4730 Mechanics 3

1 (i)	For triangle sketched with sides (0.5)2.5 and (0.5)6.3 and angle θ correctly marked OR Changes of velocity in i and j directions $2.5\cos\theta - 6.3$ and $2.5\sin\theta$, respectively. For sides 0.5x2.5, 0.5x6.3 and 2.6 (or 2.5, 6.3 and 5.2) OR $-2.6\cos\alpha = 0.5(2.5\cos\theta - 6.3)$ and $2.6\sin\alpha = 0.5(2.5\sin\theta)$ $[5.2^2 = 2.5^2 + 6.3^2 - 2x2.5x6.3\cos\theta$ OR $2.6^2 = 0.5^2\{(2.5\cos\theta - 6.3)^2 + (2.5\sin\theta)^2]$ $\cos\theta = 0.6$	B1 B1ft M1 A1 [4]	May be implied in subsequent working. May be implied in subsequent working. For using cosine rule in triangle or eliminating α. AG
(ii)	$\sin\alpha = 2.5x0.8/5.2$ OR $-2.6\cos\alpha = 0.5(2.5x0.6 - 6.3)$ Impulse makes angle of 157^o or 2.75^c with original direction of motion of P.	M1 A1 M1 A1 [4]	For appropriate use of the sine rule or substituting for θ in one of the above equations in θ and α For evaluating $(180 - \alpha)^o$ or $(\pi - \alpha)^c$ SR (relating to previous 2 marks; max 1 mark out of 2) $\alpha = 23^o$ or 0.395^C B1

2 (i)	[70x2 = 4X – 4Y] X – Y = 35	M1 A1 [2]	For taking moments about A for AB (3 terms needed)
(ii)	[110x3 = -4X + 6Y] 2X – 3Y + 165 = 0	M1 A1 [2]	For taking moments about C for BC (3 terms needed) AG
(iii)	 X = 270, Y = 235 Magnitude is 358N	M1 A1ft M1 A1ft [4]	For attempting to solve for X and Y ft any (X, Y) satisfying the equation given in (ii) For using magnitude = $\sqrt{X^2 + Y^2}$ ft depends on all 4 Ms

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3 (i)	$[T_A = (24x0.45)/0.6, T_B = (24x0.15)/0.6]$ $T_A - T_B = 18 - 6 = 12 = W \rightarrow$ P in equil'm.	M1 A1 [2]	For using $T = \lambda x/L$ for PA or PB
(ii)	Extensions are $0.45 + x$ and $0.15 - x$ Tensions are $18 + 40x$ and $6 - 40x$	B1 B1 [2]	AG From $T = \lambda x/L$ for PA and PB
(iii)	$[12 + (6 - 40x) - (18 + 40x) = 12\ddot{x}/g]$ $\ddot{x} = -80gx/12 \rightarrow$ SHM Period is 0.777s	M1 A1 A1 [3]	For using Newton's second law (4 terms required) AG From Period $= 2\pi\sqrt{12/(80g)}$
(iv)	$[v_{max} = 0.15\sqrt{80g/12}$ or $v_{max} = 2\pi x0.15/0.777$ or $\frac{1}{2}(12/g)v_{max}^2 + mg(0.15)$ $+24\{0.45^2 + 0.15^2 - 0.6^2\}/(2x0.6) = 0]$ Speed is $1.21ms^{-1}$	M1 A1 [2]	For using $v_{max} = An$ or $v_{max} = 2\pi A/T$ or conservation of energy (5 terms needed)

4 (i)	Loss in PE $= mg(0.5\sin\theta)$ $[\frac{1}{2}mv^2 - \frac{1}{2}m3^2 = mg(0.5\sin\theta)]$ $v^2 = 9 + 9.8\sin\theta$	B1 M1 A1 [3]	For using KE gain = PE loss (3 terms required) AG
(ii)	$a_r = 18 + 19.6\sin\theta$ $[ma_t = mg\cos\theta]$ $a_t = 9.8\cos\theta$	B1 M1 A1 [3]	Using $a_r = v^2/0.5$ For using Newton's second law tangentially
(iii)	$[T - mg\sin\theta = ma_r]$ $T - 1.96\sin\theta = 0.2(18 + 19.6\sin\theta)$ $T = 3.6 + 5.88\sin\theta$ $\theta = 3.8$	M1 A1 A1 B1 [4]	For using Newton's second law radially (3 terms required) AG

