4754 (C4) Applications of Advanced Mathematics

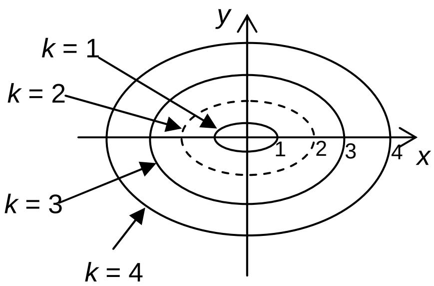
Section A

1 $3\cos\theta + 4\sin\theta = R\cos(\theta-\alpha)$ $= R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = 4$ $\Rightarrow R^2 = 3^2+4^2 = 25 \Rightarrow R = 5$ $\tan\alpha = 4/3 \Rightarrow \alpha = 0.9273$ $5\cos(\theta-0.9273) = 2$ $\Rightarrow \cos(\theta-0.9273) = 2/5$ $\theta-0.9273 = 1.1593, -1.1593$ $\Rightarrow \theta = 2.087, -0.232$	M1 B1 M1A1 M1 A1 A1 [7]	$R = 5$ cwo and no others in the range
2(i) $(1-2x)^{-\frac{1}{2}} = 1 - \frac{1}{2}(-2x) + \frac{(-\frac{1}{2}).(-\frac{3}{2})}{2!}(-2x)^2 + \dots$ $= 1 + x + \frac{3}{2}x^2 + \dots$ Valid for $-1 < -2x < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$	M1 A1 A1 M1 A1 [5]	binomial expansion with $p = -\frac{1}{2}$ correct expression cao
(ii) $\frac{1+2x}{\sqrt{1-2x}} = (1+2x)(1+x+\frac{3}{2}x^2+\dots)$ $= 1 + x + \frac{3}{2}x^2 + 2x + 2x^2 + \dots$ $= 1 + 3x + \frac{7}{2}x^2 + \dots$	M1 A1ft A1 [3]	substitituting their $1+x+\frac{3}{2}x^2+\dots$ and expanding cao
3 $V = \int_1^2 \pi x^2 dy$ $y = 1 + x^2 \Rightarrow x^2 = y - 1$ $\Rightarrow V = \int_1^2 \pi(y-1)dy$ $= \pi\left[\frac{1}{2}y^2 - y\right]_1^2$ $= \pi(2 - 2 - \frac{1}{2} + 1)$ $= \frac{1}{2}\pi$	 B1 M1 B1 M1 A1 [5]	 $\left[\frac{1}{2}y^2 - y\right]$ substituting limits into integrand

4(i) $\sin(\theta + 45°) = \cos\theta$ $\Rightarrow \sin\theta\cos 45 + \cos\theta\sin 45 = \cos\theta$ $\Rightarrow (1/\sqrt{2})\sin\theta + (1/\sqrt{2})\cos\theta = \cos\theta$ $\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\cos\theta$ $\Rightarrow \sin\theta = (\sqrt{2} - 1)\cos\theta$ $\Rightarrow \dfrac{\sin\theta}{\cos\theta} = \tan\theta = \sqrt{2} - 1$ *	M1 B1 A1 M1 E1 [5]	compound angle formula sin 45 = $1/\sqrt{2}$, cos 45 = $1/\sqrt{2}$ collecting terms
(ii) $\tan\theta = \sqrt{2} - 1$ $\Rightarrow \theta = 22.5°$, $202.5°$	B1 B1 [2]	 and no others in the range
5 $\dfrac{4}{x(x^2+4)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+4}$ $= \dfrac{A(x^2+4)+(Bx+C)x}{x(x^2+4)}$ $\Rightarrow 4 = A(x^2+4)+(Bx+C)x$ $x = 0 \Rightarrow 4 = 4A \Rightarrow A = 1$ coefft of x^2: $0 = A + B \Rightarrow B = -1$ coeffts of x: $0 = C$ $\Rightarrow \dfrac{4}{x(x^2+4)} = \dfrac{1}{x} - \dfrac{x}{x^2+4}$	M1 M1 B1 DM1 A1 A1 [6]	correct partial fractions A=1 Substitution or equating coeffts B= –1 C= 0
6 cosec $\theta = 3$ $\Rightarrow \sin\theta = 1/3$ $\Rightarrow \theta = 19.47°$, $160.53°$	M1 A1 A1 [3]	 and no others in the range

