OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS

4726

Further Pure Mathematics 2

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Friday 9 January 2009
Morning

Duration: 1 hour 30 minutes

4726

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 **(i)** Write down and simplify the first three terms of the Maclaurin series for e^{2x}. **[2]**

(ii) Hence show that the Maclaurin series for

$$\ln(e^{2x} + e^{-2x})$$

begins $\ln a + bx^2$, where a and b are constants to be found. **[4]**

2 It is given that α is the only real root of the equation $x^5 + 2x - 28 = 0$ and that $1.8 < \alpha < 2$.

(i) The iteration $x_{n+1} = \sqrt[5]{28 - 2x_n}$, with $x_1 = 1.9$, is to be used to find α. Find the values of x_2, x_3 and x_4, giving the answers correct to 7 decimal places. **[3]**

(ii) The error e_n is defined by $e_n = \alpha - x_n$. Given that $\alpha = 1.891\,574\,9$, correct to 7 decimal places, evaluate $\frac{e_3}{e_2}$ and $\frac{e_4}{e_3}$. Comment on these values in relation to the gradient of the curve with equation $y = \sqrt[5]{28 - 2x}$ at $x = \alpha$. **[3]**

3 **(i)** Prove that the derivative of $\sin^{-1} x$ is $\frac{1}{\sqrt{1 - x^2}}$. **[3]**

(ii) Given that

$$\sin^{-1} 2x + \sin^{-1} y = \tfrac{1}{2}\pi,$$

find the exact value of $\frac{dy}{dx}$ when $x = \frac{1}{4}$. **[4]**

4 **(i)** By means of a suitable substitution, show that

$$\int \frac{x^2}{\sqrt{x^2 - 1}}\,dx$$

can be transformed to $\int \cosh^2 \theta \, d\theta$. **[2]**

(ii) Hence show that $\int \frac{x^2}{\sqrt{x^2 - 1}}\,dx = \frac{1}{2}x\sqrt{x^2 - 1} + \frac{1}{2}\cosh^{-1} x + c$. **[4]**

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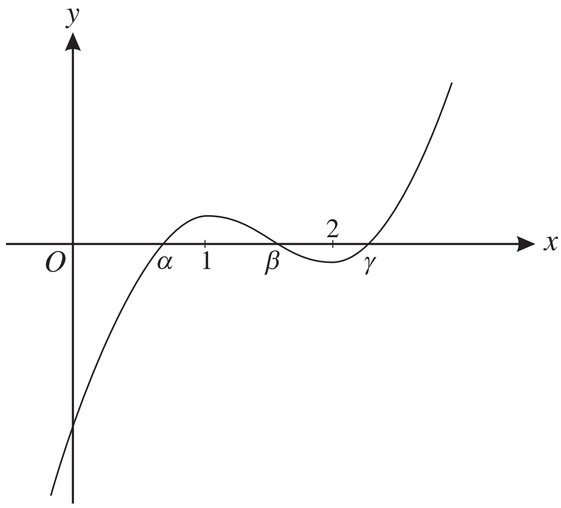

The diagram shows the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2x^3 - 9x^2 + 12x - 4.36.$$

The curve has turning points at $x = 1$ and $x = 2$ and crosses the x-axis at $x = \alpha$, $x = \beta$ and $x = \gamma$, where $0 < \alpha < \beta < \gamma$.

(i) The Newton-Raphson method is to be used to find the roots of the equation $\mathrm{f}(x) = 0$, with $x_1 = k$.

