

4734 Probability & Statistics 3

<p>1</p>	<p>T has a Poisson distribution</p> <p>$E(T)=28 \times 0.75 + 4 \times 6.4$ $= 46.6$ $\text{Var}(T)=46.6$</p>	<p>B1</p> <p>M1 A1 B1√ 4</p>	<p>From sum of Poissons</p> <p>Ft $E(T)$ only if Poisson</p>
<p>2 (i)</p> <p>-----</p> <p>(ii)</p> <p>---</p>	<p>Use $F(Q_3)=0.75$ or $\int_{Q_3}^{\infty} \frac{1}{5} e^{-\frac{1}{4}u} du = 0.25$</p> <p>Solve to obtain $Q_3 = 4.65$ AEF eg $4 \ln(16/5)$</p> <p>-----</p> $f(u) = \begin{cases} \frac{1}{5} e^u & u < 0, \\ \frac{1}{5} e^{-\frac{1}{4}u} & u \geq 0. \end{cases}$	<p>M1</p> <p>M1A1 3</p> <p>-----</p> <p>B1</p> <p>B1 2</p>	<p>M1 for solving similar eqn A0 for ≥ 4.65</p> <p>-----</p> <p>$u < 0$ unless evidence of \int</p> <p>$u \geq 0$</p>
<p>3 (i)</p> <p>-----</p> <p>(ii)</p>	<p>Use $28 \pm z_s$ $z=2.326$ $s^2 = 28 \times 72/1200$ $(25.0, 31.0)$</p> <p>-----</p> <p>$2 \times 2.326 \sqrt{(0.28 \times 0.72/n)} \leq 0.05$ AEF</p> <p>Solve to obtain n Smallest $n = 1745$ e.g. Variance is an approximation</p>	<p>M1 B1 B1 A1 4</p> <p>-----</p> <p>M1 M1 A1 B1 4</p>	<p>Accept $s=c/\sqrt{n}$ for M1 Accept 0.28 with corresponding s</p> <p>Or 1199 Accept (25, 31)</p> <p>-----</p> <p>Or = or \geq Solving similar eqn Accept 1746 ,1750 Or normal is approx or Or p only an estimate</p>
<p>4 (i)</p> <p>-----</p> <p>(ii)</p> <p>-----</p> <p>(iii)</p>	<p>$c = 1/20$</p> <p>-----</p> $\int_{25}^{45} \frac{400\sqrt{x} - 240}{20} dx$ $= \left[\frac{40}{3} x^{3/2} - 12x \right]$ <p>$= 2118(\pounds)$</p> <p>-----</p> <p>$400\sqrt{X} - 240 > 2000, X > 31.36$ $P(X > 31.36) = (45 - 31.36)/20$ $= 0.682$</p>	<p>B1 1</p> <p>-----</p> <p>M1</p> <p>A1</p> <p>A`1 3</p> <p>-----</p> <p>M1 M1 A1 3</p>	<p>-----</p> <p>Correct indefinite integral</p> <p>2120 or better than 2118</p> <p>-----</p> <p>Or 31.4</p> <p>cao</p>

