

4754 (C4) Applications of Advanced Mathematics

Section A

1 $\frac{3x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+1)}$ $\Rightarrow 3x+2 = A(x^2+1) + (Bx+C)x$ $x=0 \Rightarrow 2=A$ coefft of x^2: $0 = A+B \Rightarrow B=-2$ coefft of x: $3=C$ $\Rightarrow \frac{3x+2}{x(x^2+1)} = \frac{2}{x} + \frac{3-2x}{(x^2+1)}$	M1 M1 B1 M1 A1 A1 [6]	correct partial fractions equating coefficients at least one of B,C correct
2(i) $(1+2x)^{1/3} = 1 + \frac{1}{3} \cdot 2x + \frac{\frac{1}{3} \cdot (-\frac{2}{3})}{2!} (2x)^2 + \dots$ $= 1 + \frac{2}{3}x - \frac{2}{18}4x^2 + \dots$ $= 1 + \frac{2}{3}x - \frac{4}{9}x^2 + \dots$ * Next term $= \frac{\frac{1}{3} \cdot (-\frac{2}{3}) \cdot (-\frac{5}{3})}{3!} (2x)^3$ $= \frac{40}{81}x^3$ Valid for $-1 < 2x < 1$ $\Rightarrow -\frac{1}{2} < x < \frac{1}{2}$	M1 A1 E1 M1 A1 B1 [6]	binomial expansion correct unsimplified expression simplification www
3 $4\mathbf{j} - 3\mathbf{k} = \lambda \mathbf{a} + \mu \mathbf{b}$ $= \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k}) + \mu(4\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ $\Rightarrow 0 = 2\lambda + 4\mu$ $4 = \lambda - 2\mu$ $-3 = -\lambda + \mu$ $\Rightarrow \lambda = -2\mu, 2\lambda = 4 \Rightarrow \lambda = 2, \mu = -1$	M1 M1 A1 A1, A1 [5]	equating components at least two correct equations
4 LHS $= \cot \beta - \cot \alpha$ $= \frac{\cos \beta}{\sin \beta} - \frac{\cos \alpha}{\sin \alpha}$ $= \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\sin \alpha \sin \beta}$ $= \frac{\sin(\alpha - \beta)}{\sin \alpha \sin \beta}$ OR RHS $= \frac{\sin(\alpha - \beta)}{\sin \alpha \sin \beta} = \frac{\sin \alpha \cos \beta}{\sin \alpha \sin \beta} - \frac{\cos \alpha \sin \beta}{\sin \alpha \sin \beta}$ $= \cot \beta - \cot \alpha$	M1 M1 E1 M1 M1 E1 [3]	cot = cos / sin combining fractions www using compound angle formula splitting fractions using cot=cos/sin

5(i) Normal vectors $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ Angle between planes is θ, where $\cos \theta = \frac{2 \times 1 + (-1) \times 0 + 1 \times (-1)}{\sqrt{2^2 + (-1)^2 + 1^2} \sqrt{1^2 + 0^2 + (-1)^2}}$ $= 1/\sqrt{12}$ $\Rightarrow \theta = 73.2^\circ$ or 1.28 rads	B1 M1 M1 A1 [4]	scalar product finding invcos of scalar product divided by two modulae
(ii) $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 2+2\lambda \\ -\lambda \\ 1+\lambda \end{pmatrix}$ $\Rightarrow 2(2+2\lambda) - (-\lambda) + (1+\lambda) = 2$ $\Rightarrow 5 + 6\lambda = 2$ $\Rightarrow \lambda = -\frac{1}{2}$ So point of intersection is $(1, \frac{1}{2}, \frac{1}{2})$	B1 M1 A1 A1 [4]	
6(i) $\cos \theta + \sqrt{3} \sin \theta = r \cos(\theta - \alpha)$ $= R \cos \theta \cos \alpha + R \sin \theta \sin \alpha$ $\Rightarrow R \cos \alpha = 1, R \sin \alpha = \sqrt{3}$ $\Rightarrow R^2 = 1^2 + (\sqrt{3})^2 = 4, R = 2$ $\tan \alpha = \sqrt{3}$ $\Rightarrow \alpha = \pi/3$	B1 M1 M1 A1 [4]	$R = 2$ equating correct pairs $\tan \alpha = \sqrt{3}$ o.e.
(ii) derivative of $\tan \theta$ is $\sec^2 \theta$ $\int_0^{\frac{\pi}{3}} \frac{1}{(\cos \theta + \sqrt{3} \sin \theta)^2} d\theta = \int_0^{\frac{\pi}{3}} \frac{1}{4} \sec^2(\theta - \frac{\pi}{3}) d\theta$ $= \left[\frac{1}{4} \tan(\theta - \frac{\pi}{3}) \right]_0^{\frac{\pi}{3}}$ $= \frac{1}{4} (0 - (-\sqrt{3}))$ $= \sqrt{3}/4 *$	B1 M1 A1 E1 [4]	ft their α $\frac{1}{R^2} [\tan(\theta - \pi/3)]$ ft their R, α (in radians) www