5	Initial **i** components of velocity for A and B are $4ms^{-1}$ and $3ms^{-1}$ respectively. $3x4 + 4x3 = 3a + 4b$ $0.75(4 – 3) = b – a$ $a = 3$ Final **j** component of velocity for A is $3ms^{-1}$ Angle with l.o.c. is 45^o or 135^O	B1 M1 A1 M1 A1 M1 A1 B1 M1 A1ft [10]	May be implied. For using p.c.mmtm. parallel to l.o.c. For using NEL For attempting to find a Depends on all three M marks May be implied For using $\tan^{-1}(v_j/v_i)$ for A ft incorrect value of a ($\neq 0$) only
			SR for consistent sin/cos mix (max 8/10) $3x3 + 4x4 = 3a + 4b$ and $b – a = 0.75(3 – 4)$ M1 M1 as scheme and A1 for *both* equ's $a = 4$ M1 as scheme A1 j component for A is $4ms^{-1}$ B1 Angle $\tan^{-1}(4/4) = 45^o$ M1 as scheme A1

6(i)	Initial speed in medium is $\sqrt{2g \times 10}$ (= 14) [0.125dv/dt = 0.125g – 0.025v] $\int \frac{5dv}{5g - v} = \int dt$ $-5 \ln(5g – v) = t (+A)$ [$-5 \ln 35 = A$] $t = 5 \ln\{35/(49 – v)\}$ $v = 49 – 35e^{-0.2t}$	B1 M1 M1 A1 M1 A1 M1 A1 [8]	 For using Newton's second law with a = dv/dt (3 terms required) For separating variables and attempt to integrate For using v(0) = 14 For method of transposition AG
(ii)	$x = 49t + 175e^{-0.2t} (+B)$ [$x(3) = (49x3 + 175e^{-0.6}) – (0 + 175)$] Distance is 68.0m	M1 A1 M1 A1 [4]	For integrating to find x(t) For using limits 0 to 3 or for using x(0) = 0 and evaluating x(3)

7(i)	Gain in EE = $20x^2/(2x2)$	B1	
	Loss in GPE = $0.8g(2 + x)$	B1	Accept $0.8gx$ if gain in KE is $\frac{1}{2}\ 0.8(v^2 - 19.6)$
	$[\frac{1}{2}\ 0.8v^2 = (15.68 + 7.84x) - 5x^2]$	M1	For using the p.c.energy
	$v^2 = 39.2 + 19.6x - 12.5x^2$	A1	AG
		[4]	
(ii)	**(a)**	M1	For attempting to solve $v^2 = 0$
	Maximum extension is 2.72m	A1	
		[2]	
	(b)		For solving $20x/2 = 0.8g$ or for differentiating and attempting to solve $d(v^2)/dx = 0$ or $dv/dx = 0$ or for expressing v^2 in the form $c - a(x - b)^2$.
	$[19.6 - 25x = 0,$ $v^2 = 46.8832 - 12.5(x - 0.784)^2]$	M1	
	$x = 0.784$ or $c = 46.9$	A1	
			For substituting $x = 0.784$ in the expression for v^2 or for evaluating $\sqrt{c}$
	$[v_{max}^{\ 2} = 39.2 + 15.3664 - 7.6832]$	M1	
	Maximum speed is $6.85ms^{-1}$	A1	
		[4]	
	(c)		For using Newton's second law (3 terms required) or $a = v\ dv/dx$
	$\pm(0.8g - 20x/2) = 0.8a$ or $2v\ dv/dx = 19.6 - 25x$	M1	
	$a = \pm(9.8 - 12.5x)$ or $\ddot{y} = -12.5y$ where $y = x - 0.784$	A1	
	$[\lvert a\rvert_{max} = \lvert 9.8 - 12.5x2.72\rvert$ or $\lvert \ddot{y}_{max}\rvert = \lvert -12.5(2.72 - 0.784)\rvert]$	A1	
			For substituting $x =$ ans(ii)(a) into a(x) or $y =$ ans(ii)(a) $- 0.784$ into $\ddot{y}$ (y)
	Maximum magnitude is $24.2ms^{-2}$	M1 A1	
		[5]	