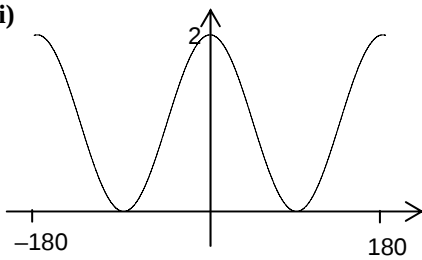


4753 (C3) Methods for Advanced Mathematics

Section A

1 $ x-1 < 3 \Rightarrow -3 < x-1 < 3$ $\Rightarrow -2 < x < 4$	M1 A1 B1 [3]	or $x-1 = \pm 3$, or squaring $\Rightarrow$ correct quadratic $\Rightarrow (x+2)(x-4)$ (condone factorising errors) or correct sketch showing $y = 3$ to scale $-2 < x < 4$ (penalise $\leq$ once only)
2(i) $y = x \cos 2x$ $\Rightarrow \frac{dy}{dx} = -2x \sin 2x + \cos 2x$	M1 B1 A1 [3]	product rule $d/dx (\cos 2x) = -2 \sin 2x$ oe cao
(ii) $\int x \cos 2x dx = \int x \frac{d}{dx} \left(\frac{1}{2} \sin 2x \right) dx$ $= \frac{1}{2} x \sin 2x - \int \frac{1}{2} \sin 2x dx$ $= \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + c$	M1 A1 A1ft A1 [4]	parts with $u = x$, $v = \frac{1}{2} \sin 2x$ $+\frac{1}{4} \cos 2x$ cao – must have $+ c$
3 Either $y = \frac{1}{2} \ln(x-1) \quad x \leftrightarrow y$ $\Rightarrow x = \frac{1}{2} \ln(y-1)$ $\Rightarrow 2x = \ln(y-1)$ $\Rightarrow e^{2x} = y-1$ $\Rightarrow 1 + e^{2x} = y$ $\Rightarrow g(x) = 1 + e^{2x}$	M1 M1 E1	or $y = e^{(x-1)/2}$ attempt to invert and interchanging x with y o.e. (at any stage) $e^{\ln y - 1} = y - 1$ or $\ln(e^y) = y$ used www
or $gf(x) = g(\frac{1}{2} \ln(x-1))$ $= 1 + e^{\ln(x-1)}$ $= 1 + x - 1$ $= x$	M1 M1 E1 [3]	or $fg(x) = \dots$ (correct way round) $e^{\ln(x-1)} = x-1$ or $\ln(e^{2x}) = 2x$ www
4 $\int_0^2 \sqrt{1+4x} dx \quad \text{let } u = 1+4x, \quad du = 4dx$ $= \int_1^9 u^{1/2} \cdot \frac{1}{4} du$ $= \left[\frac{1}{6} u^{3/2} \right]_1^9$ $= \frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3} \text{ or } 4\frac{1}{3}$	M1 A1 B1 M1 A1cao	$u = 1+4x$ and $du/dx = 4$ or $du = 4dx$ $\int u^{1/2} \cdot \frac{1}{4} du$ $\int u^{1/2} du = \frac{u^{3/2}}{3/2}$ soi substituting correct limits (u or x) dep attempt to integrate
or $\frac{d}{dx} (1+4x)^{3/2} = 4 \cdot \frac{3}{2} (1+4x)^{1/2} = 6(1+4x)^{1/2}$ $\Rightarrow \int_0^2 (1+4x)^{1/2} dx = \left[\frac{1}{6} (1+4x)^{3/2} \right]_0^2$ $= \frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3} \text{ or } 4\frac{1}{3}$	M1 A1 A1 M1 A1cao [5]	$k(1+4x)^{3/2}$ $\int (1+4x)^{1/2} dx = \frac{2}{3} (1+4x)^{3/2} \dots$ $\times \frac{1}{4}$ substituting limits (dep attempt to integrate)

5(i) period 180°	B1 [1]	condone $0 \leq x \leq 180^\circ$ or π
(ii) one-way stretch in x -direction scale factor $\frac{1}{2}$ translation in y -direction through $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	M1 A1 M1 A1 [4]	[either way round...] condone 'squeeze', 'contract' for M1 stretch used and s.f $\frac{1}{2}$ condone 'move', 'shift', etc for M1 'translation' used, +1 unit $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ only is M1 A0
(iii) 	M1 B1 A1 [3]	correct shape, touching x -axis at -90° , 90° correct domain (0, 2) marked or indicated (i.e. amplitude is 2)
6(i) e.g. $p = 1$ and $q = -2$ $p > q$ but $1/p = 1 > 1/q = -\frac{1}{2}$	M1 E1 [2]	stating values of p, q with $p \geq 0$ and $q \leq 0$ (but not $p = q = 0$) showing that $1/p > 1/q$ - if 0 used, must state that $1/0$ is undefined or infinite
(ii) Both p and q positive (or negative)	B1 [1]	or $q > 0$, 'positive integers'
7(i) $\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{\frac{2}{3}x^{-1/3}}{\frac{2}{3}y^{-1/3}}$ $= -\frac{y^{1/3}}{x^{1/3}} = -\left(\frac{y}{x}\right)^{\frac{1}{3}} *$	M1 A1 M1 E1 [4]	Implicit differentiation (must show = 0) solving for dy/dx www. Must show, or explain, one more step.
(ii) $\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$ $= -\left(\frac{8}{1}\right)^{\frac{1}{3}} \cdot 6$ $= -12$	M1 A1 A1cao [3]	any correct form of chain rule

