# 4727 Further Pure Mathematics 3

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| **1** | METHOD 1 | | | |
| | line segment between $l_1$ and $l_2$ = $\pm[4, -3, -9]$ | B1 | | For correct vector |
| | $\mathbf{n} = [1, -1, 2] \times [2, 3, 4] = (\pm)[-2, 0, 1]$ | M1* A1 | | For finding vector product of direction vectors |
| | distance = $\dfrac{\lvert[4, -3, -9] \cdot [-2, 0, 1]\rvert}{\left(\sqrt{2^2+0^2+1^2}\right)} = \dfrac{17}{\left(\sqrt{5}\right)}$ | M1 (*dep) | | For using numerator of distance formula |
| | $\neq 0$, so skew | A1 | **5** | For correct scalar product and correct conclusion |
| | METHOD 2 lines would intersect where | | | |
| | $\left.\begin{array}{l} 1+s=-3+2t \\ -2-s=1+3t \\ -4+2s=5+4t \end{array}\right\} \Rightarrow \left\{\begin{array}{l} s-2t=-4 \\ s+3t=-3 \\ 2s-4t=9 \end{array}\right.$ | B1 | | For correct parametric form for either line |
| | | M1* | | For 3 equations using 2 different parameters |
| | | A1 | | |
| | | M1 (*dep) | | For attempting to solve to show (in)consistency |
| | $\Rightarrow$ contradiction, so skew | A1 | | For correct conclusion |
| | | $\boxed{5}$ | | |
| **2 (i)** | $\left(a+b\sqrt{5}\right)\left(c+d\sqrt{5}\right)$ | M1 | | For using product of 2 distinct elements |
| | $= ac+5bd+(bc+ad)\sqrt{5} \in H$ | A1 | **2** | For correct expression |
| **(ii)** | $(e=)\ 1$ *OR* $1+0\sqrt{5}$ | B1 | **1** | For correct identity |
| **(iii)** | *EITHER* $\dfrac{1}{a+b\sqrt{5}} \times \dfrac{a-b\sqrt{5}}{a-b\sqrt{5}}$ | M1 | | For correct inverse as $\left(a+b\sqrt{5}\right)^{-1}$ and multiplying top and bottom by $a-b\sqrt{5}$ |
| | *OR* $\left(a+b\sqrt{5}\right)\left(c+d\sqrt{5}\right)=1 \Rightarrow \left\{\begin{array}{l} ac+5bd=1 \\ bc+ad=0 \end{array}\right.$ | | | *OR* for using definition and equating parts |
| | inverse $= \dfrac{a}{a^2-5b^2} - \dfrac{b}{a^2-5b^2}\sqrt{5}$ | A1 | **2** | For correct inverse. Allow as a single fraction |
| **(iv)** | 5 is prime *OR* $\sqrt{5} \notin \mathbb{Q}$ | B1 | **1** | For a correct property (or equivalent) |
| | | $\boxed{6}$ | | |
| **3** | Integrating factor = $e^{\int 2\,dx} = e^{2x}$ | B1 | | For correct IF |
| | $\Rightarrow \dfrac{d}{dx}\left(ye^{2x}\right) = e^{-x}$ | M1 | | For $\dfrac{d}{dx}(y.\text{their IF}) = e^{-3x}$ . their IF |
| | $\Rightarrow ye^{2x} = -e^{-x}(+c)$ | A1 | | For correct integration both sides |
| | $(0, 1) \Rightarrow c = 2$ | M1 | | For substituting $(0, 1)$ into their GS and solving for $c$ |
| | | A1√ | | For correct $c$ f.t. from their GS |
| | $\Rightarrow y = -e^{-3x} + 2e^{-2x}$ | A1 | **6** | For correct solution |
| | | $\boxed{6}$ | | |
| **4 (i)** | $(z=)\ 2, -2,\ 2i, -2i$ | M1 | | For at least 2 roots of the form $k\{1,\ i\}$ **AEF** |
