# 4726 Further Pure Mathematics 2

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| **1** | **(i)** | Give $1 + 2x + (2x)^2/2$ | M1 | Reasonable 3 term attempt e.g. allow $2x^2/2$ |
| | | Get $1 + 2x + 2x^2$ | A1 | cao |
| | | | | SC Reasonable attempt at f′(0) and f″(0) M1 |
| | | | | Get $1+2x+2x^2$ cao A1 |
| | **(ii)** | $\ln( (1+2x+2x^2) + (1-2x+2x^2))$ = | M1 | Attempt to sub for $e^{2x}$ and $e^{-2x}$ |
| | | $\ln(2+4x^2)$ = | A1√ | On their part **(i)** |
| | | $\ln 2 + \ln(1 + 2x^2)$ | M1 | Use of log law in reasonable expression |
| | | $\ln 2 + 2x^2$ | A1 | cao |
| | | | | SC Use of Maclaurin for f′(x) and f″(x) M1 |
| | | | | One correct A1 |
| | | | | Attempt f(0), f′(0) and f″(0) M1 |
| | | | | Get cao A1 |
| **2** | **(i)** | $x_2 = 1.8913115$ | B1 | $x_2$ correct; allow answers which round |
| | | $x_3 = 1.8915831$ | B1√ | For any other from their working |
| | | $x_4 = 1.8915746$ | B1 | For all three correct |
| | **(ii)** | $e_3/e_2 = -0.031(1)$ | M1 | Subtraction and division on their values; allow ± |
| | | $e_4/e_3 = -0.036(5)$ | A1 | Or answers which round to −0.031 and −0.037 |
| | | State $f'(\alpha) \approx e_3/e_2 \approx e_4/e_3$ | B1√ | Using their values but only if approx. equal; allow differentiation if correct conclusion; allow gradient for f′ |
| **3** | **(i)** | Diff. $\sin y = x$ | M1 | Implicit diff. to $dy/dx = \pm(1/\cos y)$ |
| | | Use $\sin^2 + \cos^2 = 1$ to A.G. | A1 | Clearly derived; ignore ± |
| | | Justify + | B1 | e.g graph/ principal values |
| | **(ii)** | Get $2/(\sqrt{(1-4x^2)} + 1/(\sqrt{(1-y^2)}\, dy/dx = 0$ | M1 | Attempt implicit diff. and chain rule; allow e.g. $(1-2x^2)$ or $a/\sqrt{(1-4x^2)}$ |
| | | | A1 | |
| | | Find $y = \sqrt{3}/2$ | M1 | Method leading to $y$ |
| | | Get $-2\sqrt{3}/3$ | A1√ | AEEF; from their $a$ above |
| | | | | SC Write $\sin(\frac{1}{2}\pi - \sin^{-1}2x) = \cos(\sin^{-1}2x)$ B1 |
| | | | | Attempt to diff. as above M1 |
| | | | | Replace $x$ in reasonable $dy/dx$ and attempt to tidy M1 |
| | | | | Get result above A1 |

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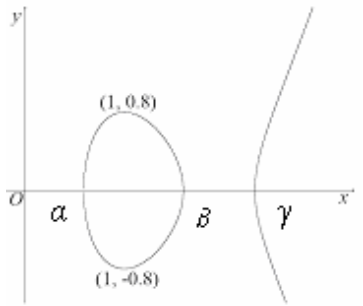
| **4** | **(i)** | Let $x=\cosh\theta$ such that<br>$dx = \sinh\theta\, d\theta$ | M1 | |
| | | Clearly use $\cosh^2 - \sinh^2 = 1$ | A1 | Clearly derive A.G. |
| | **(ii)** | Replace $\cosh^2\theta$ | M1 | Allow $a(\cosh 2\theta \pm 1)$ |
| | | Attempt to integrate their expression | M1 | Allow $b\sinh 2\theta \pm a\theta$ |
| | | Get $\frac{1}{4}\sinh 2\theta + \frac{1}{2}\theta\ (+c)$ | A1 | |
| | | Clearly replace for $x$ to A.G. | B1 | Condone no $+c$ |
| | | | | SC Use expo. def$^n$; three terms M1<br>Attempt to integrate M1<br>Get $\frac{1}{8}(e^{2\theta}-e^{-2\theta}) + \frac{1}{2}\theta\ (+c)$ A1<br>Clearly replace for $x$ to A.G. B1 |
| **5** | **(i)** | (**a**) State ($x=$) $\alpha$ | B1 | |
| | | None of roots | B1 | No explanation needed |
| | | (**b**) Impossible to say | B1 | |
| | | All roots can be derived | B1 | Some discussion of values close to 1 or 2 or central leading to correct conclusion |
| | **(ii)** |  | B1 | Correct $x$ for $y=0$; allow 0.591, 1.59, 2.31 |
| | | | B1 | Turning at (1,0.8) and/or (1,-0.8) |
| | | | B1 | Meets $x$-axis at $90^0$ |
| | | | B1 | Symmetry in $x$-axis; allow |
| **6** | **(i)** | Correct definitions used | B1 | |
| | | Attempt at $(e^x-e^{-x})^2/4 + 1$ | M1 | Allow $(e^x+e^{-x})^2+1$; allow /2 |
| | | Clearly derive A.G. | A1 | |
| | **(ii)** | Form a quadratic in $\sinh x$ | M1 | |
| | | Attempt to solve | M1 | Factors or formula |
| | | Get $\sinh x = -\frac{1}{2}$ or 3 | A1 | |
| | | Use correct ln expression | M1 | On their answer(s) seen once |
| | | Get $\ln(-\frac{1}{2}+\frac{\sqrt{5}}{2})$ and $\ln(3+\sqrt{10})$ | A1 | |
| **7** | **(i)** | $OP = 3 + 2\cos\alpha$ | | |
| | | $OQ = 3 + 2\cos(\frac{1}{2}\pi+\alpha)$<br>$= 3 - 2\sin\alpha$ | M1 | Any other unsimplified value |
| | | Similarly $OR = 3 - 2\cos\alpha$<br>$OS = 3 + 2\sin\alpha$ | M1 | Attempt at simplification of at least two correct expressions |
| | | Sum = 12 | A1 | cao |
| | **(ii)** | Correct formula with attempt at $r^2$ | M1 | Need not be expanded, but three terms if it is |
| | | Square $r$ correctly | A1 | |
| | | Attempt to replace $\cos^2\theta$ with $a(\cos 2\theta \pm 1)$ | M1 | |
| | | Integrate their expression | A1√ | Need three terms |
| | | Get $\frac{11\pi}{4} - 1$ | A1 | cao |

