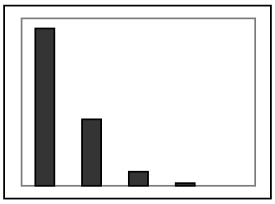
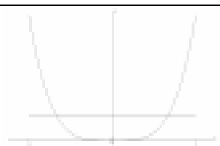
4733 Probability & Statistics 2

1		$U \sim B(800, 0.005) \approx Po(4)$ $P(U \le 6)$ $= \mathbf{0.8893}$ $n > 50$/large, $np < 5/p$ small	B1 M1 A1 B1	 **4**	Po(*np*) stated or implied Tables or formula ± 1 term, e.g. 0.7851, 0 .9489, 0.1107, *not* 1– Answer 0.889 or a.r.t. 0.8893 Both conditions
2		$\frac{23.625 - 23}{5/\sqrt{n}} = 2$ $\sqrt{n} = 16$ $n = \mathbf{256}$	M1 A1 M1 A1	 **4**	Standardise with √*n*, allow √/^{2} errors Equate to 2 or a.r.t. 2.00, signs correct Solve for √*n*, needs Φ^{-1}, *not* from /*n* 256 only, allow from wrong signs
3	**(i)**	(a) $e^{-0.42}$ $= \mathbf{0.657}$ (b) $0.42\, e^{-0.42} = \mathbf{0.276}$	M1 A1 A1	 **3**	Correct formula for *R* = 0 or 1 P(0), a.r.t. 0.657 P(1), a.r.t. 0.276
	(ii)	Po(2.1): $1 - P(\le 3) = 1 - 0.8386$ $= \mathbf{0.1614}$	M1 M1 A1	 **3**	Po(2.1) stated or implied Tables or formula, e.g. 0.8386 or 0.6496 or 0.9379 or complement; Answer, in range [0.161, 0.162]
	(iii)		B2	**2**	At least 3 separate bars, all decreasing *Allow histogram. Allow convex* P(0) < P(1) but otherwise OK: B1 Curve: B1 *[no hint of normal allowed]*
4	**(i)**	$H_0 : p = 0.14$ $H_1 : p < 0.14$ B(22, 0.14) $P(\le 2) = .86^{22} + (22 \times .86^{21} \times .14) +$ $(231 \times .86^{20} \times .14^2) = \mathbf{0.3877}$ > 0.1 Do not reject H_0. Insufficient evidence that company overestimates viewing proportion	B2 M1 A1 A1 B1 M1 A1	 **8**	Both correct. 1 error, B1, but *x* or *r* or $\bar{x}$ etc: 0 B(22, 0.14) stated or implied, e.g. N(3.08, 2.6488) or Po(3.08) Correct formula for 2 or 3 terms, *or* P(≤ 0) = 0.036 and CR Correct answer, a.r.t. 0.388, *or* CR is = 0 Explicitly compare 0.1 or CR with 2, OK from Po but *not* from N Correct comparison type and conclusion, needs binomial, at least 2 terms, *not* from P(< 2) Contextualised, some acknowledgement of uncertainty [SR: Normal: B2 M1 A0 B0 M0] [SR: 2-tailed, or *p* > 0.14, P(≥ 2): B1M1A2B0M1A1]
	(ii)	Selected independently Each adult equally likely to be chosen	B1 B1	 **2**	Independent *selection* Choice of sample elements equally likely (no credit if not focussed on selection) [Only "All samples of size *n* equally likely": B1 only unless related to Binomial conditions]
5	**(i)**		B1 B1 B1	 **3**	Horizontal straight line Symmetrical U-shaped curve Both correct, including relationship between the two and not extending beyond [–2, 2], curve through (0,0)
	(ii)	*S* is equally likely to take any value *T* is more likely at extremities	B2	**2**	Correct statement about both distributions, √ on their graph [Correct for one only, or partial description: B1] *Not* "probability of *S* is constant", etc.