| | | A1 | **2** | For correct values |

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| **(ii)** | $\dfrac{w}{1-w} = 2, -2,\ 2\mathrm{i}, -2\mathrm{i}$ | M1 | For $\dfrac{w}{1-w}$ = any one solution from **(i)** |
| | $w = \dfrac{z}{1+z}$ | M1 | For attempting to solve for $w$, using any solution or in general |
| | $w = \frac{2}{3}, 2$ | B1<br>A1 | For any one of the 4 solutions<br>For both real solutions |
| | $w = \frac{4}{5} \pm \frac{2}{5}\mathrm{i}$ | A1 **5** | For both complex solutions<br>**SR** Allow B1√ and one A1√ from $k \neq 2$ |
| | | $\boxed{7}$ | |
| **5 (i)** | $\mathbf{AB} = k\left[\frac{2}{3}\sqrt{3}, 0, -\frac{2}{3}\sqrt{6}\right]$,<br>$\mathbf{BC} = k\left[-\sqrt{3}, 1, 0\right]$, $\mathbf{CA} = k\left[\frac{1}{3}\sqrt{3}, -1, \frac{2}{3}\sqrt{6}\right]$ | B1<br>B1 | For any one edge vector of $\Delta ABC$<br>For any other edge vector of $\Delta ABC$ |
| | $\mathbf{n} = k_1\left[\frac{2}{3}\sqrt{6}, \frac{2}{3}\sqrt{18}, \frac{2}{3}\sqrt{3}\right] = k_2\left[1, \sqrt{3}, \frac{1}{2}\sqrt{2}\right]$ | M1<br>M1 | For attempting to find vector product of any two edges<br>For substituting $A$, $B$ or $C$ into $\mathbf{r.n}$ |
| | substitute $A$, $B$ or $C$ $\Rightarrow$ $x + \sqrt{3}y + \frac{1}{2}\sqrt{2}z = \frac{2}{3}\sqrt{3}$ | A1 **5** | For correct equation **AG**<br>**SR** For verification only allow M1, then A1 for 2 points and A1 for the third point |
| **(ii)** | Symmetry<br>in plane $OAB$ or $Oxz$ or $y = 0$ | B1*<br>B1<br>(*dep)**2** | For quoting symmetry or reflection<br>For correct plane<br>Allow "in $y$ coordinates" or "in $y$ axis"<br>**SR** For symmetry implied by reference to opposite signs in $y$ coordinates of $C$ and $D$, award B1 only |
| **(iii)** | $\cos\theta = \dfrac{\left\lvert\left[1, \sqrt{3}, \frac{1}{2}\sqrt{2}\right].\left[1, -\sqrt{3}, \frac{1}{2}\sqrt{2}\right]\right\rvert}{\sqrt{1+3+\frac{1}{2}}\sqrt{1+3+\frac{1}{2}}}$ | M1<br>A1 | For using scalar product of normal vectors<br>For correct scalar product |
| | $= \dfrac{\left\lvert 1-3+\frac{1}{2}\right\rvert}{\frac{9}{2}} = \dfrac{\frac{3}{2}}{\frac{9}{2}} = \dfrac{1}{3}$ | M1<br>A1 **4** | For product of both moduli in denominator<br>For correct answer. Allow $-\frac{1}{3}$ |
| | | $\boxed{11}$ | |
| **6 (i)** | $\left(m^2 + 16 = 0 \Rightarrow\right)\ m = \pm 4\mathrm{i}$ | M1 | For attempt to solve correct auxiliary equation (may be implied by correct CF) |
| | CF = $A\cos 4x + B\sin 4x$ | A1 **2** | For correct CF<br>(**AEtrig** but not $A\mathrm{e}^{4\mathrm{i}x} + B\mathrm{e}^{-4\mathrm{i}x}$ only) |
| **(ii)** | $\dfrac{\mathrm{d}y}{\mathrm{d}x} = p\sin 4x + 4px\cos 4x$ | M1<br>A1 | For differentiating PI twice, using product rule<br>For correct $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ |
