4730 Mechanics 3

1	(i) $[0.5(v_x - 5) = -3.5, 0.5(v_y - 0) = 2.4]$	M1		For using $I = m(v - u)$ in x or y direction
	Component of velocity in x-direction is $-2ms^{-1}$	A1		
	Component of velocity in y-direction is 4.8ms ⁻¹	A1		
	Speed is 5.2ms ⁻¹	A1	4	AG
SR For	candidates who obtain the speed without finding the required	componen	ts of v	elocity (max 2/4)
	Components of momentum after impact are -1 and 2.4 Ns	B1		
	Hence magnitude of momentum is 2.6 Ns and required	B1		
	speed is 2.6/0.5 = 5.2 ms^{-1}	21		
	(ii)	M1		For using $I = m(0 - y)$ or
	(11)			$I = -v_{z}$ component of 1 st impulse
	Component is -2.4 Ns	Δ1	2	ry = y component of 1 mipulse
	Component is -2.4143	Π	2	
2	(i)	M1		For 2 term equation each term
-				representing a relevant moment
	$50 \times 1 \sin \beta = 75 \times 2 \cos \beta$	A1		representation and the same moment
	$50x1\sin\rho = 75x2\cos\rho$			
	$\tan \beta = 3$	Al	3	AG
	(ii) Horizontal force is 75N	B1		
	Vertical force is 50N	B1	2	
+	(iii)	M1		For taking moments about A for the
	()			whole or for AB only
	For not more than one error in	Δ1		Where $\tan \alpha = 0.75$
	Wy Leip $\alpha + 50(2 \sin \alpha + 1 \sin \beta) =$	111		
	$wx1\sin\alpha + 30(2\sin\alpha + 1\sin\beta) =$			
	$75(2\cos\alpha + 2\cos\beta)$ or Wx1sin α +			
	$50x2\sin \alpha = 75x2\cos \alpha$			
	0.6W + 107.4 = 167.4 or $0.6W + 60 = 120$	Δ1		
	W = 100	Δ1	4	
	11 - 100	711		
3	(i)	M1		For using the principle of conservation
-				of momentum in the i direction
	6x4 - 3x8 = 6a + 3b (0 = 2a + b)	A1		
		M1		For using NEL
	(4+8)e = b - a $(12e = b - a)$	A1		
	Component is $4e \text{ ms}^{-1}$ to the left	A1	5	'to the left' may be implied by
	component is to mis to the fert	111	5	a = -4e and arrow in diagram
+	(ii) $h - 8e ms^{-1}$	B1ft		$ft h = 2a \text{ or } h = a \pm 12a$
	(1) $0 = 00 ms$	M1		For using 'i component of A's valority
		IVII		romains unchanged'
	$(8a)^2 - (4a)^2 + y^2$	A 1 ft		ft $b^2 - a^2 + y^2$
	$ \begin{array}{c} (00) - (40) \pm v \\ v - \Lambda \end{array} $		Λ	$10 - a \pm v$
	v – 1	AI	4	1
4	(i) $[mg - 0.49mv = ma]$	M1		For using Newton's second law
1	dv	A1		i or using reason is second law
	$mv \frac{dv}{dv} = mg - 0.49 mv$	111		
	$\begin{bmatrix} ux \\ y(dy + dx) \end{bmatrix}$	M1		For relevant manipulation
	$\left \frac{v(uv / ux)}{g - 0} \frac{49}{49} \right _{u} = 1$	1111		For relevant manipulation
		M1		For synthetic division of y by
	$\left \frac{v}{0.8 - 0.49} = \frac{-1}{0.40} \left \frac{(9.8 - 0.49)(-9.8)}{0.8 - 0.49} \right \right $	1011		a = 0.40 v or equivalent
	$\begin{bmatrix} 2.0 & -0.47 & 0.47 & 0.47 & 9.0 & -0.49 & 0 \end{bmatrix}$	A 1	5	
	$\left[\frac{20}{20 - v} - 1 \right] \frac{dv}{dx} = 0.49$	AI	5	AU
<u>}</u>	(ii)	M1	+	For separating the variables and
	(11)	1411		integrating
	. 20	B1		Integrating
