RECOGNISING ACHIEVEMENT

ADVANCED GCE **4727/01**

MATHEMATICS

Further Pure Mathematics 3

THURSDAY 24 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 **(a)** A group G of order 6 has the combination table shown below.

	e	a	b	p	q	r
e	e	a	b	p	q	r
a	a	b	e	r	p	q
b	b	e	a	q	r	p
p	p	q	r	e	a	b
q	q	r	p	b	e	a
r	r	p	q	a	b	e

(i) State, with a reason, whether or not G is commutative. [1]

(ii) State the number of subgroups of G which are of order 2. [1]

(iii) List the elements of the subgroup of G which is of order 3. [1]

(b) A multiplicative group H of order 6 has elements e, c, c^2, c^3, c^4, c^5, where e is the identity. Write down the order of each of the elements c^3, c^4 and c^5. [3]

2 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 4x.$$ [7]

3 Two fixed points, A and B, have position vectors $\mathbf{a}$ and $\mathbf{b}$ relative to the origin O, and a variable point P has position vector $\mathbf{r}$.

(i) Give a geometrical description of the locus of P when $\mathbf{r}$ satisfies the equation $\mathbf{r} = \lambda\mathbf{a}$, where $0 \leqslant \lambda \leqslant 1$. [2]

(ii) Given that P is a point on the line AB, use a property of the vector product to explain why $(\mathbf{r} - \mathbf{a}) \times (\mathbf{r} - \mathbf{b}) = \mathbf{0}$. [2]

(iii) Give a geometrical description of the locus of P when $\mathbf{r}$ satisfies the equation $\mathbf{r} \times (\mathbf{a} - \mathbf{b}) = \mathbf{0}$. [3]

4 The integrals C and S are defined by

$$C = \int_0^{\frac{1}{2}\pi} e^{2x} \cos 3x \, dx \qquad \text{and} \qquad S = \int_0^{\frac{1}{2}\pi} e^{2x} \sin 3x \, dx.$$

By considering $C + iS$ as a single integral, show that

$$C = -\tfrac{1}{13}(2 + 3e^{\pi}),$$

and obtain a similar expression for S. [8]

(You may assume that the standard result for $\int e^{kx}\,dx$ remains true when k is a complex constant, so that $\int e^{(a+ib)x}\,dx = \dfrac{1}{a+ib}e^{(a+ib)x}$.)

5 **(i)** Find the general solution of the differential equation

$$\frac{dy}{dx} + \frac{y}{x} = \sin 2x,$$

expressing y in terms of x in your answer. [6]

In a particular case, it is given that $y = \dfrac{2}{\pi}$ when $x = \frac{1}{4}\pi$.

(ii) Find the solution of the differential equation in this case. [2]

(iii) Write down a function to which y approximates when x is large and positive. [1]

6 A tetrahedron $ABCD$ is such that AB is perpendicular to the base BCD. The coordinates of the points A, C and D are $(-1, -7, 2)$, $(5, 0, 3)$ and $(-1, 3, 3)$ respectively, and the equation of the plane BCD is $x + 2y - 2z = -1$.

(i) Find, in either order, the coordinates of B and the length of AB. [5]

(ii) Find the acute angle between the planes ACD and BCD. [6]

7 **(i)** **(a)** Verify, without using a calculator, that $\theta = \frac{1}{8}\pi$ is a solution of the equation $\sin 6\theta = \sin 2\theta$. [1]

(b) By sketching the graphs of $y = \sin 6\theta$ and $y = \sin 2\theta$ for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$, or otherwise, find the other solution of the equation $\sin 6\theta = \sin 2\theta$ in the interval $0 < \theta < \frac{1}{2}\pi$. [2]

(ii) Use de Moivre's theorem to prove that

$$\sin 6\theta \equiv \sin 2\theta(16\cos^4\theta - 16\cos^2\theta + 3).$$ [5]

(iii) Hence show that one of the solutions obtained in part **(i)** satisfies $\cos^2\theta = \frac{1}{4}(2 - \sqrt{2})$, and justify which solution it is. [3]

8 Groups A, B, C and D are defined as follows:

A: the set of numbers $\{2, 4, 6, 8\}$ under multiplication modulo 10,

B: the set of numbers $\{1, 5, 7, 11\}$ under multiplication modulo 12,

C: the set of numbers $\{2^0, 2^1, 2^2, 2^3\}$ under multiplication modulo 15,

D: the set of numbers $\left\{\frac{1+2m}{1+2n}, \text{where } m \text{ and } n \text{ are integers}\right\}$ under multiplication.

(i) Write down the identity element for each of groups A, B, C and D. [2]

(ii) Determine in each case whether the groups

A and B,

B and C,

A and C

are isomorphic or non-isomorphic. Give sufficient reasons for your answers. [5]

(iii) Prove the closure property for group D. [4]

(iv) Elements of the set $\left\{\frac{1+2m}{1+2n}, \text{where } m \text{ and } n \text{ are integers}\right\}$ are combined under **addition**. State which of the four basic group properties are **not** satisfied. (Justification is not required.) [2]