

4725 Further Pure Mathematics 1

1 (i)	$\begin{pmatrix} a-4 & 2 \\ 3 & 0 \end{pmatrix}$	B1	Two elements correct
		B1	2 Remaining elements correct
(ii)	$4a - 6$	B1	Correct determinant
		M1	Equate det A to 0 and solve
	$a = \frac{3}{2}$	A1	3 Obtain correct answer a. e. f.
		5	
2 (i)	$u^3 - 3u^2 + 3u - 1$	B1	Correct unsimplified expansion of $(u-1)^3$
		M1	Substitute for x
	$2u^3 - 6u^2 + 9u - 8 = 0$	A1	3 Obtain correct equation
(ii)		M1	Use $(\pm)\frac{d}{a}$ of new equation
	4	A1ft	2 Obtain correct answer from their equation
		5	
3	$x - iy$	B1	Conjugate known
		M1	Equate real and imaginary parts
	$x + 2y = 12 \quad 2x + y = 9$	A1	Obtain both equations, OK with factor of i
		M1	Solve pair of equations
	$z = 2 + 5i$	A1	5 Obtain correct answer as a complex number
			S.C. Solving $z + 2iz = 12 + 9i$ can get max 4/5, not first B1
		5	
4		M1	Express as sum of three series
		M1	Use standard results
	$\frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1) - n(n+1)$	A1	Obtain correct unsimplified answer
		M1	Attempt to factorise
		A1	Obtain at least factor of $n(n+1)$
	$\frac{1}{12}n(n+1)(n+2)(3n-7)$	A1	6 Obtain fully factorised correct answer
		6	

5 (i)	B1 B1	2	Rotation 90° (about origin) Anticlockwise
(ii) <i>Either</i>	M1		Show image of unit square after reflection in $y = -x$
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	A1		Deduce reflection in x-axis
<i>Or</i>	B1ft B1ft M1	4	Each column correct ft for matrix of their transformation Post multiply by correct reflection matrix
	A1 B1B1		Obtain correct answer State reflection, in x-axis
			S.C. If pre-multiplication, M0 but B1 B1 Available for correct description of their matrix
	6		
6 (i)	B1 M1		State or use $5 + i$ as a root Use $\sum \alpha\beta = 6$
$x = -2$	A1	3	Obtain correct answer
(ii) <i>Either</i>	M1		Use $p = -\sum \alpha$
$p = -8$	A1ft M1		Obtain correct answer, from their root Use $q = -\alpha\beta\gamma$
$q = 52$	A1ft	4	Obtain correct answer, from their root
<i>Or</i>	M1 M1 A1A1		Attempt to find quadratic factor Attempt to expand quadratic and linear Obtain correct answers
<i>Or</i>	M1 M1 A1 A1ft		Substitute $(5 - i)$ into equation Equate real and imaginary parts Obtain correct answer for p Obtain correct answer for q , ft their p
	7		
7 (i)	B1	1	Obtain given answer correctly
(ii)	M1		Express at least 1 st two and last term using (i)
	A1 M1		All terms correct Show that correct terms cancel
$1 - \frac{1}{(n+1)^2}$	A1	4	Obtain correct answer, in terms of n
(iii) $\frac{1}{4}$	B1		Sum to infinity seen or implied
	B1	2	Obtain correct answer S.C. $-\frac{3}{4}$ scores B1
	7		

8 (i)	$x^2 - y^2 = 5$ and $xy = -6$	M1	Attempt to equate real and imaginary parts of $(x + iy)^2$ & $5 - 12i$
		A1	Obtain both results, a.e.f
		M1	Obtain quadratic in x^2 or y^2
		M1	Solve to obtain $x = (\pm)3$ or $y = (\pm)2$
	$\pm(3 - 2i)$	A1	5 Obtain correct answers as complex nos
(ii) square root			B1ft Circle with centre at their
		B1	Circle passing through origin
		B1ft	2 nd circle centre correct relative to 1 st
		B1	4 Circle passing through origin
		9	
9 (i)	$\det \mathbf{A} = \Delta = 6a - 6$	M1	Show correct expansion process for 3×3 or multiply adjoint by $\mathbf{A}$
		M1	Correct evaluation of any 2×2 at any stage
		A1	Obtain correct answer
	$\mathbf{A}^{-1} = \frac{1}{\Delta} \begin{pmatrix} 3a-1 & a+1 & -4 \\ 1 & 2a-1 & -2 \\ -3 & -3 & 6 \end{pmatrix}$	M1	Show correct process for adjoint entries
		A1	Obtain at least 4 correct entries in adjoint
		B1	Divide by their determinant
		A1	7 Obtain completely correct answer
(ii)	$\frac{1}{\Delta} \begin{pmatrix} 5a-7 \\ 4a-5 \\ 3 \end{pmatrix}$	M1	Attempt product of form $\mathbf{A}^{-1}\mathbf{C}$ or eliminate to get 2 equations and solve
		A1A1A1 ft all 3	Obtain correct answer
		4	S.C. if det now omitted, allow max A2 ft
		11	
10 (i)	$\mathbf{M}^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} \quad \mathbf{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$	B1	Correct $\mathbf{M}^2$ seen
		M1	Convincing attempt at matrix
		A1	multiplication for $\mathbf{M}^3$ Obtain correct answer
(ii)	$\mathbf{M}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$	B1ft	1 State correct form, consistent with (i)

10 (iii)

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|----|---|--|
| M1 | | Correct attempt to multiply $\mathbf{M}$ & $\mathbf{M}^k$
or v.v. |
| A1 | | Obtain element $2(k + 1)$ |
| A1 | | Clear statement of induction step, from
correct working |
| B1 | 4 | Clear statement of induction conclusion,
following their working |

(iv)

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|-----|---|---|
| B1 | | Shear |
| DB1 | | x-axis invariant |
| DB1 | 3 | e.g. $(1, 1) \rightarrow (21, 1)$ or equivalent
using scale factor or angles |

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