| | $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 8p\cos 4x - 16px\sin 4x$ | A1√ | For unsimplified $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. f.t. from $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ |
| | $\Rightarrow 8p\cos 4x = 8\cos 4x$ | M1 | For substituting into DE |
| | $\Rightarrow p = 1$ | A1 | For correct $p$ |
| | $\Rightarrow (y =) A\cos 4x + B\sin 4x + x\sin 4x$ | B1√ **6** | For using GS = CF + PI, with 2 arbitrary constants in CF and none in PI |

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| **(iii)** | $(0, 2) \Rightarrow A = 2$ | B1√ | For correct $A$. f.t. from their GS |
| | $\frac{dy}{dx} = -4A\sin 4x + 4B\cos 4x + \sin 4x + 4x\cos 4x$ | M1 | For differentiating their GS |
| | $x = 0,\ \frac{dy}{dx} = 0 \Rightarrow B = 0$ | M1 | For substituting values for $x$ and $\frac{dy}{dx}$ to find $B$ |
| | $\Rightarrow y = 2\cos 4x + x\sin 4x$ | A1 **4** | For stating correct solution **CAO** including $\boxed{y=}$ |
| | | $\boxed{\mathbf{12}}$ | |
| **7 (i)** | $\cos 6\theta = 0 \Rightarrow 6\theta = k \times \frac{1}{2}\pi$ | M1 | For multiples of $\frac{1}{2}\pi$ seen or implied |
| | $\Rightarrow \theta = \frac{1}{12}\pi\{1, 3, 5, 7, 9, 11\}$ | A1 | A1 for any 3 correct |
| | | A1 **3** | A1 for the rest, and no extras in $0 < \theta < \pi$ |
| **(ii)** | METHOD 1 | | |
| | $\text{Re}(c + i\,s)^6 = \cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 - s^6$ | M1 | For expanding $(c + i\,s)^6$ at least 4 terms and 2 binomial coefficients needed |
| | | A1 | For 4 correct terms |
| | $\cos 6\theta = c^6 - 15c^4(1 - c^2) + 15c^2(1 - c^2)^2 - (1 - c^2)^3$ | M1 | For using $s^2 = 1 - c^2$ |
| | $\Rightarrow \cos 6\theta = 32c^6 - 48c^4 + 18c^2 - 1$ | A1 | For correct expression for $\cos 6\theta$ |
| | $\Rightarrow \cos 6\theta = \left(2c^2 - 1\right)\left(16c^4 - 16c^2 + 1\right)$ | A1 **5** | For correct result **AG** (may be written down from correct $\cos 6\theta$ ) |
| | METHOD 2 | | |
| | $\text{Re}(c + i\,s)^3 = \cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$ | M1 | For expanding $(c + i\,s)^3$ at least 2 terms and 1 binomial coefficient needed |
| | | A1 | For 2 correct terms |
| | $\Rightarrow \cos 6\theta = \cos 2\theta\left(\cos^2 2\theta - 3\sin^2 2\theta\right)$ | M1 | For replacing $\theta$ by $2\theta$ |
| | $\Rightarrow \cos 6\theta = \left(2\cos^2\theta - 1\right)\left(4\left(2\cos^2\theta - 1\right)^2 - 3\right)$ | A1 | For correct expression in $\cos\theta$ (unsimplified) |
| | $\Rightarrow \cos 6\theta = \left(2c^2 - 1\right)\left(16c^4 - 16c^2 + 1\right)$ | A1 | For correct result **AG** |
