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ADVANCED GCE

4754/01A

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

TUESDAY 22 JANUARY 2008

Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

This document consists of **4** printed pages.

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Section A (36 marks)

1   Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $3\cos\theta + 4\sin\theta = 2$ for $-\pi \leqslant \theta \leqslant \pi$.   [7]

2   **(i)** Find the first three terms in the binomial expansion of $\dfrac{1}{\sqrt{1 - 2x}}$. State the set of values of x for which the expansion is valid.   [5]

   (ii) Hence find the first three terms in the series expansion of $\dfrac{1 + 2x}{\sqrt{1 - 2x}}$.   [3]

3   Fig. 3 shows part of the curve $y = 1 + x^2$, together with the line $y = 2$.
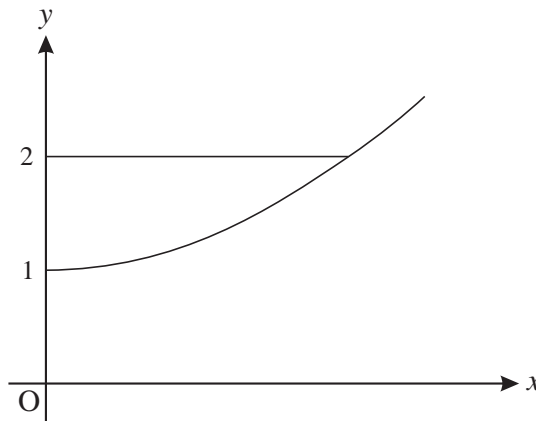

Fig. 3

The region enclosed by the curve, the y-axis and the line $y = 2$ is rotated through $360°$ about the y-axis. Find the volume of the solid generated, giving your answer in terms of π.   [5]

4   The angle θ satisfies the equation $\sin(\theta + 45°) = \cos\theta$.

   (i) Using the exact values of $\sin 45°$ and $\cos 45°$, show that $\tan\theta = \sqrt{2} - 1$.   [5]

   (ii) Find the values of θ for $0° < \theta < 360°$.   [2]

5   Express $\dfrac{4}{x(x^2 + 4)}$ in partial fractions.   [6]

6   Solve the equation $\operatorname{cosec}\theta = 3$, for $0° < \theta < 360°$.   [3]

Section B (36 marks)

7 A glass ornament OABCDEFG is a truncated pyramid on a rectangular base (see Fig. 7). All dimensions are in centimetres.
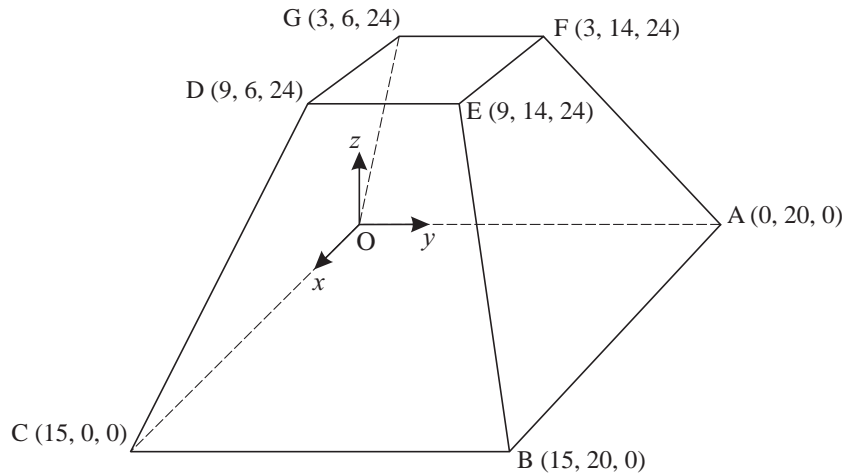
Fig. 7

   (i) Write down the vectors $\overrightarrow{CD}$ and $\overrightarrow{CB}$. [2]

   (ii) Find the length of the edge CD. [2]

   (iii) Show that the vector $4\mathbf{i} + \mathbf{k}$ is perpendicular to the vectors $\overrightarrow{CD}$ and $\overrightarrow{CB}$. Hence find the cartesian equation of the plane BCDE. [5]

   (iv) Write down vector equations for the lines OG and AF.

   Show that they meet at the point P with coordinates (5, 10, 40). [5]

You may assume that the lines CD and BE also meet at the point P.

The volume of a pyramid is $\frac{1}{3} \times$ area of base $\times$ height.

   (v) Find the volumes of the pyramids POABC and PDEFG.

   Hence find the volume of the ornament. [4]

8 A curve has equation

$$x^2 + 4y^2 = k^2,$$

where k is a positive constant.