8(i) When $x = 1$ $y = 1^2 - (\ln 1)/8 = 1$ Gradient of PR = $(1 + 7/8)/1 = 1\frac{7}{8}$	B1 M1 A1 [3]	1.9 or better
(ii) $\frac{dy}{dx} = 2x - \frac{1}{8x}$ When $x = 1$, $dy/dx = 2 - 1/8 = 1\frac{7}{8}$ Same as gradient of PR, so PR touches curve	B1 B1dep E1 [3]	cao 1.9 or better dep 1 st B1 dep gradients exact
(iii) Turning points when $dy/dx = 0$ $\Rightarrow 2x - \frac{1}{8x} = 0$ $\Rightarrow 2x = \frac{1}{8x}$ $\Rightarrow x^2 = 1/16$ $\Rightarrow x = \frac{1}{4} (x > 0)$ When $x = \frac{1}{4}$, $y = \frac{1}{16} - \frac{1}{8} \ln \frac{1}{4} = \frac{1}{16} + \frac{1}{8} \ln 4$ So TP is $(\frac{1}{4}, \frac{1}{16} + \frac{1}{8} \ln 4)$	M1 M1 A1 M1 A1cao [5]	setting their derivative to zero multiplying through by x allow verification substituting for x in y o.e. but must be exact, not $1/4^2$. Mark final answer.
(iv) $\frac{d}{dx}(x \ln x - x) = x \cdot \frac{1}{x} + 1 \cdot \ln x - 1 = \ln x$	M1 A1	product rule $\ln x$
Area = $\int_1^2 (x^2 - \frac{1}{8} \ln x) dx$ $= \left[\frac{1}{3} x^3 - \frac{1}{8} (x \ln x - x) \right]_1^2$ $= (\frac{8}{3} - \frac{1}{4} \ln 2 + \frac{1}{4}) - (\frac{1}{3} - \frac{1}{8} \ln 1 + \frac{1}{8})$ $= \frac{7}{3} + \frac{1}{8} - \frac{1}{4} \ln 2$ $= \frac{59}{24} - \frac{1}{4} \ln 2$ *	M1 M1 A1 M1 E1 [7]	correct integral and limits (soi) – condone no dx $\int \ln x dx = x \ln x - x$ used (or derived using integration by parts) $\frac{1}{3} x^3 - \frac{1}{8} (x \ln x - x)$ – bracket required substituting correct limits must show at least one step

9(i) Asymptotes when $(\sqrt{})(2x - x^2) = 0$ $\Rightarrow x(2 - x) = 0$ $\Rightarrow x = 0$ or 2 so $a = 2$ Domain is $0 < x < 2$	M1 A1 B1ft [3]	or by verification $x > 0$ and $x < 2$, not $\leq$
(ii) $y = (2x - x^2)^{-1/2}$ let $u = 2x - x^2$, $y = u^{-1/2}$ $\Rightarrow dy/du = -\frac{1}{2}u^{-3/2}$, $du/dx = 2 - 2x$ $\Rightarrow$ $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = -\frac{1}{2}(2x - x^2)^{-3/2} \cdot (2 - 2x)$ $= \frac{x-1}{(2x - x^2)^{3/2}} *$	M1 B1 A1 E1	chain rule (or within correct quotient rule) $-\frac{1}{2}u^{-3/2}$ or $-\frac{1}{2}(2x - x^2)^{-3/2}$ or $\frac{1}{2}(2x - x^2)^{-1/2}$ in quotient rule $\times (2 - 2x)$ www – penalise missing brackets here
$dy/dx = 0$ when $x - 1 = 0$ $\Rightarrow x = 1$, $y = 1/\sqrt{(2 - 1)} = 1$ Range is $y \geq 1$	M1 A1 B1 B1ft [8]	extraneous solutions M0
(iii) (A) $g(-x) = \frac{1}{\sqrt{1 - (-x)^2}} = \frac{1}{\sqrt{1 - x^2}} = g(x)$	M1 E1	Expression for $g(-x)$ – must have $g(-x) = g(x)$ seen
(B) $g(x-1) = \frac{1}{\sqrt{1 - (x-1)^2}}$ $= \frac{1}{\sqrt{1 - x^2 + 2x - 1}} = \frac{1}{\sqrt{2x - x^2}} = f(x)$	M1 E1	must expand bracket
(C) $f(x)$ is $g(x)$ translated 1 unit to the right. But $g(x)$ is symmetrical about Oy So $f(x)$ is symmetrical about $x = 1$.	M1 M1 A1	dep both M1s
or $f(1-x) = g(-x)$, $f(1+x) = g(x)$ $\Rightarrow f(1+x) = f(1-x)$ $\Rightarrow f(x)$ is symmetrical about $x = 1$.	M1 E1 A1 [7]	or $f(1-x) = \frac{1}{\sqrt{2-2x-(1-x)^2}} = \frac{1}{\sqrt{2-2x-1+2x-x^2}} = \frac{1}{\sqrt{1-x^2}}$ $f(1+x) = \frac{1}{\sqrt{2+2x-(1+x)^2}} = \frac{1}{\sqrt{2+2x-1-2x-x^2}} = \frac{1}{\sqrt{1-x^2}}$