RECOGNISING ACHIEVEMENT

ADVANCED GCE

4730/01

MATHEMATICS

Mechanics 3

THURSDAY 17 JANUARY 2008

Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

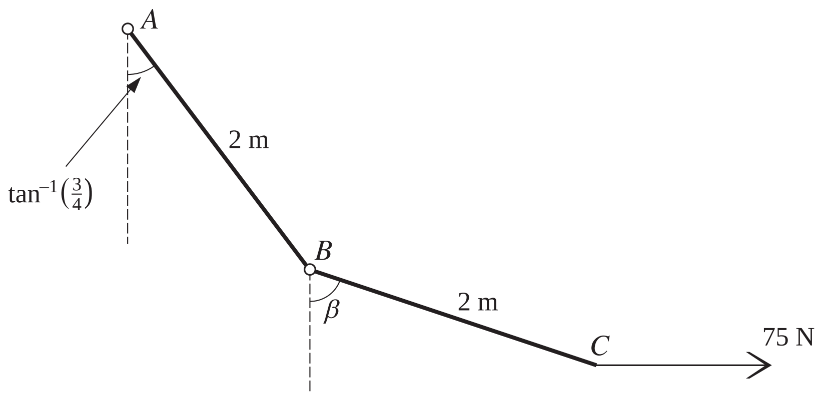
1 A smooth horizontal surface lies in the x-y plane. A particle P of mass 0.5 kg is moving on the surface with speed $5\,\mathrm{m\,s^{-1}}$ in the x-direction when it is struck by a horizontal blow whose impulse has components -3.5 N s and 2.4 N s in the x-direction and y-direction respectively.

(i) Find the components in the x-direction and the y-direction of the velocity of P immediately after the blow. Hence show that the speed of P immediately after the blow is $5.2\,\mathrm{m\,s^{-1}}$. [4]

P is struck by a second horizontal blow whose impulse is $\mathbf{I}$.

(ii) Given that P's direction of motion immediately after this blow is parallel to the x-axis, write down the component of $\mathbf{I}$ in the y-direction. [2]

2



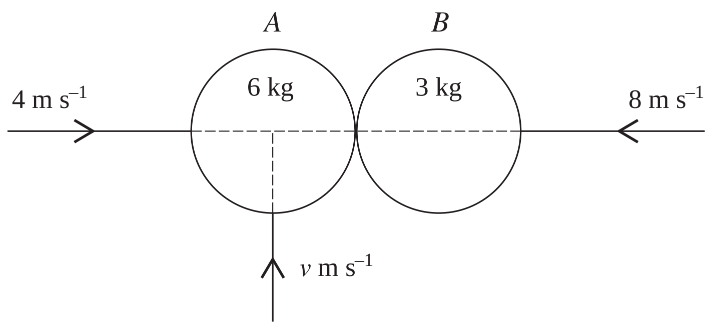
Two uniform rods AB and BC, each of length 2 m, are freely jointed at B. The weights of the rods are W N and 50 N respectively. The end A of AB is hinged at a fixed point. The rods AB and BC make angles $\tan^{-1}\left(\frac{3}{4}\right)$ and β respectively with the downward vertical, and are held in equilibrium in a vertical plane by a horizontal force of magnitude 75 N acting at C (see diagram).

(i) By taking moments about B for BC, show that $\tan\beta = 3$. [3]

(ii) Write down the horizontal and vertical components of the force acting on AB at B. [2]

(iii) Find the value of W. [4]

3



Two uniform smooth spheres A and B, of equal radius, have masses 6 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision the velocity of A has components $4\,\mathrm{m\,s^{-1}}$ along the line of centres towards B, and $v\,\mathrm{m\,s^{-1}}$ perpendicular to the line of centres. B is moving with speed $8\,\mathrm{m\,s^{-1}}$ along the line of centres towards A (see diagram). The coefficient of restitution between the spheres is e.

(i) Find, in terms of e, the component of the velocity of A along the line of centres immediately after the collision. [5]

(ii) Given that the speeds of A and B are the same immediately after the collision, and that $3e^2 = 1$, find v. [4]

4 A particle of mass m kg is released from rest at a fixed point O and falls vertically. The particle is subject to an upward resisting force of magnitude $0.49mv$ N where $v\,\mathrm{m\,s^{-1}}$ is the velocity of the particle when it has fallen a distance of x m from O.

(i) Write down a differential equation for the motion of the particle, and show that the equation can be written as $\left(\dfrac{20}{20-v} - 1\right)\dfrac{\mathrm{d}v}{\mathrm{d}x} = 0.49.$ [5]

(ii) Hence find an expression for x in terms of v. [5]

5 A particle P of mass m kg is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity $0.75mg$ N. The other end of the string is attached to a fixed point O of a smooth plane inclined at 30° to the horizontal. P is released from rest at O and moves down the plane.

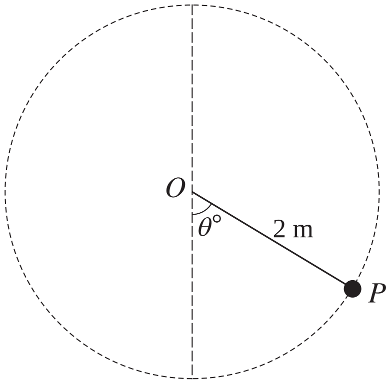
(i) Show that the maximum speed of P is reached when the extension of the string is 0.8 m. [3]

(ii) Find the maximum speed of P. [4]

(iii) Find the maximum displacement of P from O. [4]

[Questions 6 and 7 are printed overleaf.]

6



A particle P of mass 0.4 kg is attached to one end of a light inextensible string of length 2 m. The other end of the string is attached to a fixed point O. With the string taut the particle is travelling in a circular path in a vertical plane. The angle between the string and the downward vertical is $\theta°$ (see diagram). When $\theta = 0$ the speed of P is $7\,\mathrm{m\,s^{-1}}$.

(i) At the instant when the string is horizontal, find the speed of P and the tension in the string. [4]

(ii) At the instant when the string becomes slack, find the value of θ. [8]

7 A particle P, of mass m kg, is attached to one end of a light elastic string of natural length 3.2 m and modulus of elasticity $4mg$ N. The other end of the string is attached to a fixed point A. The particle is released from rest at a point 4.8 m vertically below A. At time t s after P's release P is $(4 + x)$ m below A.

(i) Show that $4\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -49x$. [3]

P's motion is simple harmonic.

(ii) Write down the amplitude of P's motion and show that the string becomes slack instantaneously at intervals of approximately 1.8 s. [4]

A particle Q is attached to one end of a light **inextensible** string of length L m. The other end of the string is attached to a fixed point B. The particle is released from rest with the string taut and inclined at a small angle with the downward vertical. At time t s after Q's release BQ makes an angle of θ radians with the downward vertical.

(iii) Show that $\dfrac{\mathrm{d}^2\theta}{\mathrm{d}t^2} \approx -\dfrac{g}{L}\theta$. [3]

The period of the simple harmonic motion to which Q's motion approximates is the same as the period of P's motion.

(iv) Given that $\theta = 0.08$ when $t = 0$, find the speed of Q when $t = 0.25$. [5]