4730 Mechanics 3

1	(i) [$0.5(v_x - 5) = -3.5$, $0.5(v_y - 0) = 2.4$]	M1		For using I = m(v – u) in x or y direction
	Component of velocity in x-direction is -2ms^{-1}	A1		
	Component of velocity in y-direction is 4.8ms^{-1}	A1		
	Speed is 5.2ms^{-1}	A1	4	AG
SR For candidates who obtain the speed without finding the required components of velocity (max 2/4)				
	Components of momentum after impact are -1 and 2.4 Ns	B1		
	Hence magnitude of momentum is 2.6 Ns and required speed is $2.6/0.5 = 5.2\text{ms}^{-1}$	B1		
	(ii)	M1		For using $I_y = m(0 - v_y)$ or I_y = -y-component of 1st impulse
	Component is –2.4Ns	A1	2	

2	(i)	M1		For 2 term equation, each term representing a relevant moment
	$50\text{x}1\sin\beta = 75\text{x}2\cos\beta$	A1		
	$\tan\beta = 3$	A1	3	AG
	(ii) Horizontal force is 75N	B1		
	Vertical force is 50N	B1	2	
	(iii)	M1		For taking moments about A for the whole or for AB only
	For not more than one error in $W\text{x}1\sin\alpha + 50(2\sin\alpha + 1\sin\beta) = 75(2\cos\alpha + 2\cos\beta)$ or $W\text{x}1\sin\alpha + 50\text{x}2\sin\alpha = 75\text{x}2\cos\alpha$	A1		Where $\tan\alpha = 0.75$
	0.6W + 107.4... = 167.4… or 0.6W + 60 = 120	A1		
	W = 100	A1	4	

3	(i)	M1		For using the principle of conservation of momentum in the **i** direction
	6x4 – 3x8 = 6a + 3b (0 = 2a + b)	A1		
		M1		For using NEL
	(4 + 8)e = b – a (12e = b – a)	A1		
	Component is $4e\text{ ms}^{-1}$ to the left	A1	5	‘to the left’ may be implied by a = -4e and arrow in diagram
	(ii) $b = 8e\text{ ms}^{-1}$	B1ft		ft b = -2a or b = a + 12e
		M1		For using ‘**j** component of A’s velocity remains unchanged’
	$(8e)^2 = (4e)^2 + v^2$	A1ft		ft $b^2 = a^2 + v^2$
	v = 4	A1	4	

4	(i) [mg – 0.49mv = ma]	M1		For using Newton’s second law
	$mv\frac{dv}{dx} = mg - 0.49mv$	A1		
	$\left[\frac{v(dv/dx)}{g - 0.49v} = 1\right]$	M1		For relevant manipulation
	$\left[\frac{v}{9.8 - 0.49v} \equiv \frac{-1}{0.49}\left(\frac{(9.8 - 0.49v) - 9.8}{9.8 - 0.49v}\right)\right]$	M1		For synthetic division of v by g - 0.49v, or equivalent
	$\left(\frac{20}{20 - v} - 1\right)\frac{dv}{dx} = 0.49$	A1	5	AG
	(ii)	M1		For separating the variables and integrating
	$\int\frac{20}{20 - v}dv = -20\ln(20 - v)$	B1		
	-20 ln(20 – v) –v = 0.49x (+C)	A1ft		
	[-20 ln20 = C]	M1		For using v = 0 when x = 0
	x = 40.8(ln20 – ln(20 – v)) – 2.04v	A1	5	Accept any correct form