Section B

7(i) $\overrightarrow{CD} = \begin{pmatrix} -6 \\ 6 \\ 24 \end{pmatrix}$ $\overrightarrow{CB} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix}$.	B1 B1 [2]	
(ii) $\sqrt{(-6)^2+6^2+24^2}$ = 25.46 cm	M1 A1 [2]	
(iii) $\overrightarrow{CD}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 6 \\ 24 \end{pmatrix}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = -24+0+24=0$ $\overrightarrow{CB}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = 0+0+0=0$ $\Rightarrow$ plane BCDE is $4x + z = c$ At C (say) $4\times15 + 0 = c \Rightarrow c = 60$ $\Rightarrow$ plane BCDE is $4x + z = 60$	M1 A1 B1 M1 A1 [5]	using scalar product or other equivalent methods
(iv) OG: $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 6 \\ 24 \end{pmatrix}$ AF: $\mathbf{r} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -6 \\ 24 \end{pmatrix}$ At (5, 10, 40) , $3\lambda = 5 \Rightarrow \lambda = 5/3$ $\Rightarrow 6\lambda = 10$, $24\lambda = 40$, so consistent. At (5, 10, 40) , $3\mu = 5 \Rightarrow \mu = 5/3$ $\Rightarrow 20 - 6\mu = 10$, $24\mu = 40$, so consistent. So lines meet at (5, 10, 40)*	B1 B1 M1 E1 E1 [5]	 evaluating parameter and checking consistency. [or other methods, e.g. solving]
(v) h=40 POABC: $V = 1/3 \times 20 \times 15 \times 40$ $= 4000$ cm^3. PDEFG: $V = 1/3 \times 8 \times 6 \times (40-24)$ $= 256$ cm^3 $\Rightarrow$ vol of ornament = $4000 - 256 = 3744$ cm^3	B1 M1 A1 A1 [4]	soi 1/3 x w x d x h used for either –condone one error both volumes correct cao

8(i) $\cos\theta = \frac{x}{k}, \sin\theta = \frac{2y}{k}$ $\cos^2\theta + \sin^2\theta = 1$ $\Rightarrow \left(\frac{x}{k}\right)^2 + \left(\frac{2y}{k}\right)^2 = 1$ $\Rightarrow \frac{x^2}{k^2} + \frac{4y^2}{k^2} = 1$ $\Rightarrow x^2 + 4y^2 = k^2$ *	M1 M1 E1 [3]	Used substitution
(ii) $\frac{dx}{d\theta} = -k\sin\theta, \frac{dy}{d\theta} = \frac{1}{2}k\cos\theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{\frac{1}{2}k\cos\theta}{k\sin\theta}$ $= -\frac{1}{2}\cot\theta$ $-\frac{x}{4y} = -\frac{2k\cos\theta}{4k\sin\theta} = -\frac{1}{2}\cot\theta = \frac{dy}{dx}$	M1 A1 E1	oe
or, by differentiating implicitly $2x + 8y\, dy/dx = 0$ $\Rightarrow dy/dx = -2x/8y = -x/4y$*	M1 A1 E1 [3]	
(iii) $k = 2$	B1 [1]	
(iv) 	B1 B1 B1 [3]	1 correct curve –shape and position 2 or more curves correct shape- in concentric form all 3 curves correct
(v) grad of stream path = –1/grad of contour $\Rightarrow \frac{dy}{dx} = -\frac{1}{(-x/4y)} = \frac{4y}{x}$ *	M1 E1 [2]	
(vi) $\frac{dy}{dx} = \frac{4y}{x} \Rightarrow \int\frac{dy}{y} = \int\frac{4dx}{x}$ $\Rightarrow \ln y = 4\ln x + c = \ln e^c x^4$ $\Rightarrow y = Ax^4$ where $A = e^c$. When $x = 2, y = 1 \Rightarrow 1 = 16A \Rightarrow A = 1/16$ $\Rightarrow y = x^4/16$ *	M1 A1 M1 M1 A1 E1 [6]	Separating variables $\ln y = 4\ln x\ (+c)$ antilogging correctly (at any stage) substituting $x = 2, y = 1$ evaluating a correct constant www

Paper B Comprehension 4754 (C4)

1	4, 1, 5, 6, 11, 17	B1 B1	for 11 and 17 for 1 and 4
2	Even, odd, odd, even, odd, odd recurs 100th term is therefore even	M1 A1	for reason www
3	$\phi^6 = (3\phi+2)+(5\phi+3) = 8\phi+5$	B1	
4	$1-EH = 1-CG = 1-(\phi-1)$ $= 2-\phi = 2-\left(\frac{1+\sqrt{5}}{2}\right)$ $= \frac{3-\sqrt{5}}{2}$	M1 A1 A1	oe
5	(i)Gradients $-\frac{1}{\phi}$ and $\frac{1}{\phi-1}$ (ii) Product of gradients: $-\frac{1}{\phi}\times\frac{1}{\phi-1} = -\frac{1}{\phi^2-\phi}$ $= -\frac{1}{1} = -1$	B1 B1 M1 E1	
6	$\frac{\phi+1}{2\phi-1} = \frac{\frac{1+\sqrt{5}}{2}+1}{1+\sqrt{5}-1}$ $= \frac{3+\sqrt{5}}{2\sqrt{5}}$ $= \frac{(3+\sqrt{5})\sqrt{5}}{2\sqrt{5}\times\sqrt{5}} = \frac{3\sqrt{5}+5}{10}$	M1 A1 E1	
7	$a+(a+d) = a+2d \quad \Rightarrow \quad a=d$ $(a+d)+(a+2d) = a+3d \quad \Rightarrow \quad a=0$ $a=d=0$ *	M1 M1 E1 [18]	