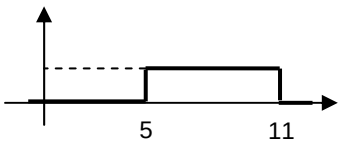
4733 Probability & Statistics 2

Penalise over-specified answers (> 6 SF) first time but only once per paper.
Use Ⓐ or Ⓒ to annotate "over-assertive" or "no context" respectively

1		$\hat{\mu} = \bar{x} = 15.16$ $\hat{\sigma}^2 = \frac{5}{4}s^2$ $= \mathbf{1.363}$	B1 M1 M1 A1	**4**	15.16 or 15.2 as answer only Use $\frac{\Sigma x^2}{5} - \bar{x}^2$ [=1.0904] Multiply by 5/4, or equiv for single formula Final answer 1.36 or 1.363 only, *not* isw
2	(i)	Not all equally likely – those in range 0 to 199 more likely to be chosen	M1 A1	**2**	Not all equally likely stated or implied Justified by reference to numbers, no spurious reasons
	(ii)	Ignore random numbers greater than 799, or 399	B1	**1**	Any valid resolution of this problem, no spurious reasons
3		B(60, 0.35) ≈ N(21, 13.65) $\Phi\left(\frac{18.5-21}{\sqrt{13.65}}\right) = \Phi(-0.6767)$ = 1 – 0.7507 $= \mathbf{0.2493}$	M1 M1 A1 M1 A1 A1	**6**	B(60, 0.35) stated or implied N(21, …) Variance or SD = 13.65 Standardise, their *np* and √*npq* or *npq*, wrong or no cc Both √*npq* and cc correct Answer, a.r.t. 0.249
4		$H_0 : \mu = 60$; $H_1 : \mu < 60$ (α) $z = \frac{58.9-60}{\sqrt{5^2/80}} = -1.967$ < -1.645	B2 M1 A1 B1		Both correct, B2 B1 for one error, but not *x*, *t*, $\bar{x}$ or $\bar{t}$ Standardise 58.9 & √80, allow – or √ errors *z*, art –1.97 or *p* in range [0.024, 0.025] Explicit comparison with –1.645 or 0.05, or +1.645 or 0.95 if 1.967 or 0.976 used
	or:	(β) $c = 60 - 1.645 \times \frac{5}{\sqrt{80}} = 59.08$ 58.9 < 59.08	M1 B1 A1✓		$60 - z \times 5/\sqrt{80}$, any $z = \Phi^{-1}$, allow √ errors or ±, not just +; *z* = 1.645 and compare 58.9 59.1 or better, ✓on wrong *z*
		Reject H_0 Significant evidence that people underestimate time	M1 A1✓	**7**	Correct first conclusion, needs essentially correct method including √80 or 80 Contextualised, uncertainty acknowledged SR: μ = 58.9: B0M1A0B1 max 2/7 SR: 2-tail: max 5/7
5	(i)	$H_0 : \lambda = 11.0$ $H_1 : \lambda > 11.0$ (α) P(≥ 19) = 1 – 0.9823 = 0.0177 < 0.05	B2 M1 A1 B1		Allow μ. Both correct, B2 One error: B1, but not *C*, *x* etc Find P(≥ 19) [or P(< 19) if later 0.95] art 0.0177 [0.9823, ditto] Compare 0.05 [0.95 if consistent], needs M1
		(β) CR ≥ 18, P(≥ 18) = 0.0322 19 > 18	M1 A1 B1		CR or CV 16/17/18/19 stated or clearly implied, but not < 18 and 0.0322 both seen, allow 0.9678 Explicit comparison with 19, needs M1
		Reject H_0 Significant evidence of an increase in number of customers	M1 A1✓	**7**	Needs essentially correct method & comparison Contextualised, uncertainty acknowledged SR: Normal, or P(= 19) or P(≤ 19) or P(> 19): First B2 only.
	(ii)	Can't deduce cause-and-effect, or there may be other factors	B1	**1**	Conclusion needed. No spurious reasons. If "DNR" in **(i)**, "couldn't deduce even if…"

