OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4753/01

Methods for Advanced Mathematics (C3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Thursday 15 January 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Solve the inequality $|x - 1| < 3$. **[3]**

2 **(i)** Differentiate $x \cos 2x$ with respect to x. **[3]**

(ii) Integrate $x \cos 2x$ with respect to x. **[4]**

3 Given that $f(x) = \frac{1}{2}\ln(x - 1)$ and $g(x) = 1 + e^{2x}$, show that $g(x)$ is the inverse of $f(x)$. **[3]**

4 Find the exact value of $\int_0^2 \sqrt{1 + 4x}\, dx$, showing your working. **[5]**

5 **(i)** State the period of the function $f(x) = 1 + \cos 2x$, where x is in degrees. **[1]**

(ii) State a sequence of two geometrical transformations which maps the curve $y = \cos x$ onto the curve $y = f(x)$. **[4]**

(iii) Sketch the graph of $y = f(x)$ for $-180° < x < 180°$. **[3]**

6 **(i)** Disprove the following statement.

'If $p > q$, then $\frac{1}{p} < \frac{1}{q}$.' **[2]**

(ii) State a condition on p and q so that the statement is true. **[1]**

7 The variables x and y satisfy the equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 5$.

(i) Show that $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$. **[4]**

Both x and y are functions of t.

(ii) Find the value of $\frac{dy}{dt}$ when $x = 1$, $y = 8$ and $\frac{dx}{dt} = 6$. **[3]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = x^2 - \frac{1}{8}\ln x$. P is the point on this curve with x-coordinate 1, and R is the point $\left(0, -\frac{7}{8}\right)$.

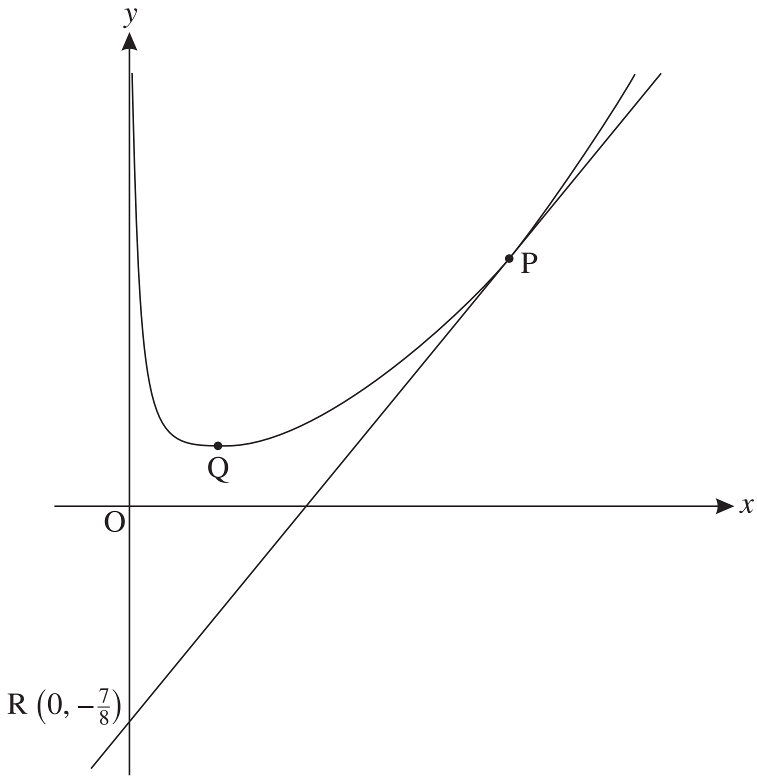

Fig. 8

(i) Find the gradient of PR. **[3]**

(ii) Find $\frac{dy}{dx}$. Hence show that PR is a tangent to the curve. **[3]**

(iii) Find the exact coordinates of the turning point Q. **[5]**

(iv) Differentiate $x\ln x - x$.

Hence, or otherwise, show that the area of the region enclosed by the curve $y = x^2 - \frac{1}{8}\ln x$, the x-axis and the lines $x = 1$ and $x = 2$ is $\frac{59}{24} - \frac{1}{4}\ln 2$. **[7]**

[Question 9 is printed overleaf.]

9 Fig. 9 shows the curve $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{\sqrt{2x - x^2}}$.

The curve has asymptotes $x = 0$ and $x = a$.

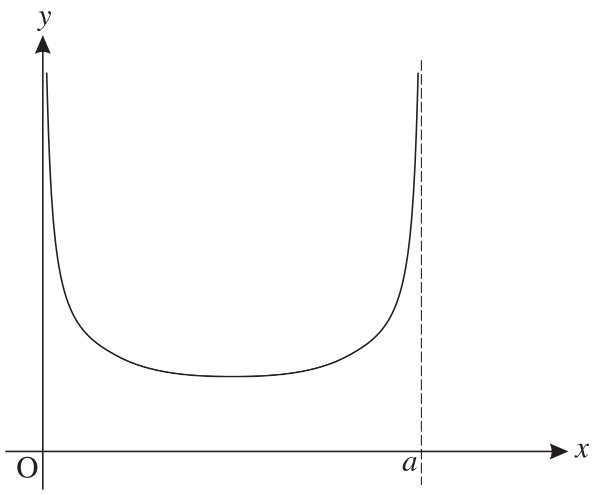

Fig. 9

(i) Find a. Hence write down the domain of the function. **[3]**

(ii) Show that $\dfrac{dy}{dx} = \dfrac{x - 1}{(2x - x^2)^{\frac{3}{2}}}$.

Hence find the coordinates of the turning point of the curve, and write down the range of the function. **[8]**

The function $\mathrm{g}(x)$ is defined by $\mathrm{g}(x) = \dfrac{1}{\sqrt{1 - x^2}}$.

(iii) (*A*) Show algebraically that $\mathrm{g}(x)$ is an even function.

(*B*) Show that $\mathrm{g}(x - 1) = \mathrm{f}(x)$.

(*C*) Hence prove that the curve $y = \mathrm{f}(x)$ is symmetrical, and state its line of symmetry. **[7]**