	(iii)	$\frac{5}{64}\int_{-2}^{2} x^6 dx = \frac{5}{64}\left[\frac{x^7}{7}\right]_{-2}^{2} \quad \left[= \frac{20}{7}\right]$ $- 0^2$ $= \frac{20}{7}$	M1 A1 B1 A1	 **4**	Integrate $x^2 g(x)$, limits –2, 2 Correct indefinite integral [= $5x^7/448$] 0 or 0^2 subtracted or E(*X*) = 0 seen, *not* $\int x^2 f(x)dx - \int x f(x)dx$ Answer $\frac{20}{7}$ or $2\frac{6}{7}$ or a.r.t. 2.86, don't need 0

6	**(i)**	$50.0 \pm 1.96\sqrt{\frac{20.25}{81}} = 50.0 \pm 0.98$ = 49.02, 50.98 $\overline{W} < 49.02$ and $\overline{W} > 50.98$	M1 B1 A1A1 A1√ **5**	50.0 ± z√(1.96/81), allow one sign only, allow √ errors z = 1.96 in equation (*not* just stated) Both critical values, min 4 SF at some stage (if both 3SF, A1) CR, allow ≤ / ≥, don't need $\overline{W}$, √ on their CVs, can't recover [Ans 50 ± 0.98: A1 only] [SR: 1 tail, M1B0A0; 50.8225 or 49. 1775: A1]
	(ii)	$\frac{50.98-50.2}{0.5} = 1.56$ $\frac{49.02-50.2}{0.5} = -2.36$ Φ(1.56) – Φ(–2.36) = **0.9315**	M1 A1 A1 M1 A1 **5**	Standardise one limit with same SD as in (i) A.r.t. 1.56, allow – } Can allow √ here A.r.t. –2.36, allow + } if very unfair Correct handling of tails for Type II error Answer in range [0.931, 0.932] [SR 1-tail M1; –1.245 or 2.045 A1; 0.893 or 0.9795 A1]
	(iii)	It would get smaller	B1 **1**	No reason needed, but withhold if definitely wrong reason seen. Allow from 1-tail
7	**(i)**	$\hat{\mu} = \bar{t} = 13.7$ $\frac{12657.28}{64} - 13.7^2$ [= 10.08]; $\times \frac{64}{63}$ = **10.24** $H_0 : \mu = 13.1$, $H_1 : \mu > 13.1$ $\frac{13.7-13.1}{\sqrt{10.24/64}} = 1.5$ or p = 0.0668 1.5 < 1.645 or 0.0668 > 0.05 Do not reject H_0. Insufficient evidence that time taken on average is greater than 13.1 min	B1 M1 M1 A1 B2 M1 A1 B1 M1 A1 **11**	13.7 stated Correct formula for biased estimate $\times \frac{64}{63}$ used, or equivalent, can come in later Variance or SD 10.24 or 10.2 Both correct. [SR: One error, B1, but x or t or $\bar{x}$ or $\bar{t}$, 0] Standardise, or find CV, with √64 or 64 z = a.r.t. 1.50, or p = 0.0668, or CV 13.758 [√ on z] Compare z & 1.645, or p & 0.05 (must be correct tail), or z = 1.645 & 13 with CV Correct comparison & conclusion, needs 64, *not* μ = 13.7 Contextualised, some acknowledgement of uncertainty [13.1 – 13.7: (6), M1 A0 B1 M0]
	(ii)	Yes, not told that dist is normal	B1 **1**	Equivalent statement, *not* "n is large", don't need "yes"
8	**(i)**	N(14.7, 4.41) Valid because np = 14.7 > 5; nq = 6.3 > 5 $1-\Phi\left(\frac{15.5-14.7}{\sqrt{4.41}}\right)$ = 1 – Φ(0.381) = 1 – 0.6484 = **0.3516**	M1 A1 B1 B1 M1 A1 A1 **7**	Normal, attempt at np Both parameters correct Check np > 5; } If both asserted but not both nq or npq > 5 } 14.7 and 6.3 seen: B1 only [Allow "n large, p close to ½"] Standardise, answer < 0.5, no √n z, a.r.t. 0.381 Answer in range [0.351, 0.352] [Exact: M0]
	(ii)	$\bar{K}$ ~ N(14.7, 4.41/36) [= N(14.7, 0.35^2)] Valid by Central Limit Theorem as 36 is large $\Phi\left(\frac{14.0+\frac{1}{72}-14.7}{\sqrt{4.41/36}}\right)$ = Φ(–1.96) = **0.025**	M1 A1√ B1 M1 A1 A1 A1 **7**	Normal, their np from (i) Their variance/36 Refer to CLT or large n (= 36, *not* 21), or "K ~ N so $\bar{K}$ ~ N", *not* same as (i), *not* np > 5, nq > 5 for $\bar{K}$ Standardise 14.0 with 36 or √36 cc included, allow 0.5 here, e.g. 14.5 – 14.7 z = –1.96 or –2.00 or –2.04, allow + if answer < 0.5 0.025 or 0.0228 [0.284 loses last 2] [Po(25.2) etc: probably 0]
	OR:	B(756, 0.7) ≈ N(529.2, 158.76) $\Phi\left(\frac{504.5-529.2}{\sqrt{158.76}}\right)$ = Φ(–1.96) = **0.025**	M1M1A1 B1 M1 A1 A1	×36; N(529.6,…); 158.76 CLT as above, or np > 5, nq > 5, can be asserted here Standardise 14×36 cc correct and √npq 0.025 or 0.0228