(i) Verify that

$$x = k \cos \theta, \qquad y = \tfrac{1}{2}k \sin \theta,$$

are parametric equations for the curve. [3]

(ii) Hence or otherwise show that $\dfrac{dy}{dx} = -\dfrac{x}{4y}$. [3]

(iii) Fig. 8 illustrates the curve for a particular value of k. Write down this value of k. [1]
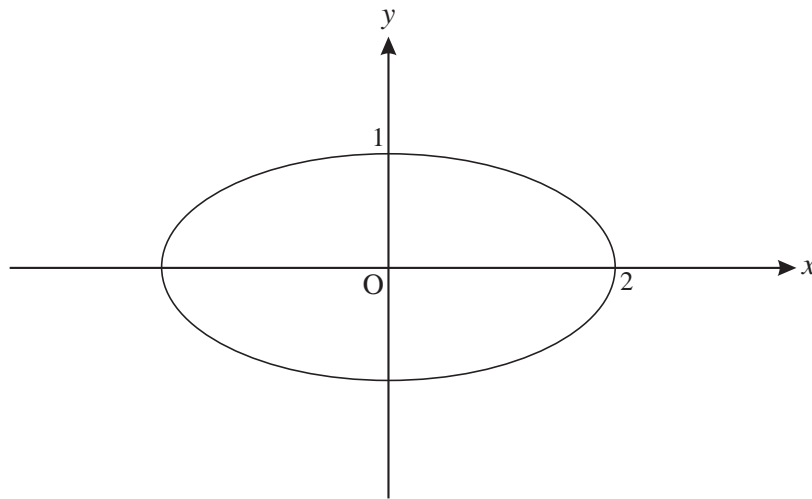
Fig. 8

(iv) Copy Fig. 8 and on the same axes sketch the curves for $k = 1$, $k = 3$ and $k = 4$. [3]

On a map, the curves represent the contours of a mountain. A stream flows down the mountain. Its path on the map is always at right angles to the contour it is crossing.

(v) Explain why the path of the stream is modelled by the differential equation

$$\frac{dy}{dx} = \frac{4y}{x}.$$ [2]

(vi) Solve this differential equation.

Given that the path of the stream passes through the point $(2, 1)$, show that its equation is $y = \dfrac{x^4}{16}$. [6]

ADVANCED GCE   **4754/01B**

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4)   Paper B: Comprehension

TUESDAY 22 JANUARY 2008   Afternoon

Time: Up to 1 hour

Additional materials: Rough paper
                    MEI Examination Formulae and Tables (MF2)

Candidate Forename		Candidate Surname	

Centre Number		Candidate Number	

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the boxes above.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Write your answers in the spaces provided on the question paper.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 18.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

FOR EXAMINER'S USE	
1	
2	
3	
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5	
6	
7	
TOTAL	

This document consists of **4** printed pages and an insert.

1 An additive sequence has third term 5 and fourth term 6. Complete the sequence up to its sixth term. [2]

$$\underline{\quad} , \quad \underline{\quad} , \quad 5, \quad 6, \quad \underline{\quad} , \quad \underline{\quad} .$$

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2 The Lucas sequence on line 32 is 2, 1, 3, 4, 7, 11, 18, 29, ...

State, with reasoning, whether the hundredth term will be odd or even. [2]

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3 On lines 56 and 57 it is stated that the sequence $1, \phi, \phi^2, \phi^3, \phi^4, \phi^5, \ldots$ can be written in the form $1, \phi, \phi + 1, 2\phi + 1, 3\phi + 2, 5\phi + 3, \ldots$.

Write down the next term, ϕ^6, in the form $a\phi + b$ where a and b are integers. [1]

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4 Find the length of HF in Fig. 3 in surd form, simplifying your answer. [3]

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5 On lines 84 to 87 it is stated that the equation of the line BD is $\phi y + x = \phi$ and the equation of the line CF is $(\phi - 1)y = x - 1$.

    (i) Write down the gradients of these lines in terms of ϕ.   [2]

    (ii) Show that these lines are perpendicular.   [2]

    (i) Gradient of line BD is ....

        Gradient of line CF is ....

    (ii) ....

        ....

        ....

        ....

6 On line 90 it is stated that the point Q has coordinates $\left(\dfrac{\phi + 1}{2\phi - 1}, \ \dfrac{\phi - 1}{2\phi - 1} \right)$, where $\phi = \dfrac{1 + \sqrt{5}}{2}$.

    Show that the x-coordinate is $\dfrac{5 + 3\sqrt{5}}{10}$.   [3]

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7 The arithmetic sequence a, $a + d$, $a + 2d$, $a + 3d$, ... is an additive sequence. Prove that $a = 0$ and $d = 0$.   [3]

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