Section B

7(i) (A) $9 / 1.5 = 6$ hours (B) $18/1.5 = 12$ hours	B1 B1 [2]	
(ii) $\frac{d\theta}{dt} = -k(\theta - \theta_0)$ $\Rightarrow \int \frac{d\theta}{\theta - \theta_0} = \int -k dt$ $\Rightarrow \ln(\theta - \theta_0) = -kt + c$ $\theta - \theta_0 = e^{-kt+c}$ $\theta = \theta_0 + Ae^{-kt}$ *	M1 A1 A1 M1 E1 [5]	separating variables $\ln(\theta - \theta_0)$ $-kt + c$ anti-logging correctly(with c) $A = e^c$
(iii) $98 = 50 + Ae^0$ $\Rightarrow A = 48$ Initially $\frac{d\theta}{dt} = -k(98 - 50) = -48k = -1.5$ $\Rightarrow k = 0.03125$ *	M1 A1 M1 E1 [4]	
(iv) (A) $89 = 50 + 48e^{-0.03125t}$ $\Rightarrow 39/48 = e^{-0.03125t}$ $\Rightarrow t = \ln(39/48)/(-0.03125) = 6.64$ hours (B) $80 = 50 + 48e^{-0.03125t}$ $\Rightarrow 30/48 = e^{-0.03125t}$ $\Rightarrow t = \ln(30/48)/(-0.03125) = 15$ hours	M1 M1 A1 M1 A1 [5]	equating taking lns correctly for either
(v) Models disagree more for greater temperature loss	B1 [1]	

8(i) $\frac{dy}{d\theta} = 2 \cos 2\theta - 2 \sin \theta, \frac{dx}{d\theta} = 2 \cos \theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $\frac{dy}{dx} = \frac{2 \cos 2\theta - 2 \sin \theta}{2 \cos \theta} = \frac{\cos 2\theta - \sin \theta}{\cos \theta}$	B1, B1 M1 A1 [4]	substituting for theirs oe
(ii) When $\theta = \pi/6$, $\frac{dy}{dx} = \frac{\cos \pi/3 - \sin \pi/6}{\cos \pi/6}$ $= \frac{1/2 - 1/2}{\sqrt{3}/2} = 0$ Coords of B: $x = 2 + 2 \sin(\pi/6) = 3$ $y = 2 \cos(\pi/6) + \sin(\pi/3) = 3\sqrt{3}/2$ BC = $2 \times 3\sqrt{3}/2 = 3\sqrt{3}$	E1 M1 A1, A1 B1ft [5]	for either exact
(iii) (A) $y = 2 \cos \theta + \sin 2\theta$ $= 2 \cos \theta + 2 \sin \theta \cos \theta$ $= 2 \cos \theta (1 + \sin \theta)$ $= x \cos \theta$ * (B) $\sin \theta = \frac{1}{2}(x - 2)$ $\cos^2 \theta = 1 - \sin^2 \theta$ $= 1 - \frac{1}{4}(x - 2)^2$ $= 1 - \frac{1}{4}x^2 + x - 1$ $= (x - \frac{1}{4}x^2)$ * (C) Cartesian equation is $y^2 = x^2 \cos^2 \theta$ $= x^2(x - \frac{1}{4}x^2)$ $= x^3 - \frac{1}{4}x^4$ *	M1 E1 B1 M1 E1 M1 E1 [7]	$\sin 2\theta = 2 \sin \theta \cos \theta$ squaring and substituting for x
(iv) $V = \int_0^4 \pi y^2 dx$ $= \pi \int_0^4 (x^3 - \frac{1}{4}x^4) dx$ $= \pi \left[\frac{1}{4}x^4 - \frac{1}{20}x^5 \right]_0^4$ $= \pi(64 - 51.2)$ $= 12.8\pi = 40.2 \text{ (m}^3\text{)}$	M1 B1 A1 [3]	need limits $\left[\frac{1}{4}x^4 - \frac{1}{20}x^5 \right]$ 12.8π or 40 or better.

Comprehension

1	$\frac{400\pi d}{1000} = 10$ $d = \frac{25}{\pi} = 7.96$	M1 E1	
2	$V = \pi 20^2 h + \frac{1}{2}(\pi 20^2 H - \pi 20^2 h)$ $= \frac{1}{2}(\pi 20^2 H + \pi 20^2 h) \text{ cm}^3 = 200\pi(H + h) \text{ cm}^3$ $= \frac{1}{5}\pi(H + h) \text{ litres}$	M1 M1 E1	divide by 1000
3	$H = 5 + 40 \tan 30^\circ \text{ or } H = h + 40 \tan \theta$ $V = \frac{1}{5}\pi(H + h) = \frac{1}{5}\pi(10 + 40 \tan 30^\circ)$ $= 20.8 \text{ litres}$	B1 M1 A1	or evaluated including substitution of values
4	$V = \frac{1}{2} \times 80 \times (40 + 5)$ $\times 30 \text{ cm}^3 = 54\,000 \text{ cm}^3$ $= 54 \text{ litres}$	M1 M1 A1	$\times 30$
5	(i) Accurate algebraic simplification to give $y^2 - 160y + 400 = 0$ (ii) Use of quadratic formula (or other method) to find other root: $d = 157.5 \text{ cm}$. This is greater than the height of the tank so not possible	B1 M1 A1 E1	
6	$y = 10$ Substitute for y in (4): $V = \frac{1}{1000} \int_0^{100} 375 dx$ $V = \frac{1}{1000} \times 37500 = 37.5 *$	B1 M1 E1 [18]	