<p>5 (i)</p> <p>(ii)</p> <p>(iii)</p>	<p>$H_0: \mu_2 = \mu_1$, $H_1: \mu_2 > \mu_1$, where μ_1 and μ_2 are the mean concentrations in the lake before and after the spillage respectively</p> <hr/> <p>$\bar{X}_2 - \bar{X}_1 \geq z s$ $z = 1.645$ $s = 0.24 \sqrt{(1/5 + 1/6)}$ ≥ 0.2391</p> <hr/> <p>$P(\bar{X}_2 - \bar{X}_1 < 0.2391)$ $z = [0.2391 - 0.3]/s$ $p = 0.3376$ This is a large probability for this error</p>	<p>B1</p> <p>B1 2</p> <hr/> <p>M1</p> <p>A1</p> <p>B1</p> <p>A1 4</p> <hr/> <p>M1</p> <hr/> <p>M1</p> <p>A1</p> <p>B1 4</p>	<p>For both hypotheses Allow in words if population mean used.</p> <hr/> <p>Accept $>$, $=$, $<$, \leq, ts</p> <hr/> <p>Or $>$; 0.239</p> <hr/> <p>May be implied</p> <hr/> <p>ART 0.337 or 0.338 Relevant comment</p>
<p>6 (i)</p> <p>(ii)</p>	<p>Use $B \sim B(29, 0.3)$, $G \sim B(26, 0.2)$ $E(F) = 29 \times 0.3 + 26 \times 0.2 = 13.9$ $\text{Var}(F) = 29 \times 0.3 \times 0.7 + 26 \times 0.2 \times 0.8 = 10.25$</p> <hr/> <p>$B: np = 8.7, nq = 20.3$ $G: np = 5.2, nq = 20.8$ All exceed 5, so normal approximation valid for each $F \sim N(13.9, 10.25)$ (approximately) (Requires $P(F \leq n) = 0.99$) $[n + 0.5 - 13.9] / \sqrt{10.25} = 2.326$, their 10.25</p> <p>$n = 20.85$ Need to have 21 spares available SR Using $B(55, 0.2527)$: B1; M1(N(13.9, 10.39)); M1B1M1A0 (Max 5/8)</p>	<p>M1</p> <p>M1A1</p> <p>M1A1 5</p> <hr/> <p>B2</p> <p>M1√</p> <hr/> <p>M1B1</p> <hr/> <p>A1</p> <p>M1</p> <p>A1 8</p>	<hr/> <p>Must check numerically B1 for checking one distribution</p> <hr/> <p>Use normal. May be implied</p> <hr/> <p>Standardise M0 if variance has divisors cc Solving similar No cc, lose last A1 (n = 22) Wrong cc, lose A1A1</p>

<p>7 (i)</p>	<p>Requires population of (2nd mark – 1st mark) to be normally distributed $H_0: \mu_d = 0, H_1: \mu_d > 0$ $T_2 - T_1 : -1 -1 2 0 -2 2 3 2$ $\bar{d} = 0.625, s^2 = 3.411 (3^{23}/56 \text{ or } 191/56)$ Use 2.998 EITHER: $t = 0.625/\sqrt{(3.411/8)}$ $= 0.957$ OR: $CV(CR), \bar{d} \geq 2.998\sqrt{3.411/8}$ $= 1.958$ EITHER $0.957 < 2.998$ OR $0.625 < 1.958$ Do not reject H_0, there is insufficient evidence of improvement</p> <hr/> <p>(ii) Use $E(X_2 - X_1 + k) = 0.625 + k$ Requires $(0.625+k) / \sqrt{(3.411/8)} \geq 2.998$ Giving $k \geq 1.33$ Increase each mark by 2</p>	<p>B1 M1 B1B1 B1 M1 A1 M1 A1 M1 8 M1 A1√ A1 3</p>	<p>M0 if clearly z</p> <hr/> <p>With comparison and conclusion</p> <hr/> <p>Allow 1.33</p>
<p>8 (i)</p>	<p>Mean = $(20+16+9)/75$ $= 0.6$ $3p = 0.6, p = 0.2$ AG</p> <hr/> <p>(ii) $H_0: B(3,p)$ fits the data $(H_1: B(3,p)$ does not fit the data) Expected values 38.4 28.8 7.2 0.6</p> <p>Combine last two cells $\chi^2 = 5.6^2/38.4 + 8.8^2/28.8 + 3.2^2/7.8$ $= 4.818$</p> <p>$4.818 > 3.841$ Reject H_0 and conclude that there is insufficient evidence that $B(3,p)$ fits the data.</p> <hr/> <p>(iii) $2.74 < 3.841$, accept H_0 conclude that $B(6, p)$ fits the data</p>	<p>M1 A1 A1 3</p> <hr/> <p>B1 M1 A1 A1 B1 M1 A1√ A1 B1√ M1 10</p> <hr/> <p>B1 1</p>	<p>Or: $X \sim B(3,p)$ or $B(3,0.2)$ Not 'Data fits model'</p> <p>Use $B(3,0.2) \times 75$ At least 2 correct All correct</p> <p>With one correct At least 2 correct Ft E values Accept 4.82 cao</p> <p>ft 4.818 SR1 If cells not combined: B1M1A1A1B0M1A1A0B1(5.991)M1 SR2:E-values rounded :B1M1A1A1 B1M1A1A0(4.865)B1M1</p> <hr/> <p>Accept with no reason if evidence of method in (ii)</p>