	$\int \frac{20}{20} dv = -20 \ln(20 - v)$	DI		
	-20 - v 20 lp(20, v), $v = 0.40v$, (+C)	A 1 f4		
	$-20 \ln(20 - V) - V = 0.49X$ (+C)	AIII M1		For using u = 0 - there = 0
	$\begin{bmatrix} -20 \ln 20 = C \end{bmatrix}$		-	For using $v = 0$ when $x = 0$
	$x = 40.8(\ln 20 - \ln(20 - v)) - 2.04v$	AI	5	Accept any correct form

5	(i)	M1		For using Newton's second law with a –
5		1411		1 of using reaction is second raw with $a = 0$
	$maxin 20^{\circ} - 0.75 max/1.2$	A 1		0
	$\frac{1100}{1000} = 0.751100 \times 1.2$		2	
		AI	3	AG
	(11) PE loss = mg(1.2 + 0.8)sin 30°	BI		
	(mg)			
	EE gain = $0.75 \text{mg}(0.8)^2 / (2 \times 1.2)$ (0.2mg)	B1		
	$[\frac{1}{2} \text{ mv}^2 = \text{mg} - 0.2\text{mg}]$	M1		For an equation with terms representing
				PE, KE and EE in linear combination
	Maximum speed is 3.96ms ⁻¹	A1	4	
	(iii) PE loss = $mg(1.2 + x)sin30^{\circ}$ or	B1ft		ft with x or $d - 1.2$ replacing 0.8 in (ii)
	mgdsin30°			
	$FE gain = 0.75 mgx^2/(2x1.2)$ or	B1ft		ft with x or $d = 1.2$ replacing 0.8 in (ii)
	$\frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}$	Din		
	$0.75 \text{ mg}(d = 1.2)7(2 \times 1.2)$	MI		For using DE loss - EE gain to obtain a
	[x - 1.0x - 1.92 = 0, u - 4u + 1.44 = 0]	IVI I		For using PE loss = EE gain to obtain a $2 \text{ terms and } 1$
				3 term quadratic in x or d
	Displacement is 3.6m	AI	4	
Alternat	ive for parts (ii) and (iii) for candidates who use Newton's se	cond law a	and $a =$	v dv/dx:
In the fo	llowing x, y and z represent displacement from equil. pos ⁿ , e	xtension, a	nd dist	ance OP respectively.
	$[mv dv/dx = mgsin30^{\circ} - 0.75mg(0.8 + x)/1.2,$	M1		For using N2 with $a = v dv/dx$
	$mv dv/dy = mgsin30^{\circ} - 0.75mgy/1.2,$			
	$mv dv/dz = mgsin30^{\circ} - 0.75mg(z - 1.2)/1.2$			
	$v^2/2 = -5gx^2/16 + C$ or	A1		
	$v^{2}/2 = gv/2 - 5gv^{2}/16 + C$ or			
	$v^{2}/2 = 5g^{2}/4 - 5g^{2}/16 + C$			
	$[C = 0.6g + 5g(-0.8)^2/16 \text{ or } C = 0.6g \text{ or}$	M1		For using $y^2(-0.8)$ or $y^2(0)$ or $y^2(1.2) =$
	C = 0.6g + 3g(-0.3)/10 of C = 0.0g of	IVI I		$2(a \sin 20^{\circ}) 1.2$ as appropriate
	C = 0.0g - 3g(1.2/4) + 3g(1.2)/10 $v^2 = (5v^2/9 + 1.6) = avv^2 = (v - 5v^2/9 + 1.2) = avv^2 = (5v/9)$	A 1		2(g sinso)1.2 as appropriate
	v = (-5x/8 + 1.0)g or v = (y - 5y/8 + 1.2)g or v = (52/2)	AI		
	$-5Z^{2}/8 - 0.9)g$			
	(ii) $[v_{max}^2 = 1.6g \text{ or } 0.8g - 0.4g + 1.2g \text{ or } 5g - 2.5g$	M1		For using $v_{max}^2 = v^2(0)$ or $v^2(0.8)$ or
	- 0.9g]			$v^{2}(2)$ as appropriate
	Maximum speed is 3.96ms ⁻¹	A1		
	(iii) $[5x^2 - 12.8 = 0 \rightarrow x = 1.6,$	M1		For solving $v = 0$
	$5y^2 - 8y - 9.6 = 0 \Rightarrow y = 2.4,$			
	$5z^2 - 20z + 7.2 = 0 \Rightarrow z = 3.6$			
	Displacement is 3.6m	A1	8	
Alternat	ive for parts (ii) and (iii) for candidates who use Newton's se	cond law a	nd SH	M analysis.