| **(iii)** | METHOD 1 | | |
| | $\cos 6\theta = 0$ | M1 | For putting $\cos 6\theta = 0$ |
| | $\Rightarrow$ 6 roots of $\cos 6\theta = 0$ satisfy $16c^4 - 16c^2 + 1 = 0$ and $2c^2 - 1 = 0$ | A1 | For association of roots with quartic and quadratic |
| | But $\theta = \frac{1}{4}\pi, \frac{3}{4}\pi$ satisfy $2c^2 - 1 = 0$ | B1 | For correct association of roots with quadratic |
| | *EITHER* Product of 4 roots *OR* $c = \pm\frac{1}{2}\sqrt{2 \pm \sqrt{3}}$ | M1 | For using product of 4 roots *OR* for solving quartic |
| | $\Rightarrow \cos\frac{1}{12}\pi\cos\frac{5}{12}\pi\cos\frac{7}{12}\pi\cos\frac{11}{12}\pi = \frac{1}{16}$ | A1 **5** | For correct value (may follow A0 and B0) |

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| | METHOD 2 | | |
| | $\cos 6\theta = 0$ | M1 | For putting $\cos 6\theta = 0$ |
| | $\Rightarrow$ 6 roots of $\cos 6\theta = 0$ satisfy $32c^6 - 48c^4 + 18c^2 - 1 = 0$ | A1 | For association of roots with sextic |
| | Product of 6 roots $\Rightarrow$ | M1 | For using product of 6 roots |
| | $\cos\frac{1}{12}\pi \cdot \frac{1}{\sqrt{2}} . \cos\frac{5}{12}\pi \cos\frac{7}{12}\pi . \frac{-1}{\sqrt{2}} . \cos\frac{11}{12}\pi = -\frac{1}{32}$ | B1 | For using $\cos\left\{\frac{3}{12}\pi, \frac{9}{12}\pi\right\} = \left\{\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right\}$ |
| | $\cos\frac{1}{12}\pi \cos\frac{5}{12}\pi \cos\frac{7}{12}\pi \cos\frac{11}{12}\pi = \frac{1}{16}$ | A1 | For correct value |
| | | **13** | |
| **8 (i)** | $g(x) = \dfrac{1}{2 - 2 . \dfrac{1}{2-2x}} = \dfrac{2-2x}{2-4x} = \dfrac{1-x}{1-2x}$ | M1<br>A1 | For use of $f\,f(x)$<br>For correct expression **AG** |
| | $gg(x) = \dfrac{1 - \dfrac{1-x}{1-2x}}{1 - 2 . \dfrac{1-x}{1-2x}} = \dfrac{-x}{-1} = x$ | M1<br>A1 **4** | For use of $gg(x)$<br>For correct expression **AG** |
| **(ii)** | Order of f = 4<br>order of g = 2 | B1<br>B1 **2** | For correct order<br>For correct order |
| **(iii)** | METHOD 1 | | |
| | $y = \dfrac{1}{2-2x} \Rightarrow x = \dfrac{2y-1}{2y}$ | M1 | For attempt to find inverse |
| | $\Rightarrow f^{-1}(x) = h(x) = \dfrac{2x-1}{2x}$ *OR* $1 - \dfrac{1}{2x}$ | A1 **2** | For correct expression |
| | METHOD 2 | | |
| | $f^{-1} = f^3 = f\,g$ or $g\,f$ | M1 | For use of $f\,g(x)$ or $g\,f(x)$ |
| | $f\,g(x) = h(x) = \dfrac{1}{2 - 2\left(\dfrac{1-x}{1-2x}\right)} = \dfrac{1-2x}{-2x}$ | A1 | For correct expression |
| **(iv)** | $\begin{array}{c\|cccc} & e & f & g & h \\ \hline e & e & f & g & h \\ f & f & g & h & e \\ g & g & h & e & f \\ h & h & e & f & g \end{array}$ | M1<br>A1<br>A1<br>A1 **4** | For correct row 1 and column 1<br>For e, f, g, h in a latin square<br>For correct diagonal e - g - e - g<br>For correct table |
| | | **12** | |