5	(i)	M1		For using Newton's second law with a = 0
	$mg\sin 30^o = 0.75mgx/1.2$	A1		
	Extension is 0.8m	A1	3	AG
	(ii) PE loss = $mg(1.2 + 0.8)\sin 30^o$ (mg)	B1		
	EE gain = $0.75mg(0.8)^2/(2\text{x}1.2)$ (0.2mg)	B1		
	[$½ mv^2 = mg – 0.2mg$]	M1		For an equation with terms representing PE, KE and EE in linear combination
	Maximum speed is $3.96ms^{-1}$	A1	4	
	(iii) PE loss = $mg(1.2 + x)\sin 30^o$ or $mgd\sin 30^o$	B1ft		ft with x or d – 1.2 replacing 0.8 in (ii)
	EE gain = $0.75mgx^2/(2\text{x}1.2)$ or $0.75mg(d – 1.2)^2/(2\text{x}1.2)$	B1ft		ft with x or d – 1.2 replacing 0.8 in (ii)
	[$x^2 – 1.6x – 1.92 = 0$, $d^2 – 4d + 1.44 = 0$]	M1		For using PE loss = EE gain to obtain a 3 term quadratic in x or d
	Displacement is 3.6m	A1	4	
Alternative for parts (ii) and (iii) for candidates who use Newton's second law and a = v dv/dx: In the following x, y and z represent displacement from equil. pos^n, extension, and distance OP respectively.				
	[$mv\,dv/dx = mg\sin 30^o – 0.75mg(0.8 + x)/1.2$, $mv\,dv/dy = mg\sin 30^o – 0.75mgy/1.2$, $mv\,dv/dz = mg\sin 30^o – 0.75mg(z – 1.2)/1.2$]	M1		For using N2 with a = v dv/dx
	$v^2/2 = – 5gx^2/16 + C$ or $v^2/2 = gy/2 – 5gy^2/16 + C$ or $v^2/2 = 5gz/4 – 5gz^2/16 + C$	A1		
	[$C = 0.6g + 5g(-0.8)^2/16$ or $C = 0.6g$ or $C = 0.6g – 5g(1.2/4) + 5g(1.2)^2/16$	M1		For using $v^2(-0.8)$ or $v^2(0)$ or $v^2(1.2) = 2(g \sin 30^o)1.2$ as appropriate
	$v^2 = (-5x^2/8 + 1.6)g$ or $v^2 = (y - 5y^2/8 + 1.2)g$ or $v^2 = (5z/2 -5z^2/8 – 0.9)g$	A1		
	(ii) [$v_{max}{}^2 = 1.6g$ or $0.8g – 0.4g + 1.2g$ or $5g – 2.5g – 0.9g$]	M1		For using $v_{max}{}^2 = v^2(0)$ or $v^2(0.8)$ or $v^2(2)$ as appropriate
	Maximum speed is $3.96ms^{-1}$	A1		
	(iii) [$5x^2 - 12.8 = 0 \Rightarrow x = 1.6$, $5y^2 – 8y – 9.6 = 0 \Rightarrow y = 2.4$, $5z^2 – 20z + 7.2 = 0 \Rightarrow z = 3.6$]	M1		For solving v = 0
	Displacement is 3.6m	A1	8	
Alternative for parts (ii) and (iii) for candidates who use Newton's second law and SHM analysis.				
	[$m\ddot{x} = mg\sin 30^o – 0.75mg(0.8 + x)/1.2 \Rightarrow$ $\ddot{x} = -\omega^2x$; $v^2 = \omega^2(a^2 – x^2)$]	M1		For using N2 with $v^2 = \omega^2(a^2 – x^2)$
	$v^2 = 5g(a^2 – x^2)/8$	A1		
		M1		For using $v^2(-0.8) = 2(g\sin 30^o)1.2$
	$v^2 = 5g(2.56 – x^2)/8$	A1		
	(ii) [$v_{max}{}^2 = 5g \text{ x } 2.56 \div 8$]	M1		For using $v_{max}{}^2 = v^2(0)$
	Maximum speed is $3.96ms^{-1}$	A1		
	(iii) [$2.56 – x^2 = 0 \Rightarrow x = 1.6$]	M1		For solving v = 0
	Displacement is 3.6m	A1		

6	(i) [$\frac{1}{2}$m7^2 = $\frac{1}{2}$mv^2 + 2mg]	M1		For using the principle of conservation of energy
	Speed is 3.13ms^{-1}	A1		
	[T = mv^2/r]	M1		For using Newton's second law horizontally and a = v^2/r
	Tension is 1.96N	A1ft	4	
	(ii) [T – mgcosθ = mv^2/r]	M1		For using Newton's second law radially
		M1		For using T = 0 (may be implied)
	v^2 = -2gcosθ	A1		
		M1		For using the principle of conservation of energy
	$\frac{1}{2}$m7^2 = $\frac{1}{2}$mv^2 +mg(2 – 2cosθ)	A1		
	[-2gcosθ = 49 –4g + 4gcosθ]	M1		For eliminating v^2
	6gcosθ = -9.8	A1		May be implied by answer
	θ = 99.6	A1	8	
Alternative for candidates who eliminate v^2 before using T = 0.				
	(ii) [T – mgcosθ = mv^2/r]	M1		For using Newton's second law radially
		M1		For using the principle of conservation of energy
	$\frac{1}{2}$m7^2 = $\frac{1}{2}$mv^2 +mg(2 – 2cosθ)	A1		
	[T - mgcosθ = m(49 –4g + 4gcosθ)2]	M1		For eliminating v^2
		M1		For using T = 0 (may be implied)
	-2g cosθ = 49 –4g + 4gcosθ	A1ft		ft error in energy equation
	6gcosθ = -9.8	A1		May be implied by answer
	θ = 99.6	A1	8	

7	(i) T = 4mg(4 + x – 3.2)/3.2	B1		
	[ma = mg – 4mg(0.8 + x)/3.2]	M1		For using Newton's second law
	4$\ddot{x}$ = -49x	A1	3	AG
	(ii) Amplitude is 0.8m	B1		(from 4 + A = 4.8)
	Period is $2\pi/\omega$ s where ω^2 = 49/4	B1		
		M1		String is instantaneously slack when shortest (4 - A = 3.2 = L). Thus required interval length = period.
	Slack at intervals of 1.8s	A1	4	AG
	(iii) [ma = -mgsinθ]	M1		For using Newton's second law tangentially
	mL$\ddot{\theta}$ = -mgsinθ	A1		
	For using sin$\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx$ –(g/L)θ	A1	3	AG
	(iv) [θ = 0.08cos(3.5x0.25)] (= 0.05127..)	M1		For using = $_o$cosωt where ω^2=12.25 (may be implied by $\dot{\vartheta}$ = -ω $_o$sinωt)
	[$\dot{\theta}$ = -3.5(0.08)sin(3.5x0.25), $\dot{\theta}^2$ = 12.25(0.08^2 – 0.05127..^{2})]	M1		For differentiating = $_o$cosωt and using $\dot{\vartheta}$ or for using $\dot{\theta}^2 = \omega^2(\theta_o{}^2 - \theta^2)$ where ω^2=12.25
	$\dot{\theta} = \mp 0.215$	A1		May be implied by final answer
	[v = 0.215x9.8/12.25]	M1		For using v = L$\dot{\vartheta}$ and L =g/ω^2
	Speed is 0.172 ms^{-1}	A1	5	