	$\int \frac{1}{2\pi} \frac{1}{x} = \frac{1}{2\pi} \frac{1}{2\pi$	M1		For using N2 with
	$\lim x = \limsup_{x \to 0} \frac{1}{2} - \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} $			$v^2 = \omega^2 (a^2 - x^2)$
	$x = -\omega^2 x; v^2 = \omega^2 (a^2 - x^2)$			$\mathbf{v} = \mathbf{\omega} \left(\mathbf{a} - \mathbf{x} \right)$
	$v^2 = 5g(a^2 - x^2)/8$	A1		
		M1		For using $v^2(-0.8) =$
				$2(gsin30^{\circ})1.2$
	$v^2 = 5g(2.56 - x^2)/8$	A1		
	(ii) $[v_{max}^2 = 5g \times 2.56 \div 8]$	M1		For using $v_{max}^2 = v^2(0)$
	Maximum speed is 3.96ms ⁻¹	A1		
	(iii) $[2.56 - x^2 - 0] \rightarrow x - 1.61$	M1		For solving $y = 0$
	$\begin{array}{c} (m) \qquad [2.30 - \Lambda - 0 \textbf{/} \Lambda - 1.0] \\ \text{Displacement is 3.6m} \end{array}$			1 or solving v = 0
	Displacement is 5.0m	AI		

6	(i) $[\frac{1}{2}m7^2 = \frac{1}{2}mv^2 + 2mg]$	M1		For using the principle of conservation
				of energy
	Speed is 3.13ms^2	Al M1		For using Newton's second low
	$\begin{bmatrix} \mathbf{I} = \mathbf{I} \mathbf{I} \mathbf{V} / \mathbf{I} \end{bmatrix}$	IVI I		horizontally and $a = v^2/r$
	Tension is 1.96N	A1ft	4	nonzontany and a = v /i
	(ii) $[T - mg\cos\theta = mv^2/r]$	M1		For using Newton's second law radially
		M1		For using $T = 0$ (may be implied)
	$v^2 = -2g\cos\theta$	A1		
		M1		For using the principle of conservation
				of energy
	$\frac{1}{2}m7^2 = \frac{1}{2}mv^2 + mg(2 - 2\cos\theta)$	AI		\mathbf{T} \mathbf{v} \mathbf{v} \mathbf{v} \mathbf{v} \mathbf{v}
	$[-2g\cos\theta = 49 - 4g + 4g\cos\theta]$	M1		For eliminating v^2
	$6g\cos\theta = -9.8$	A1		May be implied by answer
	θ = 99.6	A1	8	
Alternat	ive for candidates who eliminate v^2 before using $T = 0$.	1	1	
	(ii) $[T - mg\cos\theta = mv^2/r]$	M1		For using Newton's second law radially
		M1		For using the principle of conservation
	$1/m7^2$ $1/m2^2/mq(2-2mq(2))$	Δ1		or energy
	$\frac{72m}{2} = \frac{72m}{2} + mg(2 - 2\cos\theta)$	M1		For aliminating y^2
	$[1 - \operatorname{mgcos} \theta = \operatorname{m}(49 - 4g + 4g\cos\theta)2]$	IVI I		For eminating ∇
	$2\pi \cos \theta$ 40 $4\pi + 4\pi \cos \theta$	IVI I A 1ft		For using $I = 0$ (may be implied) ft error in energy equation
	$-2g\cos\theta = 49 - 4g + 4g\cos\theta$			May be implied by answer
	$6g\cos\theta = -9.8$		0	May be implied by answer
	$\theta = 99.6$	AI	0	
7	(i) $T = 4mg(4 + y - 2/2)/2/2$	D1		
/	$[1] 1 - 4 \ln g (4 + x - 5.2)/5.2$ [ma = mg - 4mg(0.8 + x)/3.2]	M1		For using Newton's second law
	$4\ddot{x} = -49x$	A1	3	AG
	(ii) Amplitude is 0.8m	B1		(from 4 + A = 4.8)
	Period is $2\pi/\omega$ s where $\omega^2 - 49/4$	B1		
	$1 \text{ chod is } 2\pi + 65 \text{ where } 65 = 49/4$	M1		String is instantaneously slack when
				shortest (4 - $A = 3.2 = L$). Thus required
				shortest (4 - $A = 3.2 = L$). Thus required interval length = period.
