RECOGNISING ACHIEVEMENT

ADVANCED GCE **4726/01**

MATHEMATICS

Further Pure Mathematics 2

WEDNESDAY 9 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

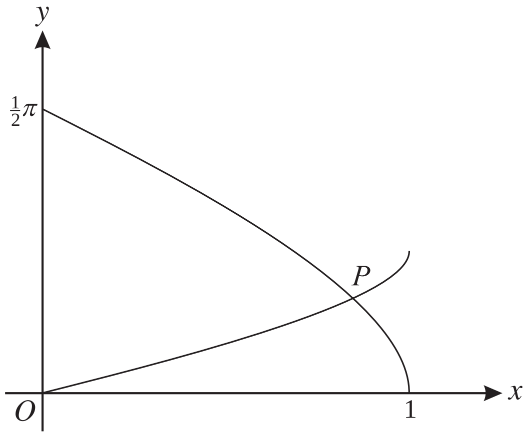
[Turn over

1 It is given that $\mathrm{f}(x) = \ln(1 + \cos x)$.

(i) Find the exact values of $\mathrm{f}(0)$, $\mathrm{f}'(0)$ and $\mathrm{f}''(0)$. [4]

(ii) Hence find the first two non-zero terms of the Maclaurin series for $\mathrm{f}(x)$. [2]

2

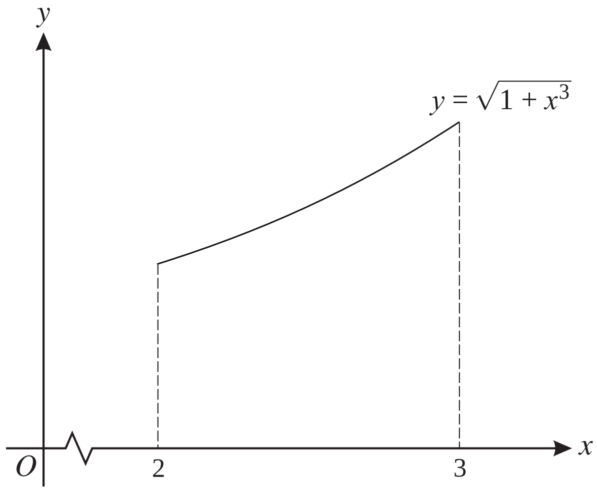


The diagram shows parts of the curves with equations $y = \cos^{-1} x$ and $y = \frac{1}{2}\sin^{-1} x$, and their point of intersection P.

(i) Verify that the coordinates of P are $\left(\frac{1}{2}\sqrt{3}, \frac{1}{6}\pi\right)$. [2]

(ii) Find the gradient of each curve at P. [3]

3



The diagram shows the curve with equation $y = \sqrt{1 + x^3}$, for $2 \leqslant x \leqslant 3$. The region under the curve between these limits has area A.

(i) Explain why $3 < A < \sqrt{28}$. [2]

(ii) The region is divided into 5 strips, each of width 0.2. By using suitable rectangles, find improved lower and upper bounds between which A lies. Give your answers correct to 3 significant figures. [4]

4 The equation of a curve, in polar coordinates, is

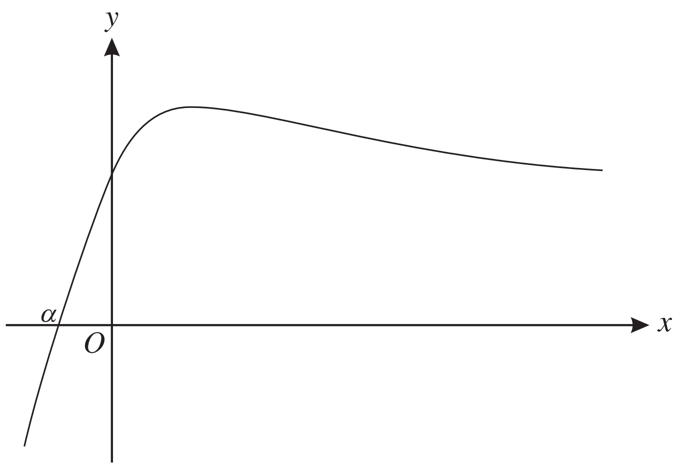
$$r = 1 + 2\sec\theta, \quad \text{for } -\tfrac{1}{2}\pi < \theta < \tfrac{1}{2}\pi.$$

(i) Find the exact area of the region bounded by the curve and the lines $\theta = 0$ and $\theta = \frac{1}{6}\pi$. [5]

[The result $\int \sec\theta \, d\theta = \ln|\sec\theta + \tan\theta|$ may be assumed.]

(ii) Show that a cartesian equation of the curve is $(x-2)\sqrt{x^2+y^2} = x$. [3]

5



The diagram shows the curve with equation $y = xe^{-x} + 1$. The curve crosses the x-axis at $x = \alpha$.

(i) Use differentiation to show that the x-coordinate of the stationary point is 1. [2]

α is to be found using the Newton-Raphson method, with $f(x) = xe^{-x} + 1$.

(ii) Explain why this method will not converge to α if an initial approximation x_1 is chosen such that $x_1 > 1$. [2]

(iii) Use this method, with a first approximation $x_1 = 0$, to find the next three approximations x_2, x_3 and x_4. Find α, correct to 3 decimal places. [5]

6 The equation of a curve is $y = \dfrac{2x^2 - 11x - 6}{x - 1}$.

(i) Find the equations of the asymptotes of the curve. [3]

(ii) Show that y takes all real values. [5]

7 It is given that, for integers $n \geqslant 1$,

$$I_n = \int_0^1 \frac{1}{(1+x^2)^n}\, dx.$$

(i) Use integration by parts to show that $I_n = 2^{-n} + 2n \int_0^1 \frac{x^2}{(1+x^2)^{n+1}}\, dx$. [3]

(ii) Show that $2nI_{n+1} = 2^{-n} + (2n-1)I_n$. [3]

(iii) Find I_2 in terms of π. [3]

8 **(i)** By using the definition of $\sinh x$ in terms of e^x and e^{-x}, show that

$$\sinh^3 x = \tfrac{1}{4}\sinh 3x - \tfrac{3}{4}\sinh x.$$ [4]

(ii) Find the range of values of the constant k for which the equation

$$\sinh 3x = k \sinh x$$

has real solutions other than $x = 0$. [3]

(iii) Given that $k = 4$, solve the equation in part **(ii)**, giving the non-zero answers in logarithmic form. [3]

9 **(i)** Prove that $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$. [3]

(ii) Hence, or otherwise, find $\int \frac{1}{\sqrt{4x^2-1}}\, dx$. [2]

(iii) By means of a suitable substitution, find $\int \sqrt{4x^2-1}\, dx$. [6]