OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI)

4754A

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Tuesday 13 January 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

Section A (36 marks)

1 Express $\dfrac{3x+2}{x(x^2+1)}$ in partial fractions. **[6]**

2 Show that $(1+2x)^{\frac{1}{3}} = 1 + \frac{2}{3}x - \frac{4}{9}x^2 + \ldots$, and find the next term in the expansion.

State the set of values of x for which the expansion is valid. **[6]**

3 Vectors $\mathbf{a}$ and $\mathbf{b}$ are given by $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

Find constants λ and μ such that $\lambda\mathbf{a} + \mu\mathbf{b} = 4\mathbf{j} - 3\mathbf{k}$. **[5]**

4 Prove that $\cot\beta - \cot\alpha = \dfrac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta}$. **[3]**

5 **(i)** Write down normal vectors to the planes $2x - y + z = 2$ and $x - z = 1$.

Hence find the acute angle between the planes. **[4]**

(ii) Write down a vector equation of the line through $(2, 0, 1)$ perpendicular to the plane $2x - y + z = 2$. Find the point of intersection of this line with the plane. **[4]**

6 **(i)** Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is acute, expressing α in terms of π. **[4]**

(ii) Write down the derivative of $\tan\theta$.

Hence show that $\displaystyle\int_0^{\frac{1}{3}\pi} \frac{1}{(\cos\theta + \sqrt{3}\sin\theta)^2}\,\mathrm{d}\theta = \frac{\sqrt{3}}{4}$. **[4]**

Section B (36 marks)

7 Scientists can estimate the time elapsed since an animal died by measuring its body temperature.

(i) Assuming the temperature goes down at a constant rate of 1.5 degrees Fahrenheit per hour, estimate how long it will take for the temperature to drop

(*A*) from 98 °F to 89 °F,

(*B*) from 98 °F to 80 °F. **[2]**

In practice, rate of temperature loss is not likely to be constant. A better model is provided by Newton's law of cooling, which states that the temperature θ in degrees Fahrenheit t hours after death is given by the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k(\theta - \theta_0),$$

where θ_0 °F is the air temperature and k is a constant.

(ii) Show by integration that the solution of this equation is $\theta = \theta_0 + A\mathrm{e}^{-kt}$, where A is a constant. **[5]**

The value of θ_0 is 50, and the initial value of θ is 98. The initial rate of temperature loss is 1.5 °F per hour.

(iii) Find A, and show that $k = 0.031\,25$. **[4]**

(iv) Use this model to calculate how long it will take for the temperature to drop

(*A*) from 98 °F to 89 °F,

(*B*) from 98 °F to 80 °F. **[5]**

(v) Comment on the results obtained in parts **(i)** and **(iv)**. **[1]**

[Question 8 is printed overleaf.]

8 Fig. 8 illustrates a hot air balloon on its side. The balloon is modelled by the volume of revolution about the x-axis of the curve with parametric equations

$$x = 2 + 2\sin\theta, \quad y = 2\cos\theta + \sin 2\theta, \quad (0 \leqslant \theta \leqslant 2\pi).$$

The curve crosses the x-axis at the point A (4, 0). B and C are maximum and minimum points on the curve. Units on the axes are metres.

