

4734 Probability & Statistics 3

1(i)	$s^2 = 0.00356/80 + 0.00340/100$ $= 7.85 \times 10^{-5}$	M1 A1 2	Sum of variances Or pooled, giving 7.81×10^{-5}
(ii)	----- (1.36-1.24) $\pm$ zs z=1.96 (0.103, 0.137) -----	M1 B1 A1 3	Must be s, accept t -----
(iii)	Not necessary since sample sizes are large	B1 1 (6)	Or equivalent. Nothing wrong
2 (i)	Use $\bar{x} \pm z \frac{\sigma}{\sqrt{n}}$ $\bar{x} = 337.5 / 20$ z = 2.326 (14.9, 18.9)	M1 B1 B1 A1 4	3 or 4 SF
(ii)	----- $1 - 0.98^3$ 0.0588 -----	M1 A1 2	Use B(3,0.02) or B(3,0.98) for M. -----
(iii)	Unbiased estimate of σ^2 required t – distribution used to obtain CV	B1 B1 2 (8)	
3 (i)	$H_0: p_W = p_N, H_1: p_W > p_N$ Pooled $\hat{p} = \frac{71+73}{80+90} \quad (= \frac{144}{170})$ $s^2 = (144/170)(26/170)(1/80+1/90)$ $z = (71/80 - 73/90)/s$ $= 1.381$ $1.381 < 1.645$ Do not reject H_0 , there is insufficient evidence that the proportion of on-time Western trains exceeds the proportion of on-time Northern trains	B1 B1 B1 M1 A1 M1 A1 7	For both hypotheses. Or π . SR: from $p_1 q_1 / n_1 + p_2 q_2 / n_2 = 0.00295$ z = 1.406 B1M1A1M1A1 Max 5/7 If no explicit comparison and correct conclusion then M1A0. Or use P-value or CR In context, not too assertive
(ii)	----- $s^2 = 71 \times 9 / 80^3 + 73 \times 17 / 90^3$ $= 0.00295$	M1 A1 2 (9)	AEF Allow one error Accept 0.0029
4(i)	Use $L - S_1 - S_2$ $\mu = 0.7$ $\sigma^2 = 0.58^2 + 0.31^2 + 0.31^2$ $= 0.5286$ (1-0.7)/ σ 0.340	M1 B1 M1 A1 M1 A1 6	Or equivalent, or implied May be implied later Correct numerator
(ii)	----- Use $L - 2S$ with $\mu = 0.7$ $\sigma^2 = 0.58^2 + 4(0.31)^2$ - 0.7/ σ - 0.824(5) 0.2048	M*1 B1 Dep*M1 A1 A1 5 (11)	M0 if as (i) unless correct Accept + 0.205 (3SF)

5(i)	Population of differences is normal $H_0: \mu_A = \mu_B$, $H_1: \mu_A < \mu_B$ where μ_A and μ_B denote the population means $\bar{x}_D = 3.222$ $s_D = 5.019$ $t = 3.222/(5.019/3)$ $= 1.926$ $CV = 1.860$ $1.926 > 1.860$ Reject H_0 , there is evidence that brand A takes less time than brand B	B1 B1 B1 M1A1 M1 A1 B1 M1 A1 10	Not “independent” Or $\mu_D = 0, \mu_D > 0$ From formula ,or B2 from calculator Accept 1.93. M1A0 if $t = -1.926$												
(ii)	One valid reason	B1 1 (11)	Data are clearly paired Data not independent												
6(i)	$37 \times 58 / 120$ 17.883.. , 17.88 AG	M1 A1 2	Or equivalent												
(ii)	H_0 : Gender and shade are independent (H_1 :--are not independent) $3.02^2(14.02^{-1} + 14.98^{-1}) +$ $6.12^2(17.88^{-1} + 19.12^{-1})$ $+ 3.1^2(26.1^{-1} + 27.9^{-1})$ $= 6.03$ EITHER: $CV = 5.991$ $6.03 > 5.991$, reject H_0 and accept that gender and shade are not independent OR: $P(\chi^2 > 6.03) = 0.049$ < 0.05 , reject H_0 and accept that gender and shade are not independent	B1 M1 A1 A1 B1 M1 A1✓ 7 B1 M1 A1✓	At least two correct All correct Ft X^2 . Can be assertive. Ft X^2												
(iii)	<table border="1"> <thead> <tr> <th></th> <th>G_1</th> <th>G_2</th> <th>G_3</th> </tr> </thead> <tbody> <tr> <td>O</td> <td>29</td> <td>37</td> <td>54</td> </tr> <tr> <td>E</td> <td>40</td> <td>40</td> <td>40</td> </tr> </tbody> </table> $121/40 + 9/40 + 196/40$ $= 8.15$ Using $df = 2$ 2.5% tables, 1.7% calculator		G_1	G_2	G_3	O	29	37	54	E	40	40	40	M1 A1 M1 A1 M1 A1 M1 A1 A1 6 (15)	For combining
	G_1	G_2	G_3												
O	29	37	54												
E	40	40	40												

7(i)	$F(t) = \begin{cases} 0 & t \leq 0, \\ t^4 & 0 < t \leq 1, \\ 1 & \text{otherwise.} \end{cases}$	B1 B1 2	For t^4 For rest
(ii)	$\begin{aligned} G(h) &= P(H \leq h) \\ &= P(T \geq 1/h^{1/4}) \\ &= 1 - F(1/h^{1/4}) \\ &= 1 - 1/h \\ g(h) &= G'(h) \\ &= 1/h^2 \\ h &\geq 1, (0 \text{ otherwise}) \end{aligned}$	M1 A1 A1 A1 M1 A1 B1 7	Accept < With attempt at differentiation Only from G obtained correctly
(iii)	$\begin{aligned} \text{EITHER: } \int_1^\infty (h^{-2} + 2h^{-3}) dh \\ &= \left[-h^{-1} - h^{-2} \right]_1^\infty \\ &= 2 \\ \text{OR: } &= 1 + 2 \int_1^\infty \frac{1}{h^3} dh \\ &= 1 + 2 \left[-\frac{1}{2h^2} \right]_1^\infty \\ &= 2 \\ \text{OR: } E(1+2T^4) &= 1 + \int_0^1 8t^7 dt \\ &= 1 + [t^8] \\ &= 2 \end{aligned}$	M1 B1 A1 M1 B1 A1 M1 B1 A1 3 (12)	For integrating $(1+2h^{-1})g(x)$, with limits from (ii) Limits not required Limits not required Limits not required