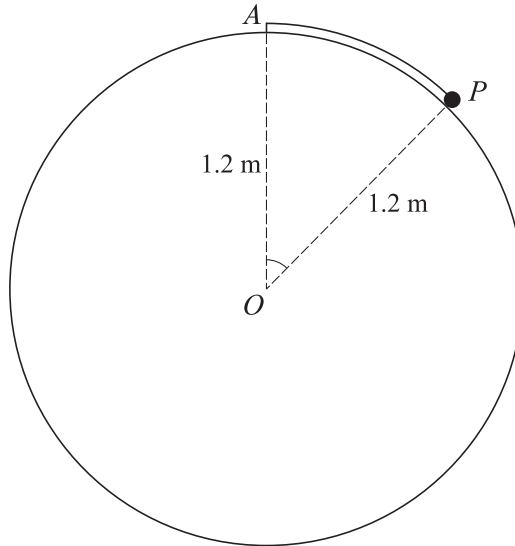
6



Two uniform smooth spheres A and B of equal radius are moving on a horizontal surface when they collide. A has mass 0.4 kg, and B has mass m kg. Immediately before the collision, A is moving with speed 4 m s⁻¹ at an acute angle θ to the line of centres, and B is moving with speed u m s⁻¹ at 30° to the line of centres. Immediately after the collision A is moving with speed v m s⁻¹ at 45° to the line of centres, and B is moving with speed 3 m s⁻¹ perpendicular to the line of centres (see diagram).

- (i) Find u . [2]
- (ii) Given that $\theta = 88.1^\circ$ correct to 1 decimal place, calculate the approximate values of v and m . [5]
- (iii) The coefficient of restitution is 0.75 . Show that the exact value of θ is a root of the equation $8 \sin \theta - 6 \cos \theta = 9 \cos 30^\circ$. [5]

[Question 7 is printed overleaf.]



The diagram shows a particle P of mass 0.5 kg attached to the highest point A of a fixed smooth sphere by a light elastic string. The sphere has centre O and radius 1.2 m . The string has natural length 0.6 m and modulus of elasticity 6.86 N . P is released from rest at a point on the surface of the sphere where the acute angle AOP is at least 0.5 radians.

(i) (a) For the case angle $AOP = \alpha$, P remains at rest. Show that $\sin \alpha = 2.8\alpha - 1.4$. [4]

(b) Use the iterative formula

$$\alpha_{n+1} = \frac{\sin \alpha_n}{2.8} + 0.5,$$

with $\alpha_1 = 0.8$, to find α correct to 2 significant figures. [2]

(ii) Given instead that angle $AOP = 0.5$ radians when P is released, find the speed of P when angle $AOP = 0.8$ radians, given that P is at all times in contact with the surface of the sphere. State whether the speed of P is increasing or decreasing when angle $AOP = 0.8$ radians. [7]