	Slack at intervals of 1.8s	A1	4	shortest (4 - A = 3.2 = L). Thus required interval length = period. AG
	Slack at intervals of 1.8s (iii) $[ma = -mgsin \theta]$	A1 M1	4	shortest (4 - A = 3.2 = L). Thus required interval length = period. AG For using Newton's second law
	Slack at intervals of 1.8s (iii) [ma = -mgsin θ]	A1 M1	4	shortest (4 - A = 3.2 = L). Thus required interval length = period. AG For using Newton's second law tangentially
	Slack at intervals of 1.8s (iii) [ma = -mgsin θ] mL $\ddot{\theta}$ = -mgsin θ	A1 M1 A1	4	shortest (4 - A = 3.2 = L). Thus required interval length = period. AG For using Newton's second law tangentially
	Slack at intervals of 1.8s (iii) [ma = -mgsin θ] mL $\ddot{\theta}$ = -mgsin θ For using sin $\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx$	A1 M1 A1 A1	4	shortest (4 - A = 3.2 = L). Thus required interval length = period. AG For using Newton's second law tangentially AG
	Slack at intervals of 1.8s (iii) [ma = -mgsin θ] mL $\ddot{\theta}$ = -mgsin θ For using sin $\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx$ -(g/L) θ	A1 M1 A1 A1	43	shortest (4 - A = 3.2 = L). Thus required interval length = period. AG For using Newton's second law tangentially AG
	Slack at intervals of 1.8s (iii) [ma = -mgsin θ] mL $\ddot{\theta}$ = -mgsin θ For using sin $\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx$ -(g/L) θ (iv) [θ = 0.08cos(3.5x0.25)] (= 0.05127)	A1 M1 A1 A1 M1	3	shortest (4 - A = $3.2 = L$). Thus required interval length = period. AG For using Newton's second law tangentially AG For using = $_{0}\cos\omega t$ where $\omega^{2}=12.25$
	Slack at intervals of 1.8s (iii) [ma = -mgsin θ] mL $\ddot{\theta}$ = -mgsin θ For using sin $\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx$ -(g/L) θ (iv) [θ = 0.08cos(3.5x0.25)] (= 0.05127)	A1 M1 A1 A1 M1	3	shortest (4 - A = 3.2 = L). Thus required interval length = period. AG For using Newton's second law tangentially AG For using = $_{o}cos\omega t$ where $\omega^{2}=12.25$ (may be implied by $\dot{\mathcal{G}} = -\omega _{o}sin\omega t$)
	Slack at intervals of 1.8s (iii) [ma = -mgsin θ] mL $\ddot{\theta}$ = -mgsin θ For using sin $\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx$ -(g/L) θ (iv) [θ = 0.08cos(3.5x0.25)] (= 0.05127) [$\dot{\theta}$ = -3.5(0.08)sin(3.5x0.25),	A1 M1 A1 A1 M1 M1	3	shortest (4 - A = $3.2 = L$). Thus required interval length = period. AG For using Newton's second law tangentially AG For using = $_{0}\cos\omega t$ where $\omega^{2}=12.25$ (may be implied by $\dot{\mathcal{G}} = -\omega_{0}\sin\omega t$) For differentiating = $_{0}\cos\omega t$ and
	Slack at intervals of 1.8s (iii) [ma = -mgsin θ] mL $\ddot{\theta}$ = -mgsin θ For using sin $\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx$ -(g/L) θ (iv) [θ = 0.08cos(3.5x0.25)] (= 0.05127) [$\dot{\theta}$ = -3.5(0.08)sin(3.5x0.25), $\dot{\theta}^2$ = 12.25(0.08 ² - 0.05127 ²)]	A1 M1 A1 A1 M1 M1	3	shortest (4 - A = 3.2 = L). Thus required interval length = period. AG For using Newton's second law tangentially AG For using = $_{o}cos\omega t$ where $\omega^{2}=12.25$ (may be implied by $\dot{\mathcal{G}} = -\omega _{o}sin\omega t$) For differentiating = $_{o}cos\omega t$ and using $\dot{\mathcal{G}}$ or for using
