

4730 Mechanics 3

1	$0.4(3\cos 60^\circ - 4) = -I \cos \theta \quad (= -1)$ $0.4(3\sin 60^\circ) = I \sin \theta \quad (= 1.03920)$ $[\tan \theta = -1.5\sqrt{3}/(1.5 - 4);$ $I^2 = 0.4^2[(1.5 - 4)^2 + (1.5\sqrt{3})^2]$ $\theta = 46.1 \text{ or } I = 1.44$ $I = 1.44 \text{ or } \theta = 46.1$	M1 A1 A1 M1 A1 M1 A1ft [7]	For using $I = \Delta mv$ in one direction SR: Allow B1 (max 1/3) for $3\cos 60^\circ - 4 = -I \cos \theta$ and $3\sin 60^\circ = I \sin \theta$ For eliminating I or θ (allow following SR case) Allow for θ (only) following SR case. For substituting for θ or for I (allow following SR case) ft incorrect θ or I; allow for θ (only) following SR case.
	Alternatively $I^2 = 1.2^2 + 1.6^2 - 2 \times 1.2 \times 1.6 \cos 60^\circ \quad \text{or}$ $'V'^2 = 3^2 + 4^2 - 2 \times 3 \times 4 \cos 60^\circ$ $I = 1.44$ $\frac{\sin \theta}{3(\text{or } 1.2)} = \frac{\sin 60}{\sqrt{13}(\text{or } 2.08)} \quad \text{or}$ $\frac{\sin \alpha}{4(\text{or } 1.6)} = \frac{\sin 60}{\sqrt{13}(\text{or } 2.08)} \text{ and } \theta = 120 - \alpha$ $\theta = 46.1$	M1 A1 M1 A1 M1 A1ft A1 [7]	For use of cosine rule For correct use of factor 0.4 (= m) For use of sine rule α must be angle opposite 1.6; ($\alpha = 73.9$) ft value of I or 'V'
2	$2a + 3b = 2 \times 4$ $b - a = 0.6 \times 4$ $[2(b - 2.4) + 3b = 8]$ $b = 2.56$ $v = 2.56$	M1 A1 M1 A1 M1 A1 B1ft [7]	For using the principle of conservation of momentum For using NEL For eliminating a ft $v = b$
3(i)	$2W(a \cos 45^\circ) = T(2a)$ $W = \sqrt{2} T$	M1 A1 A1 [3]	For using 'mmt of $2W = \text{mmt of } T$ ' AG
(ii)	Components (H, V) of force on BC at B are $H = -T/\sqrt{2} \text{ and } V = T/\sqrt{2} - 2W$ $W(a \cos \alpha) + H(2a \sin \alpha) = V(2a \cos \alpha)$ $[W \cos \alpha - T \sqrt{2} \sin \alpha = T \sqrt{2} \cos \alpha - 4W \cos \alpha]$ $T \sqrt{2} \sin \alpha = (5W - T \sqrt{2}) \cos \alpha$ $\tan \alpha = 4$	B1 M1 A1 M1 A1ft A1 [6]	For taking moments about C for BC For substituting for H and V and reducing equation to the form $X \sin \alpha = Y \cos \alpha$

	<p>Alternatively for part (ii)</p> <p>anticlockwise mmt =</p> $W(a \cos \alpha) + 2W(2a \cos \alpha + a \cos 45^\circ)$ $= T[2a \cos(\alpha - 45^\circ) + 2a]$ $[5W \cos \alpha + \sqrt{2} W = T(\sqrt{2} \cos \alpha + \sqrt{2} \sin \alpha) + 2]$ $T \sqrt{2} \sin \alpha = (5W - T \sqrt{2}) \cos \alpha$ $\tan \alpha = 4$	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1ft</p> <p>A1</p> <p>[6]</p>	<p>For taking moments about C for the whole</p> <p>For reducing equation to the form</p> $X \sin \alpha = Y \cos \alpha$
4(i)	$[-0.2(v + v^2) = 0.2a]$ $[v \, dv/dx = -(v + v^2)]$ $[1/(1 + v)] \, dv/dx = -1$	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>	<p>For using Newton's second law</p> <p>For using $a = v \, dv/dx$</p> <p>AG</p>
(ii)	$\ln(1 + v) = -x (+ C)$ $\ln(1 + v) = -x + \ln 3$ $[(1 + dx/dt)/3 = e^{-x} \rightarrow dx/dt = 3e^{-x} - 1]$ $\rightarrow e^x \, dx/dt = 3 - e^x]$ $[-e^x/(3 - e^x)] \, dx/dt = -1$	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>For integrating</p> <p>For transposing for v and using $v = dx/dt$</p> <p>AG</p>
(iii)	$[\ln(3 - e^x) = -t + \ln 2]$ $\ln(3 - e^x) = -t + \ln 2$ <p>Value of t is 1.96 (or $\ln\{2 \div (3 - e)\}$)</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p>	<p>For integrating and using $x(0) = 0$</p>
5(i)	<p>Loss of EE = $120(0.5^2 - 0.3^2)/(2 \times 1.6)$</p> <p>and gain in PE = 1.5×4</p> <p>$v = 0$ at B and loss of EE = gain in PE (= 6)</p> <p>\rightarrow distance AB is 4m</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>For using $EE = \lambda x^2/2L$ and $PE = Wh$</p> <p>For comparing EE loss and PE gain</p> <p>AG</p>
(ii)	$[120e/1.6 = 1.5]$ <p>$e = 0.02$</p> <p>Loss of EE = $120(0.5^2 - 0.02^2)/(2 \times 1.6)$</p> <p>(or $120(0.3^2 - 0.02^2)/(2 \times 1.6)$)</p> <p>Gain in PE = $1.5(2.1 - 1.6 - 0.02)$</p> <p>(or $1.5(1.9 + 1.6 + 0.02)$ loss)</p> $[KE \text{ at max speed} = 9.36 - 0.72]$ <p>(or $3.36 + 5.28$)</p> $\frac{1}{2} (1.5/9.8)v^2 = 9.36 - 0.72$ <p>Maximum speed is 10.6 ms^{-1}</p>	<p>M1</p> <p>A1</p> <p>B1ft</p> <p>B1ft</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[7]</p>	<p>For using $T = mg$ and $T = \lambda x/L$</p> <p>ft incorrect e only</p> <p>ft incorrect e only</p> <p>For using KE at max speed</p> <p>= Loss of EE - Gain (or + loss) in PE</p>
	<p>First alternative for (ii)</p> <p>x is distance AP</p> $[\frac{1}{2} (1.5/9.8)v^2 + 1.5x + 120(0.5 - x)^2/3.2 = 120 \times 0.5^2/3.2]$ <p>KE and PE terms correct</p> <p>EE terms correct</p> $v^2 = 470.4x - 490x^2$ $[470.4 - 980x = 0]$ <p>$x = 0.48$</p> <p>Maximum speed is 10.6 ms^{-1}</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>For using energy at P = energy at A</p> <p>For attempting to solve $dv^2/dx = 0$</p>