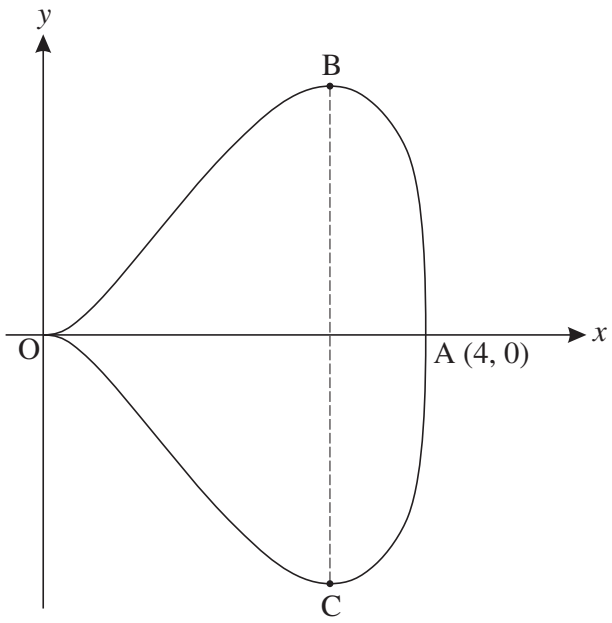

Fig. 8

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ. **[4]**

(ii) Verify that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $\theta = \frac{1}{6}\pi$, and find the exact coordinates of B.

Hence find the maximum width BC of the balloon. **[5]**

(iii) (*A*) Show that $y = x\cos\theta$.

(*B*) Find $\sin\theta$ in terms of x and show that $\cos^2\theta = x - \frac{1}{4}x^2$.

(*C*) Hence show that the cartesian equation of the curve is $y^2 = x^3 - \frac{1}{4}x^4$. **[7]**

(iv) Find the volume of the balloon. **[3]**

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ADVANCED GCE

MATHEMATICS (MEI) 4754B

Applications of Advanced Mathematics (C4) Paper B: Comprehension

Candidates answer on the question paper

OCR Supplied Materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Rough paper

Tuesday 13 January 2009
Morning

Duration: Up to 1 hour

* 4 7 5 4 B *

Candidate Forename							Candidate Surname					
Centre Number							Candidate Number					

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Write your answer to each question in the space provided, however additional paper may be used if necessary.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **4** pages. Any blank pages are indicated.

Examiner's Use Only:	
1	
2	
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For Examiner's Use

1 Show how the value $d = 8$ on line 32 is obtained. **[2]**

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2 Using the information given on lines 38 and 39, derive equation (1). **[3]**

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3 On lines 43 and 44 it is suggested that the volume of fuel in the tank in Figs. 2.1 and 2.2 could be calculated using the values of h and θ.

Calculate the volume of fuel in the case where $h = 5$ and $\theta = 30^\circ$. **[3]**

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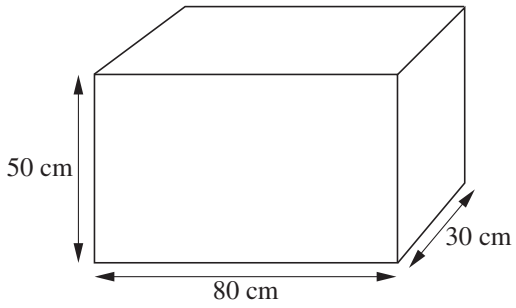
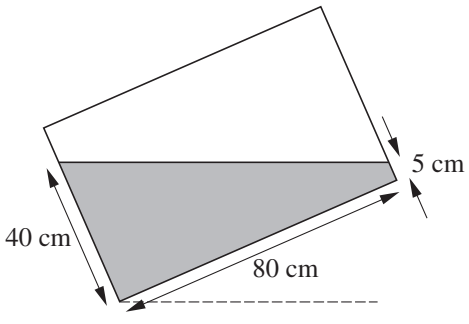
For Examiner's Use

4 A fuel tank in the shape of a cuboid is shown below.



It is partly filled with fuel and inclined at an angle to the horizontal. The side view is shown below.



Calculate the volume, in litres, of fuel in the tank. **[3]**

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For Examiner's Use

5 **(i)** Explain clearly how the equation on line 72 can be simplified to give the quadratic equation on line 74. **[1]**

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(ii) In line 76 only one root of the quadratic equation is given. Find the other root and explain why it is not relevant in the context of this problem. **[3]**

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6 On line 90 it is stated that if $H = h = 10$ then equation (4) gives a volume of 37.5 litres. Use equations (3) and (4) to show how this volume is derived. **[3]**

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