	Slack at intervals of 1.8s (iii) [ma = -mgsin θ] mL $\ddot{\theta}$ = -mgsin θ For using sin $\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx$ -(g/L) θ (iv) [θ = 0.08cos(3.5x0.25)] (= 0.05127) [$\dot{\theta}$ = -3.5(0.08)sin(3.5x0.25), $\dot{\theta}^2$ = 12.25(0.08 ² - 0.05127 ²)]	A1 M1 A1 A1 M1 M1	3	shortest (4 - A = 3.2 = L). Thus required interval length = period. AG For using Newton's second law tangentially AG For using = $_{0}cos\omega t$ where $\omega^{2}=12.25$ (may be implied by $\dot{\mathcal{Y}} = -\omega_{0}sin\omega t$) For differentiating = $_{0}cos\omega t$ and using $\dot{\mathcal{Y}}$ or for using $\dot{\mathcal{H}}^{2} = \omega^{2}(\theta_{0}^{2} - \theta^{2})$ where $\omega^{2}=12.25$
	Slack at intervals of 1.8s (iii) [ma = -mgsin θ] mL $\ddot{\theta}$ = -mgsin θ For using sin $\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx$ -(g/L) θ (iv) [θ = 0.08cos(3.5x0.25)] (= 0.05127) [$\dot{\theta}$ = -3.5(0.08)sin(3.5x0.25), $\dot{\theta}^2$ = 12.25(0.08 ² - 0.05127 ²)] $\dot{\theta}$ = \mp 0.215	A1 M1 A1 A1 M1 M1 A1	3	shortest (4 - A = 3.2 = L). Thus required interval length = period. AG For using Newton's second law tangentially AG For using = $_{0}\cos\omega t$ where $\omega^{2}=12.25$ (may be implied by $\dot{\mathcal{Y}} = -\omega_{0}\sin\omega t$) For differentiating = $_{0}\cos\omega t$ and using $\dot{\mathcal{Y}}$ or for using $\dot{\theta}^{2} = \omega^{2}(\theta_{o}^{2} - \theta^{2})$ where $\omega^{2}=12.25$ May be implied by final answer
	Slack at intervals of 1.8s (iii) [ma = -mgsin θ] mL $\ddot{\theta}$ = -mgsin θ For using sin $\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx$ -(g/L) θ (iv) [θ = 0.08cos(3.5x0.25)] (= 0.05127) [$\dot{\theta}$ = -3.5(0.08)sin(3.5x0.25), $\dot{\theta}^2$ = 12.25(0.08 ² - 0.05127 ²)] $\dot{\theta}$ = \mp 0.215 [$w = 0.215x9.8/12.25$]	A1 M1 A1 A1 M1 M1 A1 M1	3	shortest (4 - A = 3.2 = L). Thus required interval length = period. AG For using Newton's second law tangentially AG For using = $_{o}cos\omega t$ where $\omega^{2}=12.25$ (may be implied by $\dot{\mathcal{G}} = -\omega _{o}sin\omega t$) For differentiating = $_{o}cos\omega t$ and using $\dot{\mathcal{G}}$ or for using $\dot{\mathcal{G}}^{2} = \omega^{2} ({\mathcal{G}_{o}}^{2} - {\mathcal{G}}^{2})$ where $\omega^{2}=12.25$ May be implied by final answer
	Slack at intervals of 1.8s (iii) [ma = -mgsin θ] mL $\ddot{\theta}$ = -mgsin θ For using sin $\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx$ -(g/L) θ (iv) [θ = 0.08cos(3.5x0.25)] (= 0.05127) [$\dot{\theta}$ = -3.5(0.08)sin(3.5x0.25), $\dot{\theta}^2$ = 12.25(0.08 ² - 0.05127 ²)] $\dot{\theta}$ = \mp 0.215 [v = 0.215x9.8/12.25] c = 10.0172 = 1	A1 M1 A1 A1 M1 M1 A1 M1	3	shortest (4 - A = 3.2 = L). Thus required interval length = period. AG For using Newton's second law tangentially AG For using = $_{0}cos\omega t$ where $\omega^{2}=12.25$ (may be implied by $\dot{\mathcal{Y}} = -\omega_{0}sin\omega t$) For differentiating = $_{0}cos\omega t$ and using $\dot{\mathcal{Y}}$ or for using $\dot{\theta}^{2} = \omega^{2} (\theta_{0}^{2} - \theta^{2})$ where $\omega^{2}=12.25$ May be implied by final answer For using v = L $\dot{\mathcal{Y}}$ and L =g/ ω^{2}