(a) To which root, if any, would successive approximations converge in each of the cases $k < 0$ and $k = 1$? **[2]**

(b) What happens if $1 < k < 2$? **[2]**

(ii) Sketch the curve with equation $y^2 = \mathrm{f}(x)$. State the coordinates of the points where the curve crosses the x-axis and the coordinates of any turning points. **[4]**

6 **(i)** Using the definitions of $\cosh x$ and $\sinh x$ in terms of e^x and e^{-x}, show that

$$1 + 2\sinh^2 x \equiv \cosh 2x.$$ **[3]**

(ii) Solve the equation

$$\cosh 2x - 5\sinh x = 4,$$

giving your answers in logarithmic form. **[5]**

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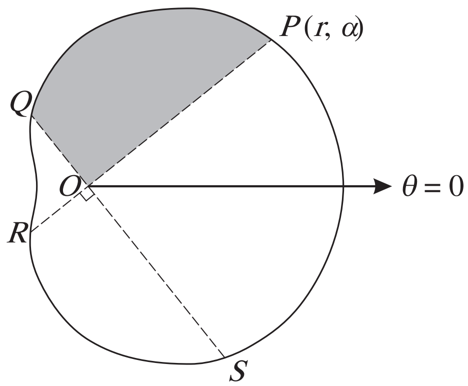


The diagram shows the curve with equation, in polar coordinates,

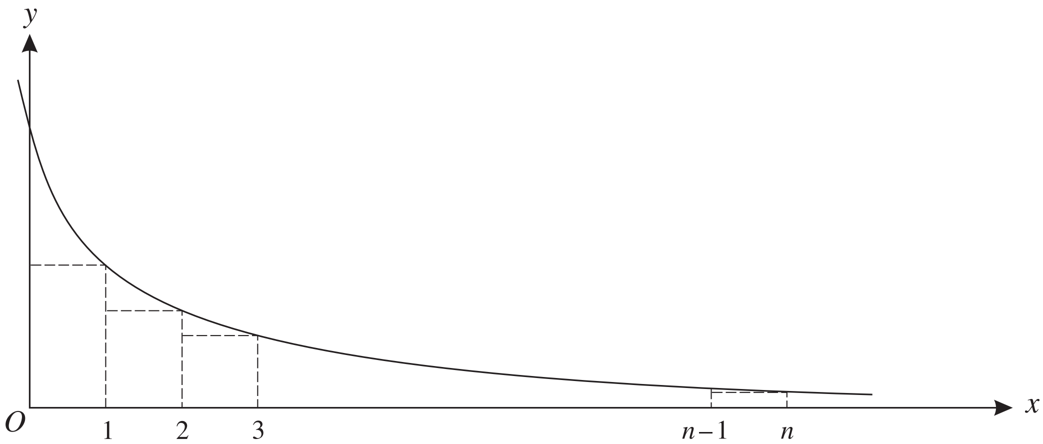
$$r = 3 + 2\cos\theta, \quad \text{for } 0 \leqslant \theta < 2\pi.$$

The points P, Q, R and S on the curve are such that the straight lines POR and QOS are perpendicular, where O is the pole. The point P has polar coordinates (r, α).

(i) Show that $OP + OQ + OR + OS = k$, where k is a constant to be found. **[3]**

(ii) Given that $\alpha = \frac{1}{4}\pi$, find the exact area bounded by the curve and the lines OP and OQ (shaded in the diagram). **[5]**

8



The diagram shows the curve with equation $y = \frac{1}{x+1}$. A set of n rectangles of unit width is drawn, starting at $x = 0$ and ending at $x = n$, where n is an integer.

(i) By considering the areas of these rectangles, explain why

$$\frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n+1} < \ln(n+1).$$ **[5]**

(ii) By considering the areas of another set of rectangles, show that

$$1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} > \ln(n+1).$$ **[2]**

(iii) Hence show that

$$\ln(n+1) + \frac{1}{n+1} < \sum_{r=1}^{n+1} \frac{1}{r} < \ln(n+1) + 1.$$ **[2]**

(iv) State, with a reason, whether $\sum_{r=1}^{\infty} \frac{1}{r}$ is convergent. **[2]**

9 A curve has equation

$$y = \frac{4x - 3a}{2(x^2 + a^2)},$$

where a is a positive constant.

(i) Explain why the curve has no asymptotes parallel to the y-axis. **[2]**

(ii) Find, in terms of a, the set of values of y for which there are no points on the curve. **[5]**

(iii) Find the exact value of $\int_a^{2a} \frac{4x - 3a}{2(x^2 + a^2)} \, dx$, showing that it is independent of a. **[5]**

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