	<p>Second alternative for (ii)</p> <p>[120e/1.6 = 1.5]</p> <p>e = 0.02</p> <p>[1.5 - 120(0.02 + x)/1.6 = 1.5 \ddot{x}/g]</p> <p>n = $\sqrt{490}$</p> <p>a = 0.48</p> <p>Maximum speed is 10.6 ms⁻¹</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p>	<p>For using T = mg and T = $\lambda x/L$</p> <p>For using Newton's second law</p> <p>For obtaining the equation in the form $\ddot{x} = -n^2x$, using (AB - L - e_{equil}) for amplitude and using v_{max} = na.</p>
6(i)	<p>PE gain by P = 0.4g × 0.8 sin θ</p> <p>PE loss by Q = 0.58g × 0.8 θ</p> <p>$\frac{1}{2}(0.4 + 0.58)v^2 = g \times 0.8(0.58\theta - 0.4\sin\theta)$</p> <p>$v^2 = 9.28\theta - 6.4\sin\theta$</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1ft</p> <p>A1</p> <p>[5]</p>	<p>For using KE gain = PE loss</p> <p>AEF</p>
(ii)	<p>0.4g sin θ - R = 0.4v²/0.8</p> <p>[0.4g sin θ - R = 4.64 θ - 3.2 sin θ]</p> <p>R = 7.12 sin θ - 4.64 θ</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>For applying Newton's second law to P and using a = v²/r</p> <p>For substituting for v²</p> <p>AG</p>
(iii)	<p>R(1.53) = 0.01(48...), R(1.54) = -0.02(9...) or simply R(1.53) > 0 and R(1.54) < 0</p> <p>R(1.53) × R(1.54) < 0 → 1.53 < α < 1.54</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>[4]</p>	<p>For substituting 1.53 and 1.54 into R(θ)</p> <p>For using the idea that if R(1.53) and R(1.54) are of opposite signs then R is zero (and thus P leaves the surface) for some value of θ between 1.53 and 1.54.</p> <p>AG</p>
7(i)	<p>T_{AP} = 19.6e/1.6 and T_{BP} = 19.6(1.6-e)/1.6</p> <p>0.5g sin30° + 12.25(1.6 - e) = 12.25e</p> <p>Distance AP is 2.5m</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1ft</p> <p>A1</p> <p>[5]</p>	<p>For using T = $\lambda e/L$</p> <p>For resolving forces parallel to the plane</p>
(ii)	<p>Extensions of AP and BP are 0.9 + x and 0.7 - x respectively</p> <p>0.5g sin 30° + 19.6(0.7 - x)/1.6 - 19.6(0.9 + x)/1.6 = 0.5 \ddot{x}</p> <p>$\ddot{x} = -49x$</p> <p>Period is 0.898 s</p>	<p>B1</p> <p>B1ft</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[5]</p>	<p>AG</p> <p>For stating k < 0 and using T = 2$\pi/\sqrt{-k}$</p>
(iii)	<p>2.8² = 49(0.5² - x²)</p> <p>x² = 0.09</p> <p>x = 0.3 and -0.3</p>	<p>M1</p> <p>A1ft</p> <p>A1</p> <p>A1ft</p> <p>[4]</p>	<p>For using v² = $\omega^2(A^2 - x^2)$ where $\omega^2 = -k$</p> <p>ft incorrect value of k</p> <p>May be implied by a value of x</p> <p>ft incorrect value of k or incorrect value of x² (stated)</p>