4734 Probability & Statistics 3

1(i)	$\int_{-a}^{0} \frac{2}{5}\,dx + \int_{0}^{\infty} \frac{2}{5}e^{-2x}\,dx = 1$ $2a/5 + 1/5 = 1$ $a = 2$	M1 A1 A1 **3**	Sum of probabilities =1
(ii)	$\int_{-2}^{0} \frac{2}{5}x\,dx + \int_{0}^{\infty} \frac{2}{5}xe^{-2x}\,dx$ $\int_{-a}^{0} \frac{2}{5}x\,dx = -\frac{a^2}{5}$ $\int_{0}^{\infty} \frac{2}{5}xe^{-2x}\,dx = \left[-\frac{1}{5}xe^{-2x}\right] + \left[-\frac{1}{10}e^{-2x}\right]$ = - 0.7	M1 A1 √ M1 A1 A1 **5** **[8]**	Σ ∫ x f(x) dx √ a By parts with 1 part correct Both parts correct CAO
2(i)	4 cartons: Total, $Y \sim N(2016, 36)$ $P(Y \le 2000) = \Phi(-16/\sqrt{36})$ = 0.00383	B1B1 M1 A1 **4**	Mean and variance
(ii)	E(V) = 0 Var(V) = 36 + 16×9 = 180	B1 M1 A1 **3**	CWO
(iii)	0.5	B1 **1** **[8]**	
3(i)	Normal distribution Mean $\mu_1 - \mu_2$; variance $2.47/n_1 + 4.23/n_2$	B1 B1B1 **3**	
(ii)	H_0: $\mu_1 = \mu_2$, H_1:$\mu_1 \ne \mu_2$ $(9.65 - 7.23)/\sqrt{(2.47/5+4.23/10)}$ =2.527 > 2.326 Reject H_0 There is sufficient evidence at the 2% significance level that the means differ	B1 M1 B1 A1 M1 A1 **6**	Or find critical region Numerator Compare with critical value **SR1:If no specific comparison but CV and conclusion correct B1. Same in Q5,6,7** **SR2: From CI: 2.42±$z\sigma$ M1, σ correct** **z = 2.326 B1, (0.193, 4.647) A1** **0 in not in CI ; reject H_0 etc M1A1 Total 6** **Conclusions not over-assertive in Q3, 5, 6**
(iii)	Any relevant comment.	B1 **1** **[10]**	e.g sample sizes too small for CLT to apply

4(i)	$G(y)=P(Y \le y)=P(1/(1+V) \le y)$ $=P(V \ge 1/y - 1)$ $= 1 - F(1/y - 1)$ $= \begin{cases} 0 & y \le 0, \\ 8y^3 & 0 < y \le 1/2, \\ 1 & y > 1/2. \end{cases}$ $g(y) = \begin{cases} 24y^2 & 0 < y \le 1/2, \\ 0 & \text{otherwise.} \end{cases}$	M1 A1 A1 A1 B1 M1 A1 **7**	 Use of F $8y^3$ obtained correctly Correct range. Condone omission of $y \le 0$ For $G'(y)$ Correct answer with range √
	$\int 24y^2/y^2\,dy$ with limits =12	M1 A1 **2** **[9]**	 With attempt at integration
5(i)	Use $p_s \pm z\sqrt{(p_s q_s/200)}$ $z=1.645$ $s = \sqrt{(0.135 \times 0.865/200)}$ (0.0952, 0.1747)	M1 B1 A1 A1 **4**	 Or /199 (0.095, 0.175) to 3DP
(ii)	H_0: $p_1 - p_2 = 0$, H_1: $p_1 - p_2 > 0$ $\dfrac{27/200 - 8/100}{\sqrt{35/300 \times 265/300 \times (200^{-1} + 100^{-1})}}$ = 1.399 > 1.282 Reject H_0. There is sufficient evidence at the 10% significance level that the proportion of faulty bars has reduced	B1 M1 B1 A1 A1 M1 A1 **7** **[11]**	Or equivalent Correct form. Pooled estimate of $p = 35/300$ Correct form of sd OR: $P(z \ge 1.399) = 0.0809 < 0.10$ **SR: No pooled estimate: B1M1B0B0 A1 for 1.514, M1A1 Max 5/7**
6(i)	Assumes that decreases have a normal distn H_0:$\mu_{O-F} = 0.2$ (or ≥), H_1:$\mu_{O-F} > 0.2$ O-F: 0.6 0.4 0.2 0.1 0.3 0.2 0.4 0.3 $\bar{D} = 0.3125$ $s^2 = 0.024107$ $(0.3125-0.2)/\sqrt{(0.024107/8)}$ =2.049 > 1.895 Reject H_o – there is sufficient evidence at the 5% significance level that the reduction is more than 0.2	B1 B1 M1 B1 A1 M1 A1 M1 A1 **9**	 B1 Use paired differences t-test Must have /8 OR: $P(t \ge 2.049)= 0.0398 < 0.05$ Allow M1 from $t_{14} = 1.761$ **SR: 2-sample test:B1B1M0B1A0 M1 using 1.761 A0 Max 4/9**
(ii)	$0.3125 \pm t\sqrt{(0.024107/8)}$ $t = 2.365$ (0.1827, 0.4423)	M1 B1 A1 **3** **[12]**	Allow with z but with /8 Rounding to (0.283, 0.442)

7(i)	H_0:Vegetable preference is independent of gender H_1: All alternatives E-Values 26 16.25 22.75 22 13.75 19.25 $\chi^2 = 5^2(26^{-1} + 22^{-1}) + 7.25^2(16.25^{-1} + 13.75^{-1})$ $+ 2.25^2(22.75^{-1} + 19.25^{-1})$ $=9.641$ $9.64 > 5.991$ Reject H_0, (there is sufficient evidence at the 5% that) vegetable preference and gender are not independent	 B1 M1 A1 M1 A1 A1 M1 A1 **8**	 For both hypotheses At least one correct All correct Correct form of any one All correct ART 9.64 OR: $P(\geq 9.641)=0.00806 <0.05$
(ii)	(H_0: Vegetables have equal preference H_1: All alternatives) Combining rows: 48 30 42 E-Values: 40 40 40 $\chi^2 = (8^2+10^2 + 2^2)/40$ $= 4.2$ $4.2 < 4.605$ Do not reject H_0, there is insufficient evidence at the 10% significance level of a difference in the proportion of preferred vegetables	 M1 A1 M1 A1 M1 A1 **6** **[14]**	 OR:$P(\geq 4.2) = 0.122 > 0.10$ AEF in context