

**ADVANCED GCE**  
**MATHEMATICS**  
Further Pure Mathematics 3

**4727**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

None

**Thursday 29 January 2009**  
**Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 In this question  $G$  is a group of order  $n$ , where  $3 \leq n < 8$ .

(i) In each case, write down the smallest possible value of  $n$ :

(a) if  $G$  is cyclic, [1]

(b) if  $G$  has a proper subgroup of order 3, [1]

(c) if  $G$  has at least two elements of order 2. [1]

(ii) Another group has the same order as  $G$ , but is not isomorphic to  $G$ . Write down the possible value(s) of  $n$ . [2]

2 (i) Express  $\frac{\sqrt{3} + i}{\sqrt{3} - i}$  in the form  $re^{i\theta}$ , where  $r > 0$  and  $0 \leq \theta < 2\pi$ . [3]

(ii) Hence find the smallest positive value of  $n$  for which  $\left(\frac{\sqrt{3} + i}{\sqrt{3} - i}\right)^n$  is real and positive. [2]

3 Two skew lines have equations

$$\frac{x}{2} = \frac{y+3}{1} = \frac{z-6}{3} \quad \text{and} \quad \frac{x-5}{3} = \frac{y+1}{1} = \frac{z-7}{5}.$$

(i) Find the direction of the common perpendicular to the lines. [2]

(ii) Find the shortest distance between the lines. [4]

4 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 65 \sin 2x. \quad [9]$$

5 The variables  $x$  and  $y$  are related by the differential equation

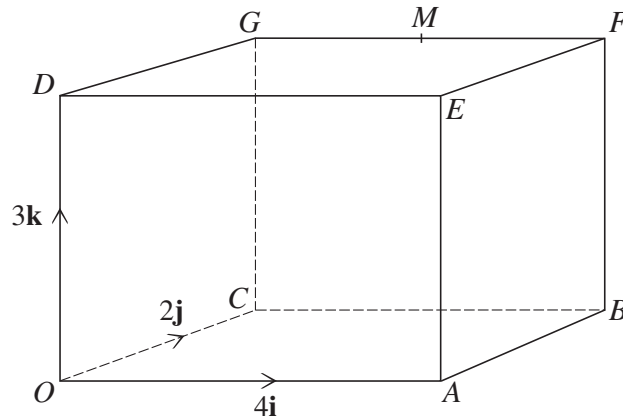
$$x^3 \frac{dy}{dx} = xy + x + 1. \quad (\text{A})$$

(i) Use the substitution  $y = u - \frac{1}{x}$ , where  $u$  is a function of  $x$ , to show that the differential equation may be written as

$$x^2 \frac{du}{dx} = u. \quad [4]$$

(ii) Hence find the general solution of the differential equation (A), giving your answer in the form  $y = f(x)$ . [5]

6



The cuboid  $OABCDEFG$  shown in the diagram has  $\overrightarrow{OA} = 4\mathbf{i}$ ,  $\overrightarrow{OC} = 2\mathbf{j}$ ,  $\overrightarrow{OD} = 3\mathbf{k}$ , and  $M$  is the mid-point of  $GF$ .

(i) Find the equation of the plane  $ACGE$ , giving your answer in the form  $\mathbf{r} \cdot \mathbf{n} = p$ . [4]

(ii) The plane  $OEFC$  has equation  $\mathbf{r} \cdot (3\mathbf{i} - 4\mathbf{k}) = 0$ . Find the acute angle between the planes  $OEFC$  and  $ACGE$ . [4]

(iii) The line  $AM$  meets the plane  $OEFC$  at the point  $W$ . Find the ratio  $AW : WM$ . [5]

7 (i) The operation  $*$  is defined by  $x * y = x + y - a$ , where  $x$  and  $y$  are real numbers and  $a$  is a real constant.

(a) Prove that the set of real numbers, together with the operation  $*$ , forms a group. [6]

(b) State, with a reason, whether the group is commutative. [1]

(c) Prove that there are no elements of order 2. [2]

(ii) The operation  $\circ$  is defined by  $x \circ y = x + y - 5$ , where  $x$  and  $y$  are **positive** real numbers. By giving a numerical example in each case, show that two of the basic group properties are not necessarily satisfied. [4]

8 (i) By expressing  $\sin \theta$  in terms of  $e^{i\theta}$  and  $e^{-i\theta}$ , show that

$$\sin^6 \theta \equiv -\frac{1}{32}(\cos 6\theta - 6 \cos 4\theta + 15 \cos 2\theta - 10). \quad [5]$$

(ii) Replace  $\theta$  by  $(\frac{1}{2}\pi - \theta)$  in the identity in part (i) to obtain a similar identity for  $\cos^6 \theta$ . [3]

(iii) Hence find the exact value of  $\int_0^{\frac{1}{4}\pi} (\sin^6 \theta - \cos^6 \theta) d\theta$ . [4]

There are no questions printed on this page.