6	(i)	(a) Probabilities don't total 1	B1	**1**	Equivalent statement
		(b) P(> 70) must be < P(> 50)	B1	**1**	Equivalent statement
		(c) P(> 50) = 0.3 $\Rightarrow \mu < 50$ P(< 70) = 0.3 $\Rightarrow \mu > 70$	B1	**1**	Any relevant valid statement, e.g. "P(< 50) = 0.7 but P(< 50) must be < P(< 70)"
	(ii)	$\mu = 60$ by symmetry	B1		$\mu = 60$ obtained at any point, allow from Φ
		$\frac{10}{\sigma} = \Phi^{-1}(0.7) = 0.524(4)$	M1		One standardisation, equate to Φ^{-1}, not 0.758
		$\sigma = 10/0.5243$	B1		$\Phi^{-1} \in [0.524, 0.5245]$ seen
		= **19.084**	A1	**4**	σ in range [19.07, 19.1], e.g. 19.073
7	(i)		M1 A1	**2**	Horizontal line Evidence of truncation *[no need for labels]*
	(ii)	$\mu = 8$	B1		8 only, cwd
		$\int_5^{11} \frac{1}{6}t^2 dt = \left[\frac{1}{18}t^3\right]_5^{11}$ [= 67]	M1		Attempt $\int kt^2$ dt, limits 5 and 11 seen
			B1		$k = 1/6$ stated or implied
		$- 8^2$	M1		Subtract their (non-zero) mean^2
		= **3**	A1	**5**	Answer 3 only, *not* from MF1
	(iii)	N(8, 3/48)	M1		Normal stated or implied
			A1		Mean 8
		$1 - \Phi\left(\frac{8.3-8}{\sqrt{3/48}}\right) = 1 - \Phi(1.2)$	A1		Variance their (non-zero) **(ii)**/48
		$= 1 - 0.8848$	M1		Standardise, $\sqrt{n}$, ignore sign or $\sqrt{}$ errors. cc: M0
		= **0.1151**	A1		Answer, art 0.115
		Normal distribution only approx.	B1	**6**	Any equivalent comment, e.g. CLT used
8	(i)	P(≤ 4) = 0.0473	M1		P(≤ r) from B(10, 0.7), r = 3/4/5, *not* N
		Therefore CR is ≤ 4	B1		"≤ 4" stated, not just "4", nothing else
		P(Type I error) = **4.73%**	A1	**3**	Answer, art 0.0473 or 4.73%, must be stated
	(ii)	B(10, 0.4) and find P(> 4)	M1		Must be this, *not* isw, ✓on **(i)**
		$1 - P(\leq 4)$	M1		Allow for 0.6177 or 0.1622
		= **0.3669**	A1	**3**	Answer, art 0.367
	(iii)	0.5×0.3669	M1		$0.5 \times$ **(ii)**
		= **0.18345**	A1✓	**2**	Ans correct to 3 SF, e.g. 0.184 from 0.367

9	(i)	$1 - P(\leq 7) = 1 - 0.9881$ $= \mathbf{0.0119}$	M1 A1	**2**	Allow for 0.0038 or 0.0335 Answer, a.r.t. 0.0119
	(ii)	Po(12) $P(\leq 14) - P(\leq 12)$ [0.7720 – 0.5760] $= \mathbf{0.196}$	M1 M1 A1	**3**	Po(12) stated or implied Formula, 2 consecutive correct terms, or tables, e.g. .0905 or .3104 or .1629 Answer, art 0.196
	(iii)	Po(60) ≈ N(60, 60) $\Phi\left(\frac{69.5-60}{\sqrt{60}}\right) = \Phi(1.226)$ $= \mathbf{0.8899}$	M1 A1 M1 A1 A1	**5**	N(60, …) Variance or SD 60 Standardise, λ & $\sqrt{\lambda}$, allow λ or wrong or no cc $\sqrt{\lambda}$ and cc both correct Answer 0.89 or a.r.t. 0.890
	(iv)	(a) $1 - e^{-3m}(1 + 3m)$	M1 A1	**2**	M1 for one error, e.g. no "1 –", or extra term, or 0^{th} term missing; answer, aesf
		(b) $m = 1.29$, $p = 0.89842$ $m = 1.3$, $p = 0.9008$ Straddles 0.9, therefore solution between 1.29 and 1.3	M1 A1 A1 A1	**4**	Substitute 1.29 or 1.3 into appropriate fn Comp | 0.9 | 0.1 | 0 1.29 | 0.898 | 0.10158 | –.00158 1.3 | 0.901 | 0.09918 | .0008146 Explicit comparison with relevant value, & conclusion, needs both *p*s correct
	or	Method for iteration; 1.296… 1.2965or better; conclusion stated	M1A1 A1A1		Can be implied by at least 1.296… Need at least 4 dp for M1A2