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| **8** | **(i)** | Area = $\int 1/(x+1)\,dx$ | B1 | Include or imply correct limits |
| | | Use limits to $\ln(n+1)$ | B1 | |
| | | Compare area under curve to areas of rectangles | B1 | Justify inequality |
| | | Sum of areas = $1\text{x}(\frac{1}{2} + \frac{1}{3} + \ldots + 1/(n+1))$ | M1 | Sum seen or implied as 1 x $y$ values |
| | | Clear detail to A.G. | A1 | Explanation required e.g. area of last rectangle at $x=n$, area under curve to $x=n$ |
| | **(ii)** | Show or explain areas of rectangles above curve | M1 | |
| | | Areas of rectangles (as above) > area under curve | A1 | First and last heights seen or implied; A.G. |
| | **(iii)** | Add 1 to both sides in **(i)** to make $\sum(1/r)$ | B1 | Must be clear addition |
| | | Add $1/(n+1)$ to both sides in **(ii)** to make $\sum(1/r)$ | B1 | Must be clear addition; A.G. |
| | **(iv)** | State divergent | B1 | Allow not convergent |
| | | Explain e.g. $\ln(n+1)\to\infty$ as $n\to\infty$ | B1 | |
| **9** | **(i)** | Require denom. = 0 | B1 | |
| | | Explain why denom. $\neq 0$ | B1 | Attempt to solve, explain always > 0 etc. |
| | **(ii)** | Set up quadratic in $x$ | M1 | |
| | | Get $2yx^2-4x+(2a^2y+3a) =0$ | A1 | |
| | | Use $b^2 \geq 4ac$ for real $x$ | M1 | Produce quadratic inequality in $y$ from their quad.; allow use of = or < |
| | | Attempt to solve their inequality | M1 | Factors or formula |
| | | Get $y > 1/2a$ and $y < -2/a$ | A1 | Justified from graph |
| | | | | SC Attempt diff. by quot./product rule M1 |
| | | | | Solve $dy/dx =0$ for two values of $x$ M1 |
| | | | | Get $x=2a$ and $x=-a/2$ A1 |
| | | | | Attempt to find two $y$ values M1 |
| | | | | Get correct inequalities (graph used to justify them) A1 |
| | **(iii)** | Split into two separate integrals | M1 | |
| | | Get $k\ln(x^2+a^2)$ | A1 | Or $p\ln(2x^2+2a^2)$ |
| | | Get $k_1\tan^{-1}(x/a)$ | A1 | $k_1$ not involving $a$ |
| | | Use limits and attempt to simplify | M1 | |
| | | Get $\ln 2.5 - 1.5\tan^{-1}2 + 3\pi/8$ | A1 | AEEF |
| | | | | SC Sub. $x = a\tan\theta$ and $dx = a\sec^2\theta\,d\theta$ M1 |
| | | | | Reduce to $\int p\tan\theta - p_1\,d\theta$ (ignore limits here) A1 |
| | | | | Integrate to $p\ln(\sec\theta) - p_1\theta$ A1 |
| | | | | Use limits (old or new) and attempt to simplify M1 |
| | | | | Get answer above A1 |